Upper Bound on the Scale of Majorana-Neutrino Mass Generation

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(Received 5 July 2000)

We derive a model-independent upper bound on the scale of Majorana-neutrino mass generation. The upper bound is $4\pi v^2/\sqrt{3}\,m_\nu$, where $v \simeq 246$ GeV is the weak scale and m_ν is the Majorana-neutrino mass. For neutrino masses implied by neutrino oscillation experiments, all but one of these bounds are less than the Planck scale, and they are all within a few orders of magnitude of the grand-unification scale.

DOI: 10.1103/PhysRevLett.86.212 PACS numbers: 14.60.Pq, 12.10.Kt, 14.60.St

There are three known types of neutrinos in nature, associated with the electron, the muon, and the tau lepton. Considerable evidence has mounted that one or more of these neutrino species has a nonzero mass, based on the observation of neutrino oscillations [1]. Since neutrinos are massless in the standard model of particle physics, the observation of nonzero neutrino masses is our first evidence of physics beyond the standard model.

The standard model of the electroweak interaction is a gauge theory based on the local symmetry group $SU(2)_L \times U(1)_Y$. The model contains three generations of quark and lepton fields and an $SU(2)_L$-doublet Higgs field which acquires a vacuum-expectation value and breaks the $SU(2)_L \times U(1)_Y$ symmetry to the $U(1)_{EM}$ symmetry of electromagnetism. There are three reasons why neutrinos are massless in this model.

(1) Only renormalizable interactions are included, i.e., terms in the Lagrangian of mass dimension four or less. The unique term of dimension five allowed by the gauge symmetry is [2]

$$\mathcal{L} = \frac{c}{M}(L^T \epsilon \phi) C (\phi^T \epsilon L) + \text{H.c.}, \tag{1}$$

where $L = (\nu_L, \ell_L)$ is an $SU(2)_L$ doublet containing the left-chiral neutrino and charged-lepton fields and $\phi = (\phi^+, \phi^0)$ is the $SU(2)_L$-doublet Higgs field [$\epsilon \equiv i\sigma_2$ is the antisymmetric 2×2 matrix in $SU(2)_L$ space; C is the charge-conjugation matrix in Dirac space]. This term would give rise to a Majorana-neutrino mass $m_\nu = cv^2/M$ when the neutral component of the Higgs field acquires a vacuum-expectation value $\langle \phi^0 \rangle = v/\sqrt{2}$, where $v = (\sqrt{2}\,G_F)^{-1/2} \simeq 246$ GeV is the weak scale.

(2) The only neutral lepton fields are in $SU(2)_L$ doublets. In particular, no $SU(2)_L \times U(1)_Y$-singlet fermion field is present. If present, the gauge symmetry would allow a Yukawa term

$$\mathcal{L} = -y_D \bar{L} \epsilon \phi^* \nu_R + \text{H.c.}, \tag{2}$$

where ν_R is the singlet field. Such a term would result in a Dirac neutrino mass $m_D = y_D v/\sqrt{2}$ when the neutral component of the Higgs field acquires a vacuum-expectation value, in the same way that the other fermions acquire Dirac masses. The gauge symmetry would also allow a Majorana mass for the singlet field,

$$\mathcal{L} = -\frac{1}{2} M_R \nu_R^T C \nu_R + \text{H.c.} \tag{3}$$

Majorana-neutrino masses may also be generated via the addition of $SU(2)_L$-triplet, $Y = 0$ fermion fields [3].

(3) The only scalar field is the $SU(2)_L$-doublet Higgs field. In particular, no $SU(2)_L$-triplet, $Y = 1$ Higgs field is present. If present, the gauge symmetry would allow a term

$$\mathcal{L} = -y_M L^T \epsilon \sigma^i C L \Phi^i + \text{H.c.}, \tag{4}$$

where Φ^i is the Higgs triplet field. Such a term would result in a Majorana-neutrino mass $m_\nu = 2 y_M u$ when the neutral component of the Higgs triplet field, $\Phi^0 = (\Phi^1 + i\Phi^2)/\sqrt{2}$, acquires a vacuum-expectation value $\langle \Phi^0 \rangle = u/\sqrt{2}$ [4–7]. Majorana-neutrino masses may also be generated via the addition of $SU(2)_L$-singlet scalar fields [4,8].

