# Spin-Orbit Effects in a GaAs Quantum Dot in a Parallel Magnetic Field

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We analyze the effects of spin-orbit coupling on fluctuations of the conductance of a quantum dot fabricated in a GaAs heterostructure. Counterintuitively we argue that spin-orbit effects may become important in the presence of a large parallel magnetic field $B_\parallel$, even if they are negligible for $B_\parallel = 0$. This should be manifest in the level repulsion of a closed dot, and in reduced conductance fluctuations in dots with a small number of open channels in each lead, for large $B_\parallel$. Our picture is consistent with the experimental observations of Folk *et al.*

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Recent experiments by Folk *et al.* [1] studied statistics of fluctuations of the conductance $g$ through a quantum dot in a GaAs heterostructure with an applied magnetic field $B_\parallel$ in the plane of the sample. In the largest dots studied, the application of $B_\parallel$ was observed to reduce the variance of the fluctuations, var($g$), by a factor of roughly 4, in contrast to a reduction factor of 2, which was originally expected. As noted by Folk *et al.,* the extra reduction might be understood if, for some reason, spin-orbit coupling increased with the application of $B_\parallel$.

While naively one might expect $B_\parallel$ to align the electron spin in such a way that makes the spin-orbit coupling less effective, in this Letter we argue that the confinement of an electron to a quantum dot in a GaAs structure leads to an opposite effect. This effect could well explain the observations of Ref. [1]. Similar effects should appear in the repulsion between energy levels in a closed dot. Our conclusions may allow for future experiments where spin and electronic properties are studied with the spin-orbit scattering rate being a controlled parameter.

In a single particle picture, conductance fluctuations through a chaotic or disordered quantum dot may be crudely understood as arising from fluctuations in the number of electronic levels in an energy window of size $2N\Delta$, and in the matrix elements coupling these levels to the leads. Here $\Delta$ is the mean level spacing in the dot, for each spin state, and $N$ is the number of channels in each lead, (i.e., each lead has conductance $2Ne^2/h$). We assume the leads to be perfectly coupled to the dot, such that Coulomb blockade effects are insignificant. The *mean* conductance in this geometry, including both spin states, is $\langle g\rangle = Ne^2/h$. In the experiments of Ref. [1], $N$ was in the range 1 to 3.

In the experiments of Ref. [1], a weak perpendicular magnetic field $B_\perp$ was applied. This field was strong enough to break time-reversal symmetry for the orbital motion, but not strong enough to produce a significant Zeeman splitting. Then, if spin-orbit coupling is absent and $B_\parallel = 0$, conductance fluctuations should satisfy

$$\mathrm{var}(g) = 4C_N, \quad (\text{no spin-orbit}, \quad B_\parallel = 0), \qquad (1)$$

where the constant $C_N$ is var($g$) for spinless electrons in a dot with $N$ open channels per lead, and the factor 4 results from the degeneracy of the two spin states. (From here on we measure all conductances in units of $e^2/h$, so that $C_N$ is dimensionless.) The factor $C_N$ depends on the temperature $T$ through the ratio $T/\hbar\Gamma$ (where $\Gamma \equiv N\Delta/\pi\hbar$ is the escape rate from the dot) and on the phase breaking rate $\tau_\phi^{-1}$ through the parameter $\Gamma\tau_\phi$. The value of $C_N$ can be calculated from random matrix theory (RMT), using the Gaussian unitary ensemble (GUE).

