$B \to \rho\pi$ Decays, Resonant and Nonresonant Contributions

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We point out that a new contribution to B decays to three pions is relevant in explaining recent data from the CLEO and BABAR Collaborations, in particular, the results on quasi-two-body decays via a ρ meson. We also discuss the relevance of these contributions to the measurement of CP violations.

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Several exclusive charmless hadronic B decays are known with good accuracy, and more will be measured in the near future, due to the large amount of data coming from e^+e^- machines such as the CLEO experiment [1] at the Cornell Electron Storage Ring, the BaBar experiment at SLAC [2], and the BELLE experiment at KEK [3], or hadron machines such as the Large Hadron Collider at CERN, with its program for B physics. In the following we will consider quasi-two-body B decays to three pions. Our motivation is twofold.

(1) $\bar{B}^0 \to \rho^\mp\pi^\pm$ and $B^- \to \rho^0\pi^-$ were recently measured by the CLEO and BABAR Collaborations. The ratios of these branchings are consistent among the two experiments and different from the theoretical expectations.

(2) Quasi-two-body B decays to three π can be used for the determination of the angles of the unitarity triangle of CP violations [4].

Concerning the first point, the CLEO Collaboration finds [5] $\mathcal{B}(B^- \to \rho^0\pi^-) = (10.4^{+3.3}_{-3.4} \pm 2.1) \times 10^{-6}$ and $\mathcal{B}(\bar{B}^0 \to \rho^\mp\pi^\pm) = (27.6^{+8.4}_{-7.4} \pm 4.2) \times 10^{-6}$; combining these numbers we find a ratio

$$R = \frac{\mathcal{B}(\bar{B}^0 \to \rho^\mp\pi^\pm)}{\mathcal{B}(B^- \to \rho^0\pi^-)} = 2.65 \pm 1.8\,, \qquad (1)$$

where the error in the ratio includes all errors from the branchings; the error may be smaller than what is indicated if part of the systematics simplifies in the ratio. The BABAR Collaboration [6] gives the preliminary results $\mathcal{B}(B^- \to \rho^0\pi^-) = (24 \pm 8 \pm 3) \times 10^{-6}$ and $\mathcal{B}(\bar{B}^0 \to \rho^\mp\pi^\pm) = (49 \pm 13^{+6}_{-5}) \times 10^{-6}$; with these numbers we find a ratio $R = 2.0 \pm 1.3$.

As discussed in [7], this ratio is rather small with respect to theoretical expectations; as a matter of fact, when computed in simple approximation schemes, as factorization with no penguins, one gets, from the Bauer-Stech-Wirbel model [8], $R \simeq 6$. A calculation including penguins in the factorization approximation was performed in [9] and gives $R \simeq 5.5$. Calculations beyond factorization were performed in [10] with similar results [11] (see Table I).

Previous papers [9,11–13] investigated the role of B^* and higher resonances in these decays, while here we will investigate a completely different mechanism that enhances selectively some of the $B \to 3\pi$ decays, namely, the possibility that a broad light scalar resonance is present in the 3-body Dalitz plot.

The light σ resonance has accumulated considerable interest after it was reintroduced as a very broad resonance into the 1996 edition of the Reviews of Particle Physics [14]. The E791 Collaboration has an evidence of a very broad scalar resonance [15] having mass $m_\sigma = 478 \pm 24$ MeV and width $\Gamma_\sigma = 324 \pm 41$ MeV taking part into the decay process $D^+ \to 3\pi$ via the resonant channel $D^+ \to \sigma\pi^+ \to 3\pi$. The broad σ was controversial due to the extreme difficulty in disentangling it from available data on $\pi\pi \to \pi\pi$. In the following we will use the experimental numbers for the mass and width of the σ given by the E791 Collaboration. In the process $D^+ \to 3\pi$ the σ is a good description of three π events not due to quasi-two-body decays of a narrow resonance as it accounts for 46% of the three π's branching [16]. We will model $B^+ \to 3\pi$ in a similar way and see that indeed such a contribution is relevant and larger than or comparable to the nonresonant long-distance contribution of the B^*. These two contributions, after the appropriate experimental cuts, constitute an irreducible background to the processes $B \to \rho\pi$, therefore adding events to some of them and modifying the ratio of branching ratios. This also has consequences on methods based on $B \to \rho\pi$ for the determination of the CP violating angles. It should be mentioned that the latest edition of the Reviews of Particle Physics [14] gives a broad range for the σ mass and width. Their precise value is not crucial to our analysis as long as the sigma is within the strip of the Dalitz plot where the ρ mass contribution (within a few hundred MeV due to the experimental cuts) is found.

