Experimental Observation of Beliaev Coupling in a Bose-Einstein Condensate

E. Hodby, O. M. Maragò, G. Hechenblaikner, and C. J. Foot
Clarendon Laboratory, Department of Physics, University of Oxford, Parks Road, Oxford OX1 3PU, United Kingdom
(Received 11 October 2000)

We report the first experimental observation of Beliaev coupling between collective excitations of a Bose-Einstein condensed gas. Beliaev coupling is not predicted by the Gross-Pitaevskii equation and so this experiment tests condensate theory beyond the mean field approximation. Measurements of the amplitude of a high frequency scissors mode show that the Beliaev process transfers energy to a lower-lying mode and then back and forth between these modes, unlike Landau processes which lead to a monotonic decrease in amplitude. To enhance the Beliaev process we adjusted the geometry of the magnetic trapping potential to give a frequency ratio of 2 to 1 between the two scissors modes.

DOI: 10.1103/PhysRevLett.86.2196 PACS numbers: 03.75.Fi, 05.30.Jp, 05.45.–a, 32.80.Pj

In the Beliaev coupling process [1], one quantum of excitation converts into two quanta of a lower frequency mode, in a manner analogous to parametric down-conversion of light in nonlinear media. This process occurs readily in a homogeneous system such as superfluid helium where there are many very closely spaced low-lying energy levels and conservation of energy is easily satisfied. Since the energy is coupled to many modes of the homogeneous system, the process is irreversible and more accurately called Beliaev damping. However, a trapped Bose-Einstein condensate (BEC) [2] has discrete modes with well-resolved excitation energies. Beliaev processes are inhibited and more difficult to detect. We have observed the Beliaev process for a scissors mode [3] of a BEC of rubidium atoms, when the mode resonantly couples to another mode at half its frequency. Beliaev coupling can be regarded as a mixing process, in which Bogoliubov quasiparticles interact with the ground state component of the condensate, to produce new quasiparticles at a lower energy.

At finite temperature, Landau damping is the primary mechanism which dissipates the energy of collective excitation into the thermal cloud [4], while Beliaev processes dominate only under resonant conditions or at zero temperature. Landau damping corresponds to a scattering process between two quasiparticles in the initial mode, which results in one particle gaining all of the energy and the other going into the condensate ground state. The Landau damping rate and its strong dependence on temperature has been measured for two of the quadrupole excitations of the condensate [5]. We observe a comparable Landau damping rate for the scissors mode in most trap geometries, except those where the frequencies of the two scissors modes were matched and the Beliaev process was dominant.

An atom in a harmonic trap has three dipole modes of oscillation along each of the principle axes, corresponding to the three trap frequencies ω_x, ω_y, and ω_z. The center of mass motion of an interacting cloud of atoms, such as the Bose condensate and any surrounding thermal cloud, has the same eigenfrequencies by Kohn's theorem [6]. However, the frequencies of other collective excitations of the condensate, including the quadrupole modes, do not correspond to those of a harmonic oscillator because of the strong interatomic interactions. Six of the low-lying collective excitations are described by the linearized set of equations:

$$\ddot{\boldsymbol{q}} = \boldsymbol{S}\boldsymbol{q}\,, \tag{1}$$

where the six components of the vector $\boldsymbol{q}$,

$$\boldsymbol{q} = (Q_{ii}, Q_{jj}, Q_{kk}, Q_{ij}, Q_{jk}, Q_{ik})\,, \tag{2}$$

are elements of the quadrupole tensor, $\boldsymbol{Q}$.

$$Q_{ij} = \langle x_i x_j \rangle\,. \tag{3}$$

The six eigenvalues of $\boldsymbol{S}$ give the frequencies of the six normal modes. Linear combinations of the diagonal elements of $\boldsymbol{Q}$ (e.g., $\langle x^2 \rangle$) describe the three normal modes with principal axes of fixed orientation. We refer to these modes as M_h, M_l, M_2. In the case of the axially symmetric trap, where the angular momentum about the axis (m) is a good quantum number, these become the high- and low-lying $m = 0$ modes and the $m = 2$ mode, respectively [7,8]. The three independent off-diagonal elements of $\boldsymbol{Q}$ (e.g., $\langle xy \rangle$) relate to the three scissors modes, which we label M_{xy}, M_{yz}, M_{xz}, with frequencies ω_{xy}, ω_{yz}, ω_{xz}. The frequency spectrum has been calculated in [9,10] and is shown, in the hydrodynamic limit, in Fig. 1 as a function of trap geometry.