For $B_\parallel \neq 0$, still in the absence of spin-orbit coupling, the Fermi levels for spin-up and spin-down electrons are split by the Zeeman energy $E_Z = g^*\mu_B B_\parallel$. When $E_Z$ is larger than both $T$ and $\Gamma$, the contributions from the two spin states become statistically independent, giving

$$\mathrm{var}(g) = 2C_N \quad (\text{no spin-orbit}, \quad E_Z \gg \Gamma, T). \qquad (2)$$

However, in the presence of a strong spin-orbit coupling, the two spin levels will be mixed, and will be described by a single GUE, with mean level spacing $\Delta/2$, and $2N$ open channels in each lead. (Recall that Kramers degeneracy is already broken by $B_\perp$.) Thus in that case,

$$\mathrm{var}(g) = C_{2N} \quad (\text{strong spin-orbit}). \qquad (3)$$

The crossover to strong spin-orbit coupling should be controlled by the dimensionless parameter $\lambda = \epsilon_{\mathrm{so}}/\Delta$ where $\epsilon_{\mathrm{so}}$ is the root-mean-square (rms) value of the matrix element $\langle i|H_{\mathrm{so}}|j\rangle$. Here the states $i, j$ have opposite spin directions and orbital energies that differ by $E_Z$. (The matrix element is to be calculated with the dot isolated from the leads. The same parameter $\lambda$ controls the repulsion between levels of opposite spins in the closed dot.) Then in the presence of $B_\parallel$ we can write

$$\mathrm{var}(g)_{B_\parallel} = F_N(\lambda, T/\hbar\Gamma, \Gamma\tau_\phi), \qquad (4)$$

where $F_N \to 2C_N$ for $\lambda \to 0$, and $F_N \to C_{2N}$ for $\lambda$ sufficiently large. Note that $\Gamma$ is unchanged if $N$ is doubled and $\Delta$ is halved, so $\Gamma$ remains constant as one varies $\lambda$. We shall also see that at least approximately, $C_N \approx C_{2N}$, so that $F_N$ decreases by a factor of 2 as $\lambda$ varies from 0 to $\infty$. Then, if the system parameters are such that $\lambda$ grows from

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zero to a large value as a parallel field $B_{||}$ is turned on, the factor-of-2 reduction in $F$, combined with the factor-of-2 reduction on breaking the spin degeneracy, should lead to overall reduction of a factor of $\approx 4$ in var$(g)$, relative to the $B_{||} = 0$ value, in Eq. (1). This is in accord with the observation of Ref. [1].

A variety of evidence, based on RMT and other approaches, suggests that for a large $N$, $C_N$ is independent of $N$, for any fixed value of $\Gamma$, $T$, and $\tau_\phi$ [2–6]. The biggest deviation from this is presumably for $N = 1$ and no dephasing. At $T = 0$, with no dephasing, the value of $C_N$ is known, within RMT, to be $(16 - 4N^{-2})^{-1}$ [2]. Thus for $N = 1$, the reduction factor $F_N(0, 0, \infty)/F_N(\infty, 0, \infty)$ is $5/2$ rather than 2.

We define a crossover value $\lambda_c$ where var$(g)$ is halfway between the values for $\lambda = 0$ and $\lambda$ large. We may estimate $\lambda_c$ as the value of $\lambda$ such that $\tau_{\rm so}^{-1} = \tau_\phi^{-1} + \Gamma$, where $\tau_{\rm so}^{-1} = 2\pi\lambda^2\Delta/\hbar$ is the rate for spin flip due to spin-orbit coupling, given by Fermi's golden rule. Writing $N_{\rm eff} \equiv N + \pi\hbar(\Delta\tau_\phi)^{-1}$, this gives

$$\lambda_c \approx 0.23 N_{\rm eff}^{1/2}. \tag{5}$$

Numerical calculations, discussed further below and illustrated in Fig. 1, are at least qualitatively consistent with this estimate, but suggest that the factor 0.23 should be replaced by 0.1.