The amplitudes we are interested in are those corresponding to the diagrams in Figs. 1 and 2. We compute

TABLE I. Estimates for the ratio R beyond the factorization approximation (the so-called charming penguins) using different sets of input data: QCD sum rules (QCDSR), lattice-QCD, and quark models (QM).

QCDSR	$R = 6.3$
Lattice	$R = 5.5$
QM	$R = 6.4$

them at the tree level using the following effective Hamiltonian [8]:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{ud} \{ a_1 (\bar{u} b)_{V-A} (\bar{d} u)_{V-A} + a_2 (\bar{d} b)_{V-A} (\bar{u} u)_{V-A} \}, \tag{2}$$

giving, in the first case,

$$\langle \sigma \pi^- | H_{\text{eff}} | B^- \rangle = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{ud} a_1 F_0^{(B\sigma)} m_\pi^2 (m_B^2 - m_\sigma^2) \frac{f_\pi g_{\sigma \pi^+ \pi^-}}{\sqrt{2}} \left(\frac{1}{u - m_\sigma^2 + i \Gamma_\sigma(u) m_\sigma} + \frac{1}{t - m_\sigma^2 + i \Gamma_\sigma(t) m_\sigma} \right), \tag{3}$$

where the calculation of the form factor $F_0^{(B\sigma)}$ proceeds as in [17], $g_{\sigma\pi^+\pi^-} = 2.52$ GeV comes from the experimental value of Γ_σ, and the comoving width $\Gamma_\sigma(x)$ is

$$\Gamma_\sigma(x) = \Gamma_\sigma \frac{m_\sigma}{\sqrt{x}} \frac{\sqrt{x/4 - m_\pi^2}}{\sqrt{m_\sigma^2/4 - m_\pi^2}}, \tag{4}$$

where x is $u = (p - p_1)^2$ or $t = (p - p_2)^2$ (as in the crossed channel, see Fig. 1, in which we have two identical π^- particles in the final state). The comoving width can be obtained comparing the usual formula for the fixed width (see, for example, [14]) with a similar formula where the σ mass is replaced by the square root of the relevant Mandelstam variable (as the σ is an intermediate state in the decay process). The coefficient a_1 is given by $a_1 = C_1 + \frac{1}{3} C_2$ where the Wilson coefficients C_1 and C_2, fitted for B decays, are $C_1(m_b) = 1.105$ and $C_2(m_b) = -0.228$.

On the other hand, the diagram in Fig. 2 is controlled by the a_2 coefficient appearing in (2) and the result is

$$\langle \sigma \pi^0 | H_{\text{eff}} | \bar{B}^0 \rangle = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{ud} a_2 F_0^{(B\sigma)} (m_\pi^2)(m_B^2 - m_\sigma^2) f_\pi \times \frac{g_{\sigma\pi^+\pi^-}}{u - m_\sigma^2 + i \Gamma_\sigma(u) m_\sigma}, \tag{5}$$

with $a_2 = C_2 + \frac{1}{3} C_1$. We have checked that the formulas (3) and (5), when applied to $D \to \sigma \pi$ with the obvious changes in the masses, form factor values, and Cabibbo-Kobayashi-Maskawa coefficients, reproduce the experimental results of [15].