These restrictions eliminate the possibility of a Dirac neutrino mass and yield an "accidental" global lepton-number symmetry, $U(1)_L$, which forbids a Majorana neutrino mass. [Lepton number guarantees masslessness of the neutrino to all orders in perturbation theory. Beyond perturbation theory, lepton number is violated; however, $B - L$ symmetry (baryon number minus lepton number) survives and suffices to enforce the masslessness of the neutrino [6].] In this paper, we will encounter examples with massive neutrinos based on relaxing each of these three restrictions. (In the minimal supersymmetric standard model, renormalizable Majorana-neutrino mass terms are allowed. Imposing R parity suffices to forbid such terms.)

Since neutrino masses are necessarily associated with physics beyond the standard model, one would like to know the energy scale at which this new physics resides. In this paper we derive a model-independent upper bound on the scale of Majorana-neutrino mass generation. We also discuss two models that exemplify, and can even saturate, this bound: one with an $SU(2)_L \times U(1)_Y$-singlet fermion field, and one with an $SU(2)_L$-triplet Higgs field. The analysis we perform is in the spirit of a similar analysis for Dirac fermions carried out in Ref. [9]. However, there is no known model that saturates the upper bound on the scale of Dirac-fermion mass generation [9,10], in contrast to the case of Majorana-neutrino masses addressed in this paper.

We assume that the neutrino masses are Majorana, unlike the other known fermions, which carry electric charge and are therefore forbidden to have Majorana masses. If there is no $SU(2)_L \times U(1)_Y$-singlet fermion field in nature, then the neutrino masses are necessarily Majorana. However, even if such a field exists, the gauge symmetry allows the Majorana mass term of Eq. (3) for this field, and there is no reason why this mass should be small. The other known fermions acquire a mass only after the $SU(2)_L \times U(1)_Y$ symmetry is broken, and thus their masses are of order the weak scale, v, or less. Since a Majorana mass for the ν_R field is not protected by the gauge symmetry, it is natural to assume that it would be much greater than the weak scale [11]. So even if the ν_R field exists it is likely to be heavy, in which case the light neutrinos are Majorana fermions.

We begin our analysis with the standard model, but with a Majorana-neutrino mass of unspecified origin. Since the neutrino mass is put in artificially, this is only an effective field theory, valid up to some energy scale at which it is subsumed by a deeper theory, which we regard as the scale of Majorana-neutrino mass generation. The effective theory yields amplitudes that are an expansion in powers of energy divided by some mass scale. A simple way to derive an upper bound on the scale at which the effective theory breaks down is to examine tree-level $2 \to 2$ scattering amplitudes and identify the ones that grow with energy. Unitarity of the S matrix ensures that partial-wave amplitudes of inelastic $2 \to 2$ scattering processes cannot exceed $1/2$ in absolute value. When that value is exceeded at tree level, it indicates that the effective field theory is no longer valid, because the energy expansion does not converge. We thereby discover the energy at which the effective field theory necessarily breaks down; this represents an upper bound on the scale of new physics. This argument has been used to derive an upper bound on the scale of new physics in the Fermi theory of the weak interaction [12], on the scale of electroweak symmetry breaking in the electroweak theory (without a Higgs field) [13], and on the scale of Dirac-fermion mass generation [9].

The scattering amplitudes that grow with energy involve Majorana neutrinos in the initial and/or intermediate state, and longitudinally polarized weak vector bosons in the final state. The Feynman diagrams that contribute to the four relevant amplitudes are shown in Fig. 1. In the high-energy limit, $s \gg M_W^2, M_Z^2, m_\nu^2, m_\ell^2$, the zeroth-partial-wave amplitudes are given by the simple expressions

$$a_0\left(\frac{1}{\sqrt{2}}\,\nu_{i\pm}\nu_{j\pm} \to W_L^+ W_L^-\right) \sim \mp \frac{m_{\nu_i}\sqrt{s}}{8\pi\sqrt{2}\,v^2}\,\delta_{ij}\,, \tag{5}$$