In general, the first and second harmonic oscillations involved in the down-conversion process are the scissors modes in the xy and xz planes, respectively (M_{xy} and M_{xz}). However, for the special case of an axially symmetric trap, the scissors mode, M_{xy}, merges with the M_2 mode to give a doubly degenerate excitation. The frequencies of these two scissors modes in the hydrodynamic limit are given by $\omega_{xy} = \sqrt{\omega_x^2 + \omega_y^2}$ and $\omega_{xz} = \sqrt{\omega_x^2 + \omega_z^2}$. Thus we observe a resonant coupling when the following condition

 0031-9007/01/86(11)/2196(4)$15.00

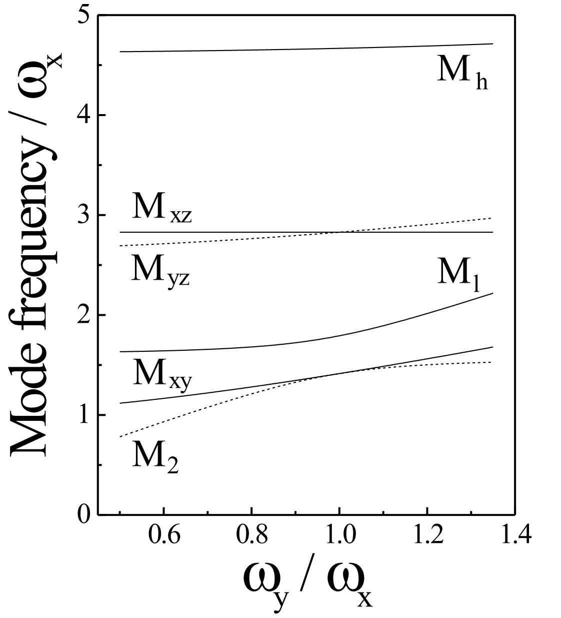

FIG. 1. The frequency spectrum of the six quadrupole modes of the condensate as a function of the ellipticity of the trapping potential in the xy plane. ω_z/ω_x is chosen to be $\sqrt{7}$, at which value M_{xz} has exactly twice the frequency of M_{xy} and M_2, in an axially symmetric trap.

is satisfied:

$$2 \times \sqrt{1 + \frac{\omega_y^2}{\omega_x^2}} = \sqrt{1 + \frac{\omega_z^2}{\omega_x^2}} . \tag{4}$$

The scissors mode is a small angle, irrotational oscillation of the condensate, about a given axis, that occurs without a change of the cloud shape [3,11]. M_2 is an out of phase oscillation in the x and y directions, with a very small motion in the z direction. This axial motion falls to zero amplitude in the cylindrically symmetric trap. References [7,11] show the geometries of the two oscillations [12].

We initially create Bose-Einstein condensates in an axially symmetric time-orbiting potential (TOP) trap ($\omega_x/2\pi = \omega_y/2\pi = 127.6 \pm 1.0$ Hz). The trap frequency ratio, ω_z/ω_x, is 2.83 and hence the condensate has an oblate shape. After rf evaporative cooling, the condensate temperature is significantly below $0.5T_c$ and the thermal cloud is no longer visible. A detailed description of our method for selectively exciting the xz scissors mode is given in [11]. In summary, we apply an additional bias field of amplitude B_z in the axial direction, oscillating in phase with the x component of the TOP bias field, B_x. This enables the symmetry axis of the trapping potential to be tilted by an angle that depends on B_z/B_x. We first form the condensate in a standard TOP trap, then adiabatically tilt the trap to an angle ϕ, and then suddenly flip it to $-\phi$. This excites a scissors mode oscillation of amplitude $\theta = 2\phi$ in the xz plane, about the new equilibrium position.

However, the application of the z bias field has a second, more subtle effect [13]. The axial trapping frequency, ω_z, and hence the xz scissors mode frequency is reduced, as B_z is increased. The radial frequencies ω_x and ω_y are also reduced, but by a negligible amount. By changing B_z/B_x we are able to tune the xz scissors frequency into resonance with twice the xy scissors frequency. Since the excitation angle is coupled to B_z/B_x, this is also changed but always remains well within the small angle scissors mode limit defined in [3]. Figure 2 shows theoretically and experimentally how the excitation amplitude, θ, and the frequency of the xz scissors mode, ω_{xz}, change as a function of B_z/B_x and hence ω_z/ω_x.