Spin-orbit coupling in GaAs heterostructures originates from the asymmetry of the potential creating the two-dimensional electron gas (2DEG) (Rashba term) and from the lack of inversion symmetry in the GaAs lattice structure (Dresselhaus term). The operator describing the spin-orbit coupling is composed of both terms:

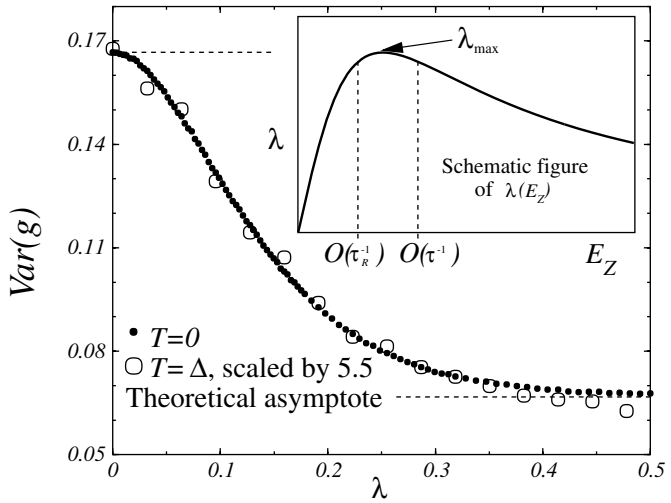


FIG. 1. Variance of conductance for the case $N = 1, \tau_\phi^{-1} = 0$, as a function of the coupling $\lambda$, for $T = 0$ and $T = \Delta$. Asymptotes show known results for $\lambda = 0$ and $\lambda = \infty$ at $T = 0$. Data for $T = \Delta$ were scaled by a factor of 5.5. The inset shows *schematic* behavior of $\lambda$ as a function of the Zeeman energy $E_Z$, following Eqs. (8) and (10).

$$H_{\rm so} = \gamma(\vec{v} \times \vec{\sigma}) \cdot \hat{z} + \eta(v_x\sigma_x - v_y\sigma_y), \tag{6}$$

where $\vec{v}$ is the velocity operator, $\vec{\sigma}$ are the Pauli spin matrices, $\gamma$ and $\eta$ are coupling constants, and we ignore terms $\propto v^3$. We assume that the 2DEG is grown on a [001] GaAs plane and $x, y$ denote the cubic axes in the plane. Note that this spin-orbit coupling is different from the one encountered in measurements of conductance fluctuations in metals. The latter is induced by impurities, and is characterized by coupling constants that strongly vary with position.

In the absence of $H_{\rm so}$, the eigenstates of the electronic Hamiltonian are products of a spatial part $|a\rangle$ and a spin part $|\sigma\rangle$, where $\sigma = \uparrow (\downarrow)$ denotes spin parallel (antiparallel) to the Zeeman field $\vec{B}_{||}$. With $H_{\rm so}$, the mean-square value of the dimensionless spin-orbit coupling $\lambda$ for states with opposite spins at the Fermi energy is

$$\lambda^2(E_Z) \equiv \sum_{ab} \overline{|(H_{\rm so})_{a\uparrow,b\downarrow}|^2}\,\delta(\epsilon_a - \epsilon_b - E_Z)\delta(\epsilon_a - \epsilon_F), \tag{7}$$

where $\epsilon_a, \epsilon_b$ are the orbital energies of the states $|a\rangle$ and $|b\rangle$ (i.e., the energies at $B_{||} = 0$), the overbar denotes averaging over disorder, and $\epsilon_F$ is the Fermi energy. Here and henceforth $O_{s1,s2} \equiv \langle s1|O|s2\rangle$. As we now show, in a quantum dot the typical matrix element $(H_{\rm so})_{a\uparrow,b\downarrow}$ depends on the energy difference $\epsilon_a - \epsilon_b$, so that $\lambda^2$ does indeed depend on $E_Z$.