The definition of the form factors $F_0^{(B\sigma)}$ and $F_1^{(B\sigma)}$ is the following:

$$\langle \sigma(q_\sigma) | A^\mu(q) | B(p) \rangle = \left[\frac{m_D^2 - m_\sigma^2}{q^2} q^\mu \right] F_0^{(B\sigma)}(q^2) + \left[(p + q_\sigma)^\mu - \frac{m_D^2 - m_\sigma^2}{q^2} q^\mu \right] \times F_1^{(B\sigma)}(q^2), \tag{6}$$

with $F_1(0) = F_0(0)$. Applying the factorization selects the form factor $F_0^{(B\sigma)}$ since

$$\langle \pi^- | A^\mu_{(\bar{d}u)} | \text{VAC} \rangle = i f_\pi q^\mu. \tag{7}$$

The *polar* and *direct* contributions to the semileptonic form factors discussed in [17] for the $D \to \sigma$ transition give in this case

$$F_0^{(B\sigma)}(m_\pi^2) \simeq F_0(0) = 0.45 \pm 0.15, \tag{8}$$

where $m_B = 5.72$ GeV, $m_B(1^+) = 5.7$ GeV, with $B(1^+)$ being the polar intermediate state connecting B to A^μ [17]. The errors include the uncertainty due to the variation of the parameters of the constiutent quark model [18] in a fixed range of values and that arising from the extrapolation of the polar form factors to $q^2 = m_\pi^2 \simeq 0$, following the same steps as in [17]. In that paper the analogous form factor for D decays was evaluated, $F_0^{(D\sigma)}$, and the result of the model closely reproduces the experimental value of [15]. The calculation does not include $1/m_b$ corrections, where m_b is the heavy quark

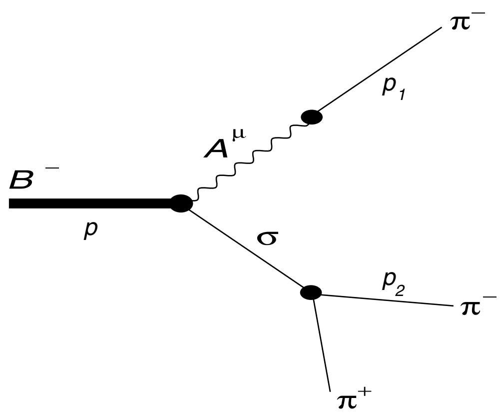

FIG. 1. Tree level diagram for a B^- decaying to three pions via the σ resonance. The two identical π^-'s in the final state require one to sum coherently two such diagrams having the momentum labels p_1 and p_2 inverted.

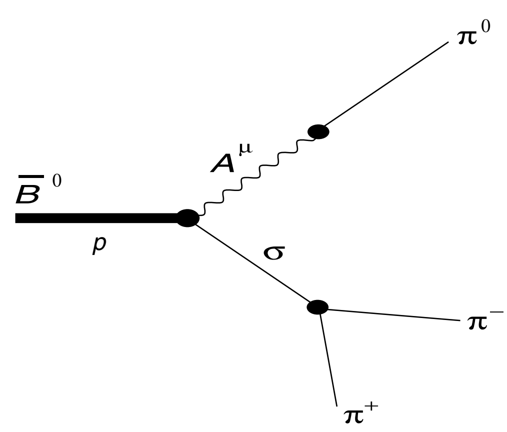

FIG. 2. As in Fig. 1 but for the $\bar{B}^0$, with no identical particles in the final state.

mass, however such corrections were estimated to be smaller than the quoted experimental error for D decays and are even smaller in the study of B decays.

The possibility of a σ contribution to CP violations was studied in $K \to \pi\pi$ decays [19]. We consider in the following the impact of a σ contribution in quasi-two-body pion decays of the B meson. In particular, the study of the $B \to \rho\pi$ channels is used for the determination of the unitarity angles α and γ [4]. These analyses make the assumption that, using cuts in the three invariant masses for the pion pairs, one can extract the ρ contribution without significant background contaminations. The ρ has spin 1, the π spin 0 as well as the initial B, and therefore the ρ has angular distribution $\cos^2\theta$ (θ is the angle of one of the ρ decay products with the other π in the ρ rest frame). This means that the Dalitz plot is mainly populated at the border, especially the corners, by this decay. Analyses following these lines were performed by the BABAR working groups [2]; Monte Carlo simulations, including the background from the narrow f_0 resonance, show that, with cuts at $m_{\pi\pi} = m_\rho \pm 200$ MeV, no significant contributions from other sources are obtained. The role of excited resonances such as the ρ' and the nonresonant background was discussed [20]. Finally the role of the off mass shell B^* contribution was discussed in [9,11,12].