$$a_0\left(\frac{1}{\sqrt{2}}\,\nu_{i\pm}\nu_{j\pm} \to \frac{1}{\sqrt{2}}\,Z_L^0 Z_L^0\right) \sim \mp \frac{m_{\nu_i}\sqrt{s}}{8\pi v^2}\,\delta_{ij}\,, \tag{6}$$

$$a_0(\nu_{i-}\ell_- \to Z_L^0 W_L^-) \sim \frac{m_{\nu_i}\sqrt{s}}{8\pi\sqrt{2}\,v^2}\,U_{\ell i}^*\,, \tag{7}$$

$$a_0\left(\frac{1}{\sqrt{2}}\,\ell_-\ell_- \to \frac{1}{\sqrt{2}}\,W_L^- W_L^-\right) \sim \frac{\sqrt{s}}{8\pi v^2}\sum_{i=1}^{3} U_{\ell i}^2 m_{\nu_i}\,, \tag{8}$$

where v is the weak scale, the indices i, j denote the three neutrino mass eigenstates, the subscripts on the neutrinos and charged leptons indicate helicity $\pm 1/2$, and the subscript on the partial-wave amplitude indicates $J = 0$. The unitary matrix $U_{\ell i}$ relates the neutrino weak and mass eigenstates. Each amplitude grows linearly with energy, and is proportional to the Majorana-neutrino mass or a linear combination of masses. (The amplitude for $\ell_-\ell_- \to W_L^- W_L^-$ involves the same linear combination of masses as the amplitude for neutrinoless double beta decay [14].)

The strongest bound on the scale of Majorana-neutrino mass generation is obtained by considering a scattering process which is a linear combination of the above amplitudes:

$$a_0\left(\frac{1}{2}\,(\nu_{i+}\nu_{i+} - \nu_{i-}\nu_{i-}) \to \frac{1}{\sqrt{3}}\,(W_L^+ W_L^- + Z_L^0 Z_L^0)\right) \sim -\frac{\sqrt{3}\,m_{\nu_i}\sqrt{s}}{8\pi v^2}\,. \tag{9}$$

The unitarity condition on inelastic $2 \to 2$ scattering amplitudes, $|a_J| \leq 1/2$ [15], implies that the scale of Majorana-neutrino mass generation is less than the scale

$$\Lambda_{\rm Maj} \equiv \frac{4\pi v^2}{\sqrt{3}\,m_\nu}\,, \tag{10}$$

which is inversely proportional to the neutrino mass. This is the principal result of this paper.

To gain some intuition for Eq. (10), we consider three different mechanisms for the generation of a Majorana-neutrino mass. First consider the addition of the dimension-five term of Eq. (1) to the standard model. The neutrino acquires a Majorana mass $m_\nu = cv^2/M$, where c/M is the coefficient of the dimension-five term. However, despite the addition of an explicit source for the Majorana-neutrino mass, the theory remains an effective field theory. The dimension-five term generates a $\nu\nu h^0$ vertex, where h^0 is the Higgs boson, which leads to the additional Feynman diagram in Fig. 2. Although this diagram cancels the term that grows with energy in the amplitude of Eq. (6), the other three amplitudes continue to grow with energy. [The amplitude of Eq. (5) undergoes a sign change when the Higgs diagram is included. The other two amplitudes have no additional contributions.] Thus there must still be new physics at or below the scale $\Lambda_{\rm Maj}$. The generation of a Majorana-neutrino mass via a nonrenormalizable dimension-five term cannot promote an effective field theory to a renormalizable one.

Consider instead the addition of an $SU(2)_L \times U(1)_Y$-singlet fermion field, ν_R, and the terms in Eqs. (2) and