For simplicity, we first discuss the case where $\eta = 0$ in $H_{\rm so}$, and we choose $\vec{B}_{||} \parallel \hat{x}$. For a macroscopic system in the diffusive regime, comparing the Kubo-Greenwood formula with the Drude formula, one finds

$$\overline{|(v_x)_{ab}|^2} \approx \frac{2D\Delta}{\pi\hbar}\,\frac{1}{1 + (\omega\tau)^2}, \tag{8}$$

where $\hbar\omega \equiv \epsilon_a - \epsilon_b$, $\tau$ is the transport lifetime, and $D = v_F^2\tau/2$ is the diffusion constant. Thus, for a large diffusive system and for $\omega\tau \ll 1$, we find $\lambda \approx \frac{\gamma v_F}{\Delta}(\frac{\Delta\tau}{\pi\hbar})^{1/2}$, which does not depend on $\omega$. A parallel field does not affect the strength of spin-orbit coupling as long as $E_Z \ll \hbar/\tau$.

In contrast, the confinement of the electron to a quantum dot suppresses the velocity matrix elements when $\omega\tau_R \ll 1$, where $\tau_R$ is a Thouless time, which we define as the time for an electron to cross from the center to the edge of the dot. This is most easily seen if we use the relation $|(v_x)_{ab}| = \omega|x_{ab}|$, and note that the matrix element of $x$ is bounded by the maximum radius $R$ of the dot. More precisely, we may use the relation

$$\begin{aligned}\overline{|x_{ab}|^2} &= \sum_a \sum_{b\neq a} |x_{ab}|^2\Delta^2\delta(\epsilon_a - \epsilon_b - \hbar\omega)\delta(\epsilon_a - \epsilon_F) \\ &= \int_0^\infty \frac{\Delta dt}{\pi\hbar}\left[\delta x^2 - \frac{1}{2}[x(0) - x(t)]^2\right]_{aa} \cos\omega t,\end{aligned} \tag{9}$$

where the last line should be averaged over all states at the Fermi energy; $(\delta x^2)_{aa} \approx R^2$ is the position uncertainty in the state $a$; and $\{[x(0) - x(t)]^2\}_{aa}$ may be approximated by averaging over the appropriate classical trajectories.

For a dot in the diffusive regime, where $v_F\tau \ll R$, we have $\tau_R = R^2/2D$. Then, $\{[x(0) - x(t)]^2\}_{aa} = v_F^2 t^2/2$ as long as $t \ll \tau$; it grows as $2Dt$ for $\tau < t < \tau_R$, and finally approaches $2\{\delta x^2\}_{aa}$ for $t > \tau_R$. Thus,

$$\overline{|(v_x)_{ab}|^2} \approx \begin{cases} c\,\dfrac{2D\Delta}{\pi\hbar}\,(\omega\tau_R)^2 & \text{for } \omega < \tau_R^{-1}, \\ c\,\dfrac{2D\Delta}{\pi\hbar} & \text{for } \tau_R^{-1} < \omega < \tau^{-1}, \end{cases} \tag{10}$$

where $c$ is a constant which depends on the dot's shape. For a roughly circular dot of radius $R$, $c \approx O(1)$, and we ignore it below. The value of $\overline{|(v_x)_{ab}|^2}$ falls off according to (8) as $\omega$ increases further.

For a ballistic chaotic dot, the time scales $\tau$ and $\tau_R \equiv R/v_F$ coincide, and $D \approx v_F R/2$. The second line of Eq. (10) does not apply. The maximum value of $\overline{|(v_x)_{ab}|^2}$ is $\approx v_F R\Delta/\pi\hbar$, obtained when $\omega\tau_R \approx 1$. From these results, we may calculate $\varepsilon_{\rm so}$ and $\lambda^2(E_Z) = \gamma^2\overline{|(v_x)_{ab}|^2}/\Delta^2$. For both types of dots the confinement leads to a $B_{||}$ dependence of $\lambda^2(\omega)$ for $E_Z\tau_R \ll \hbar$ (see the inset of Fig. 1). The maximum value of $\lambda^2$ is $\lambda^2_{\rm max} \approx \frac{\gamma^2 v_F R}{\pi\hbar\Delta}$.