The formulas derived in the present paper allow one to estimate the contribution of the σ which was not taken into account in the previous analyses. As the resonance is broad, part of the events from $B \to \sigma\pi \to 3\pi$ survive the cuts on the invariant mass of two π that reconstruct the mass of the ρ (within ± 200 MeV) in $B \to \rho\pi \to 3\pi$. We define the integrating region in the Dalitz plot around the ρ resonance:

$$\begin{aligned}\Gamma_{\rm eff}(\bar B^0 \to \rho^-\pi^+) &\\ &= \Gamma(\bar B^0 \to \pi^+\pi^-\pi^0)|_{m_\rho-\delta\le\sqrt{s}\le m_\rho+\delta}\,,\\ \Gamma_{\rm eff}(\bar B^0 \to \rho^+\pi^-) &\\ &= \Gamma(\bar B^0 \to \pi^+\pi^-\pi^0)|_{m_\rho-\delta\le\sqrt{t}\le m_\rho+\delta}\,.\end{aligned}$$

The Mandelstam variables are $s = (p_{\pi^+} + p_{\pi 0})^2$, $t = (p_{\pi^-} + p_{\pi 0})^2$, and we use $\delta = 200$ MeV.

Moreover the σ contributes to the decay $B^\pm \to \pi^+\pi^-\pi^\pm$ and only in a negligible way to the decay $\bar B^0 \to \pi^+\pi^-\pi^0$ if its contribution is restricted to the experimental cuts that reconstruct the process $\bar B^0 \to \rho^\pm\pi^\mp$. Note that in the latter case the ρ meson is charged, while the σ is neutral. This means that on average the two charged pions will have a high invariant mass from the σ resonance in the process, while in order to reconstruct a charged ρ meson a neutral and a charged pion have to be used. We have checked numerically that this is indeed correct and the σ contribution to $\bar B^0 \to \rho^\pm\pi^\mp$ (we reconstruct the ρ mass within ± 200 MeV) is 3 orders of magnitude smaller than the ρ contribution [for which we find $\mathcal{B}(\bar B^0 \to \rho^\mp\pi^\pm) = 19.9 \times 10^{-6}$]. The σ contribution to the charged B decay is on the contrary a fraction of the ρ one and comparable to or larger than the contribution from the B^* (see Table II). This provides a clear mechanism to enhance the denominator of the ratio R, giving a result closer to the experimental one:

TABLE II. Effective branching ratios (in units of 10^{-6}) for the charged $B^- \to \pi^+\pi^-\pi^-$ decay into three pions. Cuts as indicated in the text. Set I refers to the choice of the hadronic parameter $g = 0.4$ for the coupling $BB^*\pi$ relevant for the s evaluation of the B^* contribution, while Set II is calculated using $g = 0.6$. The σ contribution is less dependent on hadronic uncertainties as the coupling $g_{\sigma\pi^+\pi^-}$ is obtained from data.

$B^- \to \pi^+\pi^-\pi^-$	ρ	σ	B^*	$\rho + \sigma$	All
Set I	3.8	1.5	0.8	5.5	4.9
Set II	3.8	1.5	1.8	5.5	5.1

$$R = \frac{\mathcal{B}(\bar B^0 \to \rho^\mp\pi^\pm)}{\mathcal{B}(B^- \to \rho^0\pi^-)} = 3.6\,, \tag{9}$$

including only the ρ and σ contributions, or $R \simeq 4$ including also the B^*, which however is less precisely known. A possibility to disentangle these contributions from the ρ is to vary the experimental cuts and see the effect on the effective branching ratios. We find that the process $B^- \to \sigma\pi^-$ has a total branching ratio of 4.3×10^{-6}, therefore comparable to the one of the ρ. Allowing the cuts around the ρ mass to be ± 300 MeV, the σ contribution grows to 2.7×10^{-6}, to be compared to what is indicated in Table II for a cut of ± 200 MeV.

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