Our first observation of Beliaev coupling occurred in an axially symmetric trap, $\omega_x = \omega_y$. In this case the xy scissors mode becomes degenerate with, and indistinguishable from, the $|m = 2|$ quadrupole mode, with frequency $\sqrt{2}\,\omega_x$ (Fig. 1). The resonance condition of Eq. (4) is then fulfilled when $\omega_z/\omega_x = \sqrt{7}$.

We recorded the xz scissors mode oscillation using destructive absorption imaging. The angle of the cloud as a function of oscillation evolution time was extracted from 2D Gaussian fits to absorption images of the expanded condensate. We fit an exponentially decaying sine wave to the angle versus time data and use the damping rate as a measure of the rate at which energy is flowing out of the mode. For values of ω_z/ω_x far from the resonance condition, we observe only a slow Landau damping rate of ~18 Hz (Fig. 3a). This background damping rate increases with the mode frequency, ω_{xz}, as predicted in [15].

Close to resonance we observe two new features in the xz scissors mode. First, there is a sharp increase in the

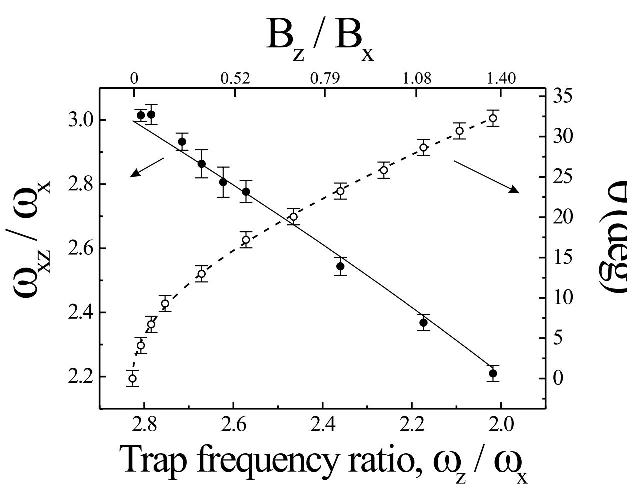

FIG. 2. The various effects of increasing the z bias field, B_z. B_z/B_x is shown along the top axis and the trap frequency ratio, ω_z/ω_x, that results is shown along the lower x axis. The solid line is the theoretical value of the xz scissors frequency, ω_{xz}, scaled against ω_x. The solid circles show experimental values obtained by fitting sine waves to data such as in Figs. 3a and 3b (left axis). The dotted line shows the theoretical excitation angle, θ, while the open circles show the values obtained from measuring the tilt angle of the static trap (right axis).

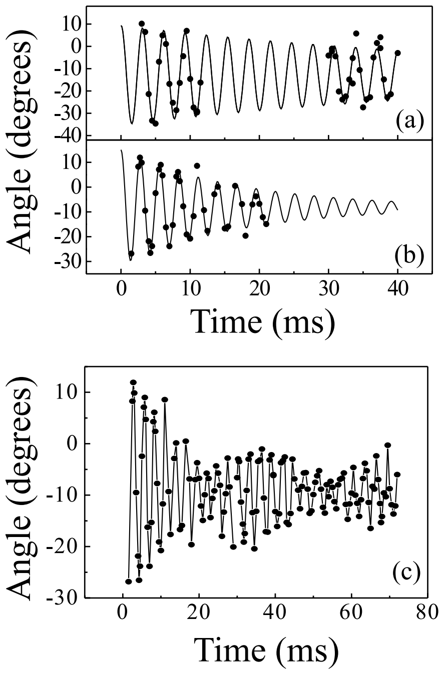

FIG. 3. Experimental data showing the angle of a condensate in the xz scissors mode as a function of time. (a) and (b) are typical plots from cylindrically symmetric traps with different aspect ratios, fitted with exponentially decaying sine waves, from which the coupling rates, Γ, were extracted. (a) $\omega_z/\omega_x = 2.36$; $\Gamma = 17 \pm 5$ Hz. (b) $\omega_z/\omega_x = 2.62$; $\Gamma = 57 \pm 8$ Hz. In (b) the trap geometry is very close to the resonance condition of Eq. (4) and the fast decay is primarily due to Beliaev damping. The conditions of (a) are far from resonance and so only the slow underlying Landau damping rate is observed. (c) has the same initial data as (b) but continues to track the oscillation over a longer period. The transfer of energy back and forth between the two modes is clearly observed, as the amplitude of M_{xz} falls and rises [14].

initial damping rate of the oscillation, as the Beliaev process becomes significant (Fig. 3b). Second, over longer observation periods ($t > 30$ ms) the amplitude envelope grows and then falls again, as energy continues to be transferred back and forward between the two modes, in a manner characteristic of nonlinear coupling (Fig. 3c) [16]. To obtain a measure of the initial damping rate, we fit the decaying sine wave up to, but not beyond, the first amplitude minimum [14]. The maximum coupling rate occurs when $\omega_z/\omega_x = 2.65 \pm 0.04$, in very good agreement with the predicted value of $\sqrt{7}$ (Fig. 4).