If $\gamma$ and $\eta$ are both nonzero, $\lambda$ depends on the direction of $B_{||}$ within the $x$-$y$ plane. It is different, e.g., for $B_{||}||(110)$ and $B_{||}||(1\bar{1}0)$, even for a dot which is roughly circular. However, the average of $\lambda^2$ over all directions of $B_{||}$ will be $(\gamma^2 + \eta^2)\,\overline{|(v_x)_{ab}|^2}/\Delta^2$.

In order to make a comparison to the experiments of Ref. [1], we consider a ballistic dot where $\gamma, \eta$ are such that $\lambda(\Delta) \ll \lambda_c$ and $\lambda_{\rm max}$ is greater than the crossover value $\lambda_c$ (see Fig. 1). At low temperatures $T \ll \hbar/\pi\tau_R$, the variance var$(g)$ should decrease in two stages as the Zeeman energy $E_Z$ is increased. In the first it would drop from $4C_N^0$ to $2C_N^0$ over the range $0 < E_Z < \pi\max(T, \hbar\Gamma)$, due to the removal of spin degeneracy of the levels. Then var$(g)$ would drop by an additional factor of approximately 2, resulting from the turning on of spin-orbit coupling, over the larger range $\pi\max(T, \hbar\Gamma) < E_Z < \hbar/\tau_R$. (For a still larger value of $E_Z$, the conductance fluctuations would increase again.) In Ref. [1], $T$ was comparable to $\hbar/\pi\tau_R$. Under such conditions we expect the factor-of-4 decrease in var$(g)$ to occur smoothly over the range $0 < E_Z < \hbar/\tau_R$. (Recall that time-reversal invariance is broken by $B_\perp$ in all cases.)

A quantitative comparison of this scenario to the experiment of Ref. [1] requires information regarding the strength of spin-orbit coupling, which we parametrize by the dimensionless parameter $Q_{\rm so} \equiv (\gamma^2 + \eta^2)^{1/2} v_F/E_F$. In terms of $Q_{\rm so}$, for a ballistic dot $\lambda_{\rm max} \approx Q_{\rm so} N_e^{3/4}/3$, where $N_e$ is the number of electrons in the dot. For our scenario to be consistent with the experiment, we need $\lambda_{\rm max}$ to be at least comparable to $\lambda_c \approx 0.2 N_{\rm eff}^{1/2}$ for the large measured dot (where $N_e = 16\,000$ and $N_{\rm eff} \approx 6$) and smaller than $\lambda_c$ for the small measured dot (where $N_e = 2000$ and $N_{\rm eff} \approx 6$). These requirements suggest $5 \times 10^{-3} > Q_{\rm so} > 10^{-3}$. There are additional numerical uncertainties, however, because our application to ballistic chaotic systems of formulas derived for diffusive systems [e.g., Eq. (10)] involved several unknown numbers of order unity.

Although $\gamma$ and $\eta$ have been measured previously in other GaAs heterostructures, the parameters depend on details of the structure, and are difficult to extrapolate from one system to another. Values of $Q_{\rm so}$ extracted from existing data on GaAs 2DEGs include $Q_{\rm so} \approx 1.6 \times 10^{-2}$, from optical measurements [7], in a sample with $n = 4 \times 10^{11}$ cm$^{-2}$, and $Q_{\rm so} \approx 5 \times 10^{-3}$, from Shubnikov–de Haas measurements [8], in a sample with $n = 1.2 \times 10^{12}$ cm$^{-2}$. Magnetoresistance measurements [9] in 2DEGs extract the spin-orbit scattering rate by studying the crossover from weak localization to weak antilocalization as the density is increased. At the densities where the crossover occurs, typically around $n \sim 6 \times 10^{11}$ cm$^{-2}$, values around $\tau_{\rm so}^{-1} \sim 4 \times 10^{10}$ sec$^{-1}$ are found in the moderate mobility samples ($l \sim 0.5\ \mu$m), corresponding to $Q_{\rm so} \sim 4 \times 10^{-3}$. Our estimates of $Q_{\rm so}$ for the samples of Ref. [1], which had $n = 2 \times 10^{11}$ cm$^{-2}$, are not incompatible with the range of previous measurements.