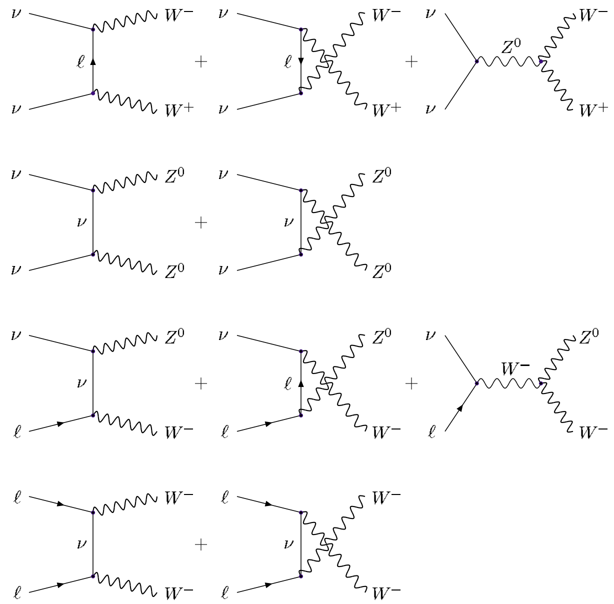

FIG. 1. Feynman diagrams that contribute to the amplitudes in Eqs. (5)–(8). The source of the Majorana-neutrino mass is unspecified, so there are no diagrams involving the coupling of the Majorana neutrino to the Higgs boson. Unitary gauge is used throughout.

(3), which are allowed by the gauge symmetry. Let us consider the limit $M_R \gg m_D$, motivated by our earlier argument that M_R should be much greater than the weak scale while the Dirac mass m_D is of order or less than the weak scale. This "seesaw" model yields a Majorana neutrino of mass $m_\nu \approx m_D^2/M_R$, which is much less than the Dirac mass, and thus provides an attractive explanation of why neutrinos are so much lighter than the other known fermions [16]. There is also a heavy Majorana neutrino, $N \approx \nu_R$, approximately of mass M_R. This particle leads to the additional Feynman diagrams obtained by replacing any intermediate ν state in Fig. 1 with N. A Higgs diagram analogous to Fig. 2 must also be included. One finds that the terms that grow with energy are canceled in all four amplitudes of Eqs. (5)–(8), so the addition of these two dimension-four terms has promoted the effective field theory to a renormalizable one. The scale of Majorana-neutrino mass generation is the mass of the heavy Majorana neutrino, $M_R \approx m_D^2/m_\nu$, and since $m_D = y_D v/\sqrt{2}$, one finds $M_R \approx y_D^2 v^2/2m_\nu$. This respects the upper bound of Eq. (10) provided the Yukawa coupling $y_D \lesssim \sqrt{8\pi}$, as it must [17]. The bound is saturated when the Yukawa coupling takes its largest allowed value.

The third mechanism introduces an $SU(2)_L$-triplet Higgs field and the term of Eq. (4) to generate a Majorana-neutrino mass [4–7]. The vacuum-expectation value of this field must be much less than the weak scale, because the relation $M_W^2 \simeq M_Z^2 \cos^2\theta_W$, which is satisfied experimentally, is obtained if the weak bosons acquire their mass dominantly from the vacuum-expectation value of an $SU(2)_L$ doublet, but not a triplet. In any case, a small vacuum-expectation value for the triplet is desirable in order to generate small Majorana-neutrino masses ($m_\nu = 2y_M u$). This model contains three neutral scalars, one singly charged scalar, and one doubly charged scalar. The term of Eq. (4) gives rise to new interactions that yield the additional Feynman diagrams in Fig. 3 involving these Higgs scalars in the intermediate state. (We impose CP conservation in this model, in which case one of the neutral scalars is CP odd and does not contribute to the amplitudes.) These diagrams cancel the terms that grow with energy in the amplitudes of Eqs. (5)–(8), so once again the addition of a dimension-four term has rendered an effective field theory renormalizable. The scale of Majorana-neutrino mass generation is the mass of these Higgs scalars. We have shown that their mass respects, and can even saturate, the model-independent upper bound on the scale of Majorana-neutrino mass generation, Eq. (10).

These two models demonstrate that M, the inverse coefficient of the dimension-five term of Eq. (1), is the scale of Majorana-neutrino mass generation. If one integrates out the heavy Majorana neutrino in the seesaw model, one obtains this dimension-five term, with $c/M = -y_D^2/2M_R$. The same thing happens if one integrates out the Higgs triplet. In both cases, M is equal to the scale at which new physics appears, and c is a dimensionless product of coupling constants and mass ratios. These models can naturally saturate our bound, Eq. (10), precisely because they generate a Majorana-neutrino mass term of dimension five in the low-energy theory.