At first glance, the data in Fig. 3c could be interpreted as a beating of the xz scissors mode against a mode of very similar frequency. However, beating is characterized by a coupling rate Γ that *increases* on either side of resonance, as the frequency difference between the two modes increases. This is not consistent with the data of Fig. 4, which show a coupling rate that *decreases* away from resonance. The investigation of the coupling rate in different trap geometries described below (Fig. 5) gives further evidence that we are not observing a beating process.

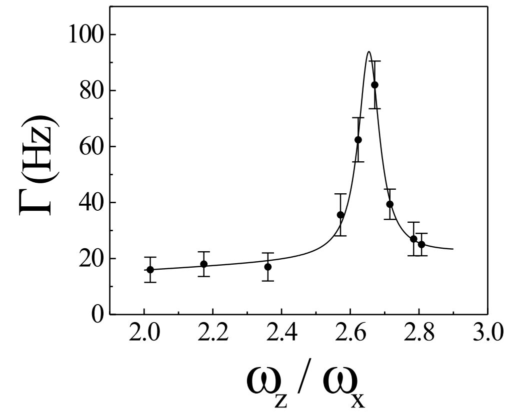

FIG. 4. The initial damping rate of the xz scissors mode as a function of the trap frequency ratio ω_z/ω_x, in an axially symmetric trap. The data are fitted with a Lorentzian peak plus a linear function. The Lorentzian gives the trap geometry for peak Beliaev coupling. Far from resonance, the Landau damping rate is observed. The increase of the Landau damping rate with mode frequency is modeled by the linear part of the fit.

In the axially symmetric trap described so far, M_2 and M_{xy} are degenerate, but in a totally anisotropic trapping potential, $\omega_x \neq \omega_y$, this degeneracy is broken (Fig. 1). To investigate coupling to the pure xy scissors mode, we repeated the experiment, looking for resonant behavior in a range of triaxial traps. The trap geometry was chosen to be close to the resonance condition between M_{xy} and M_{xz} given in Eq. (4).

The condensate was formed in a symmetric TOP trap as before. However, while it was slowly tilted in the xz plane, it was also adiabatically deformed by ramping the amplitude of the y component of the TOP bias field. This made the trapping potential elliptical in the xy plane, with ω_y/ω_x between 0.6 and 1.05 [17]. For each value of ω_y/ω_x, we determined the value of ω_z/ω_x for peak coupling. Figure 5 shows these data, with the resonant coupling condition superimposed. The agreement between

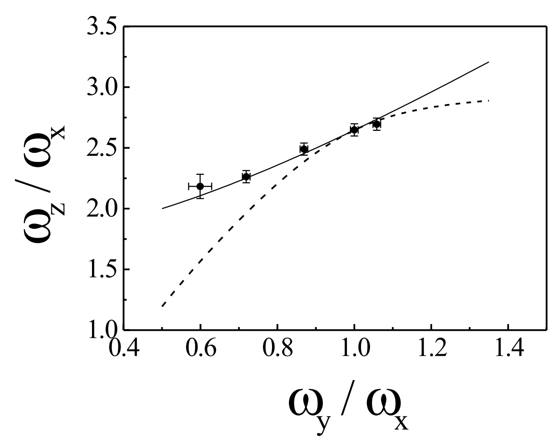

FIG. 5. Trap geometries for which M_{xz} is resonant with twice the frequency of M_{xy} (solid line) and M_2 (dashed line). The data points show the value of ω_z/ω_x at which the peak coupling rate was observed, for a given ω_y/ω_x.

experiment and theory over a range of trap geometries confirms that the correct coupling process has been identified.

These observations do not rule out the possibility of down-conversion into the M_2 mode in an anisotropic potential, since our traps were specifically chosen for M_{xz} to be resonant with M_{xy} and not M_2. In fact, for traps with a very small anisotropy in the xy plane, so that *both* modes are close to resonance with the xz scissors mode, we notice a broadening of the peak in the initial damping rate. This is expected when a second down-conversion process, into the M_2 mode, is resonant in a slightly different trap geometry.