The suppression of spin-orbit matrix elements by the confinement to a dot affects also the scattering rate due to spin-orbit coupling, $\tau_{\rm so}^{-1}$. Note that spin-orbit scattering processes do not necessarily result in a spin flip of the electron. The probability of a spin flip in a spin-orbit scattering process depends on the ratio $\gamma/\eta$ and on the initial direction of the spin. We focus on the case $\eta = 0$ and initial spin state in the $x$-$y$ plane, in which half of spin-orbit scattering processes involve a spin flip. We also set $E_Z = 0$ for this part of the discussion. The rate of spin flip due to a spin-orbit scattering process of a state $|a\rangle$ is $\frac{\hbar}{\tau_{\rm so}} = {\rm Im}\Sigma(a, \varepsilon_a)$, where $\Sigma(a, \varepsilon_a)$ is the on-shell self-energy of the state $|a\rangle$ due to spin-orbit scattering events, irrespective of the final spin state.

To second order in the spin-orbit interaction, ${\rm Im}\Sigma(a, \varepsilon) = 2\pi\sum_b \overline{|(H_{\rm so})_{a\uparrow,b\downarrow}|^2}\,\delta(\epsilon - \epsilon_b)$. Because of the finite escape rate $\Gamma \gtrsim \Delta$, the $\delta$ functions are broadened enough to allow the sum to be replaced by an integral. Then, in view of Eqs. (6), (8), and (10), the *on-shell* self-energy $\Sigma(a, \varepsilon_a)$ *vanishes*. We go beyond this order, to a self-consistent self-energy, where

$$\Sigma(a, \varepsilon_a) = 2\int \frac{d\varepsilon_b}{\Delta}\,\frac{\overline{|(H_{\rm so})_{a\uparrow b\downarrow}|^2}}{\varepsilon_a - \varepsilon_b - \Sigma(b, \varepsilon_a) - i\Gamma}, \tag{11}$$

and approximate the solution of (11) by substituting the second order expression in its right-hand side. In the

diffusive limit we find

$$\frac{1}{\tau_{\rm so}} \approx \begin{cases} \dfrac{\tau_R}{\tau_{\rm so}^{\infty}}\left(\dfrac{1}{4\pi\tau_{\rm so}^{\infty}}\right) + \Gamma & \text{for } \dfrac{\tau_R}{\tau_{\rm so}^{\infty}} \ll 1, \Gamma\tau_R \ll 1, \\ \dfrac{1}{\tau_{\rm so}^{\infty}} & \text{for } \dfrac{\tau_R}{\tau_{\rm so}^{\infty}} \gg 1, \end{cases} \tag{12}$$

where $\frac{1}{\tau_{\rm so}^{\infty}} = 4\gamma^2 D/\hbar^2$ is the spin-flip scattering rate in an open system. As expected, when $\tau_R \gg \tau_{\rm so}^{\infty}$ the confinement of electrons to the dot does not significantly affect spin-orbit scattering rate. In contrast, for a small dot spin-orbit scattering rate is suppressed. When $\Gamma \ll \frac{1}{\tau_{\rm so}^{\infty}}$ it becomes of fourth order in the coupling constants, $\frac{1}{\tau_{\rm so}} \approx \frac{\tau_R}{4\pi(\tau_{\rm so}^{\infty})^2}$. At this order, the smallness of spin-orbit matrix elements at close energies is overcome by virtual transitions of high energy difference ($\sim\tau_R^{-1}$).