Neutrino oscillation experiments do not measure the neutrino mass, but rather the absolute value of the mass-squared difference of two species of neutrinos, Δm^2. This implies a lower bound of $m_\nu \geq \sqrt{\Delta m^2}$ on the mass of one of the two participating neutrino species. Using Eq. (10), one finds the upper bounds on the scale $\Lambda_{\rm Maj}$ given in Table I for a variety of neutrino oscillation experiments. These upper bounds are all within a few orders of magnitude of the Planck scale, $G_N^{-1/2} \simeq 1.2 \times 10^{19}$ GeV, which is the scale before which quantum gravity must become

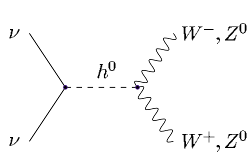

FIG. 2. Additional diagram that contributes to the amplitudes in Eqs. (5) and (6) when the Majorana neutrino acquires its mass via a coupling to the Higgs field.

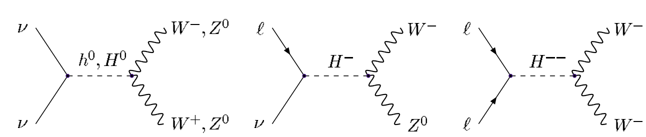

FIG. 3. Additional diagrams that contribute to the amplitudes in Eqs. (5)–(8) when the Majorana neutrino acquires its mass via a coupling to an $SU(2)_L$-triplet Higgs field.

TABLE I. Neutrino mass-squared differences from a variety of neutrino oscillation experiments, and their interpretations. The last column gives the upper bound on the scale of Majorana-neutrino mass generation, Eq. (10), for each interpretation. Table adapted from Ref. [1].

Experiment	Favored channel	Δm^2 (eV^2)	$\Lambda_{\rm Maj}$ (GeV) <
LSND	$\bar{\nu}_\mu \to \bar{\nu}_e$	0.2–2.0	9.8×10^{14}
Atmospheric	$\nu_\mu \to \nu_\tau$	3.5×10^{-3}	7.4×10^{15}
Solar			
MSW (large angle)	$\nu_e \to \nu_\mu$ or ν_τ	$(1.3-18) \times 10^{-5}$	1.2×10^{17}
MSW (small angle)	$\nu_e \to$ anything	$(0.4-1) \times 10^{-5}$	2.2×10^{17}
Vacuum	$\nu_e \to \nu_\mu$ or ν_τ	$(0.05-5) \times 10^{-10}$	2.0×10^{20}

relevant. However, only the vacuum-oscillation interpretation of the solar neutrino deficit yields a scale that could be as large as the Planck scale. In all other cases, we find that the physics of Majorana-neutrino mass generation must be below the Planck scale. Thus, if these neutrino masses arise from quantum gravity, then the scale of quantum gravity must be somewhat less than the Planck scale.

The upper bounds on $\Lambda_{\rm Maj}$ are also within a few orders of magnitude of the grand-unification scale, $\mathcal{O}(10^{16})$ GeV. The LSND (Liquid Scintillator Neutrino Detector) and atmospheric neutrino experiments yield an upper bound on $\Lambda_{\rm Maj}$ slightly below the grand-unification scale, but the scale of Majorana-neutrino mass generation could be less than the unification scale in a grand-unified model. For example, in a grand-unified model that makes use of the seesaw mechanism, the mass of the heavy Majorana-neutrino N could be equal to a small Yukawa coupling times the vacuum-expectation value of the Higgs field that breaks the grand-unified group.

In this paper we have derived a model-independent upper bound on the scale of Majorana-neutrino mass generation, Eq. (10). The upper bounds on this scale implied by a variety of neutrino oscillation experiments are listed in Table I. All but one of these bounds are less than the Planck scale, and they are all within a few orders of magnitude of the grand-unification scale.

We are grateful for conversations with K. Babu and D. Dicus. This work was supported in part by the U.S. Department of Energy under Contract No. DOE DE-FG02-91ER40677. We gratefully acknowledge the support of GAANN, under Grant No. DE-P200A980724, from the U.S. Department of Education for J. M. N.

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