In nonlinear optics, it is well known that down-conversion produces output beams of squeezed light and analogous effects with matter waves may be explored using BEC. In previous work we observed up-conversion between the two $m = 0$ modes of a BEC in a cylindrically symmetric potential [18] and interesting comparisons may be made between the two processes. While the up-conversion process showed strong dispersion of the driving oscillation close to resonance, this was not observed in the measured values of ω_{xz} in Fig. 2. Second, the return of energy to the original mode, shown strikingly in Fig. 3c, was not observed for the up-conversion process. Finally, while spontaneous up-conversion is predicted by the Gross-Pitaevskii equation (GPE), the reverse process of spontaneous down-conversion may not be described within this framework. This may be explained by the following simple argument [16]. The GPE contains a nonlinear term proportional to $|\Psi|^2$, so excitation of a pure mode with time dependence $e^{i\omega t}$ leads to terms of angular frequency 2ω, but not to terms in $\omega/2$ and hence no down-conversion. However, if there is initially some amplitude at frequency $\omega/2$, then nonlinear mixing with the oscillation at frequency ω leads to a transfer of energy between the modes at ω and $\omega/2$.

In conclusion, we have observed Beliaev coupling of the xz scissors mode. Measurements over a range of trap geometries confirm that this corresponds to resonant down-conversion into the xy scissors mode. In future experiments, we intend to investigate the role of Bose stimulation in the Beliaev coupling process by increasing the population of the initial state. This can be achieved by increasing the temperature or by using different initial conditions.

We thank all the members of the Oxford theoretical BEC group, in particular, K. Burnett, M. Rusch, and S. Morgan for their help and advice. This work was supported by the EPSRC and the TMR program (No. ERB FMRX-CT96-0002). O. M. M. acknowledges the support of a Marie Curie Fellowship, TMR program (No. ERB FMBI-CT98-3077).

[1] S. T. Beliaev, Sov. Phys. JETP **34**, 323 (1958).
[2] M. H. Anderson *et al.,* Science **269**, 198 (1995); K. B. Davis *et al.,* Phys. Rev. Lett. **75**, 3969 (1995).
[3] D. Guéry-Odelin and S. Stringari, Phys. Rev. Lett. **83**, 4452 (1999).
[4] S. A. Morgan, J. Phys. B **33**, 3847 (2000).
[5] D. S. Jin, M. R. Matthews, J. R. Ensher, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **78**, 764 (1997); D. M. Stamper-Kurn, H-J. Miesner, S. Inouye, M. R. Andrews, and W. Ketterle, Phys. Rev. Lett. **81**, 500 (1998).
[6] W. Kohn, Phys. Rev. **123**, 1242 (1961).
[7] D. S. Jin, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **77**, 420 (1996).
[8] M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. **77**, 988 (1996).
[9] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999).
[10] M. O. da C. Pires and E. J. V. de Passos, J. Phys. B **33**, 3929 (2000).
[11] O. M. Maragò, S. A. Hopkins, J. Arlt, E. Hodby, G. Hechenblaikner, and C. J. Foot, Phys. Rev. Lett. **84**, 2056 (2000).
[12] Note that in an axially symmetric trap, down-conversion occurs into an equal superposition of the $m = +2$ and $m = -2$ modes, ensuring that the net angular momentum about the z axis is zero at all times. For a triaxial trap, angular momentum is not conserved and need not be considered in the coupling process.
[13] E. Hodby, G. Hechenblaikner, O. M. Maragò, J. Arlt, S. Hopkins, and C. J. Foot, J. Phys. B **33**, 4087 (2000).
[14] Beliaev coupling is a stimulated process and the Bose enhancement depends on the initial population of the final state. Detailed modeling of this process would be required to find a function to fit all of the data in Fig. 3c, rather than just the initial part.
[15] L. P. Pitaevskii and S. Stringari, Phys. Lett. A **235**, 398 (1997); P. O. Fedichev, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **80**, 2269 (1998).
[16] S. A. Morgan, S. Choi, K. Burnett, and M. Edwards, Phys. Rev. A **57**, 3818 (1998).
[17] J. R. Ensher, Ph.D. thesis, University of Colorado, 1998; J. Arlt, O. Maragò, E. Hodby, S. A. Hopkins, G. Hechenblaikner, S. Webster, and C. J. Foot, J. Phys. B **32**, 5861 (1999).
[18] G. Hechenblaikner, O. M. Maragò, E. Hodby, J. Arlt, S. Hopkins, and C. J. Foot, Phys. Rev. Lett. **85**, 692 (2000).