The suppression of $\tau_{\rm so}^{-1}$ in small dots has implications for electronic transport through the dots. Normally, for an open system, when $\tau_{\rm so}^{-1}$ gets larger than $\tau_{\phi}^{-1}$, weak localization turns into weak antilocalization, and the magnetoresistance becomes positive. The corresponding criterion for a quantum dot compares the reduced $\tau_{\rm so}^{-1}$ with $(\tau_{\phi}^{-1} + \Gamma)$. In fact, the spin-orbit scattering rate relevant for transport may be even smaller than the $\tau_{\rm so}^{-1}$ of (12). Even in open 2D electron structures in GaAs, there are subtleties due to the fact that spin rotations induced by $H_{\rm so}$ are correlated with spatial displacements of the electron. For example, as analyzed in [10], when $\eta = \pm\gamma$, there is no weak antilocalization, if the cubic term, $\propto v^3$ in $H_{\rm so}$, is ignored [10].

Before concluding, we explain the random matrix calculations leading to Fig. 1. These calculations are aimed at studying the $\lambda$ dependence of var($g$) in the presence of a strong $B_{||}$. The Hamiltonian of the closed dot was modeled by a $2M \times 2M$ random matrix $H$ of the form $H_{ij} = \eta_{ij}K_{ij}$ where $1 < i < M$ labels states with spin up, $M + 1 < i < 2M$ labels states with spin down, $\eta_{ij} = 1$ if $i$ and $j$ are states with the same spin, $\eta_{ij} = \varepsilon_{\rm so} = \pi\lambda/M^{1/2}$ if $i$ and $j$ are states with opposite spin, and $K$ is a random Hermitian matrix from the GUE, with the distribution $P(K) \propto e^{-(1/2)\,{\rm tr}K^{\dagger}K}$. This matches the definitions in the text because $\varepsilon_{\rm so}$ is the rms value of the matrix element connecting two states in the different spin blocks, and $\Delta = \pi/M^{1/2}$ is the average level spacing at the center of the band for one block. For the case $N = 1$, we connect the system to "leads" with perfect conducting channels at two sites for spin up and two sites for spin down, and calculate the $2 \times 2$ transmission matrix $t$ for energies near the center of the band [11].

At $T = 0$ one finds the conductance by using the Landauer formula $g = {\rm tr}tt^{\dagger}$. To obtain results at finite $T$, for each realization of the random matrix we first calculate the transmission matrix and thus the $T = 0$ conductivity $g^{(0)}(E)$ for a range of energies $E$. This conductance is weighted by the derivative of the Fermi function and integrated to give $g(T) = \int_{-\infty}^{\infty} \frac{df}{dE}\, g^{(0)}(E)\, dE$.

The conductance fluctuations as a function of $\lambda$ for zero temperature and temperature $\Delta$ are shown in Fig. 1. The data for $T = \Delta$ were accumulated from 5000 realizations with $M = 60$, while the data for $T = 0$ were obtained from $10^6$ realizations with $M = 20$. More limited calculations at $T = 0$ with $M = 60$ showed differences of less than 10% from $M = 20$.

It is clear from the figure that the $T = \Delta$ results have the same dependence on $\lambda$ as the $T = 0$ results, and we have evidence that this remains true for higher $T$. The reduction in var($g$) by a factor of $\approx$6 is consistent with the theoretical expectation [4] that $C_N(T) \approx C_N(T = 0)\pi\hbar\Gamma/6T$, for $T \geq \pi\hbar\Gamma$, if dephasing is absent. The middle of the crossover occurs roughly at $\lambda = 0.1$, which is somewhat less than the value 0.23 given by Eq. (5).

Calculations for $2 \leq N \leq 4$ at $T = 0$ (not shown) are consistent with a crossover value $\lambda_c$ scaling as $N^{1/2}$, as predicted by (5). Dephasing can also be included by using a "third-lead" model of the type discussed by Brouwer and Beenakker [12]. Results for $N = 1$ and $T = 0$ seem to show a variation of $\lambda_c$ somewhat slower than $N_{\rm eff}^{1/2}$, at least in the range $1 < N_{\rm eff} < 4$.

In summary, we presented a theory by which the effect of spin orbit on conductance fluctuations in a quantum dot depends strongly on an applied parallel magnetic field. This theory may well explain the experimental observation of Ref. [1].

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