

## Relativistic Symmetry Suppresses Quark Spin-Orbit Splitting

P. R. Page,\* T. Goldman,<sup>†</sup> and J. N. Ginocchio<sup>‡</sup>

*MS B283, Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*  
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Experimental data indicate small spin-orbit splittings in hadrons. For heavy-light mesons we identify a relativistic symmetry that suppresses these splittings. We suggest an experimental test in electron-positron annihilation. Furthermore, we argue that the dynamics necessary for this symmetry are possible in QCD.

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*I. Introduction.*—Recently, Isgur [1] has reemphasized the experimental fact that spin-orbit splittings in meson and baryon systems, which might be expected to originate from one-gluon-exchange (OGE) effects between quarks, are absent from the observed spectrum. He conjectures that this is due to a fairly precise, but accidental, cancellation between OGE and Thomas precession effects, each of which has “splittings of hundreds of MeV” [1]. Taking the point of view that precise cancellations reflect symmetries rather than accidents, we have examined what dynamical requirements would lead to such a result. One of us recently observed [2] that a relativistic symmetry is the origin of pseudospin degeneracies first observed in nuclei more than thirty years ago [3,4]. We find that a close relative of that dynamics can account for the spin degeneracies observed in hadrons composed of one light quark (antiquark) and one heavy antiquark (quark).

Below, we first elucidate the experimental evidence for small spin-orbit splittings. Then we identify the symmetry involved in terms of potentials in the Dirac Hamiltonian for heavy-light quark systems and note the relation to the symmetry for pseudospin. We show that the former symmetry predicts that the Dirac momentum space wave functions will be identical for the two states in the doublet, leading to a proposed experimental test. Finally, we argue that the required relation between the potentials may be plausible from known features of QCD.

*II. Experimental and lattice QCD spectrum.*—In the limit where the heavy (anti)quark is infinitely heavy, the angular momentum of the light degrees of freedom,  $j$ , is separately conserved [5]. The states can be labeled by  $l_j$ , where  $l$  is the orbital angular momentum of the light degrees of freedom. In nonrelativistic models of conventional mesons the splitting between  $l_{l+(1/2)}$  and  $l_{l-(1/2)}$  levels, e.g., the  $p_{3/2}$  and  $p_{1/2}$  or  $d_{5/2}$  and  $d_{3/2}$  levels, can *only* arise from spin-orbit interactions [1]. The  $p_{1/2}$  level corresponds to two degenerate broad states with different total angular momenta  $J = j \pm s_Q$  (here  $j = \frac{1}{2}$ ), where  $s_Q$  is the spin of the heavy (anti)quark [5]. For example, in the case of  $D$  mesons,  $s_Q = \frac{1}{2}$  and the two states are called  $D_0^*$  and  $D_1'$ . There are also two degenerate narrow  $p_{3/2}$  states  $D_1$  and  $D_2^*$  [5]. The degenerate states separate as one moves slightly away from the heavy quark limit, and

their spin-averaged mass remains approximately equal to the mass before separation.

For the  $D$  mesons, the CLEO Collaboration claims a broad  $J^P = 1^+$  state at  $2461_{-34}^{+41} \pm 10 \pm 32$  MeV [6], belonging to the  $p_{1/2}$  level, in close vicinity to the  $D_2^*$  at  $2459 \pm 2$  MeV [7], belonging to the  $p_{3/2}$  level, indicating a remarkable  $p_{3/2}$ - $p_{1/2}$  spin-orbit degeneracy of  $-2 \pm 50$  MeV. It is appropriate to extract the spin-orbit splitting this way since first, the charm quark behaves like a heavy quark. Second, the difference between the  $D_1'$  and  $D_2^*$  levels is the best indicator [8] of the  $p_{3/2}$ - $p_{1/2}$  splitting in the absence of experimental data [9] on the  $D_0^*$ , as opposed to the difference between the  $D_1'$  and spin-averaged  $p_{3/2}$  level at  $2446 \pm 2$  MeV. Spin-averaged masses are determined from experiment [7].

For the  $K$  mesons, the  $p_{1/2}$  level is at  $1409 \pm 5$  MeV, with  $p_{3/2}$  nearby at  $1371 \pm 3$  MeV, corresponding to a  $p_{3/2}$ - $p_{1/2}$  splitting of  $-38 \pm 6$  MeV. The splitting between the higher lying  $d_{5/2}$  and  $d_{3/2}$  levels is  $-4 \pm 14$  MeV or  $41 \pm 13$  MeV, depending on how the states are paired into doublets. These results indicate a near spin-orbit degeneracy if the strange quark can be treated as heavy, although it has certainly not been established that such a treatment is valid.

For  $B$  mesons, both L3 [8] and OPAL [10] have performed analyses, using input from theoretical models and heavy quark effective theory, to determine that the  $p_{3/2}$ - $p_{1/2}$  splitting is  $97 \pm 11$  MeV (L3) or  $-109 \pm 14$  MeV (OPAL). Note that these are *not* model-independent experimental results. In the same analyses the mass difference between  $B_2^*$  and  $B_0^*$ , an approximate indicator of the  $p_{3/2}$ - $p_{1/2}$  splitting, is  $110 \pm 11$  MeV (L3) or  $-89 \pm 14$  MeV (OPAL). The L3 result agrees with lattice QCD estimates of  $155_{-13}^{+9} \pm 32$  MeV [11] and  $183 \pm 34$  MeV [12]. However, according to other estimates [13], the splitting is less than 100 MeV, and consistent with zero. Recently,  $31 \pm 18$  MeV was calculated [14]. One lattice QCD study found evidence for a change of sign in the splitting somewhere between the charm and bottom quark masses, albeit with large error bars [15]. A splitting of 40 MeV serves as a typical example of model predictions [16], although there is variation summarized in Ref. [14].

In order to more quantitatively measure the spin-orbit splitting, define

$$r = \frac{(p_{3/2} - p_{1/2})}{[(4p_{3/2} + 2p_{1/2})/6 - s_{1/2}]}, \quad (1)$$

where all entries refer to masses. The experimental data on  $D$ ,  $K$ , and  $B$  mesons give, respectively,  $r = 0.00 \pm 0.10$ ,  $-0.06 \pm 0.00$  and  $0.23 \pm 0.04$  (L3) or  $-0.23 \pm 0.03$  (OPAL). For the Dirac equation with arbitrary vector and scalar Coulomb potentials, the only cases for which the relevant analytic solutions are known,  $-0.7 \lesssim r \lesssim 0.6$ . It is hence evident that the spin-orbit splittings extracted from experimental results are indeed small.

There is also evidence in light quark mesons and baryonic systems that the spin-orbit interaction is small [1]. In nonrelativistic models, meson and “two-body” baryon spin-orbit interactions are related and, for a specific class of baryons, the spin-orbit interaction is small for exactly the same reasons that it is small in mesons [1].

*III. A dynamical symmetry for the Dirac Hamiltonian.*—If we consider a system of a (sufficiently) heavy antiquark (quark) and light quark (antiquark), the dynamics may well be represented by the motion of the light quark (antiquark) in a fixed potential provided by the heavy antiquark (quark). Let us assume that both vector and scalar potentials are present. Then the Dirac Hamiltonian describing the motion of the light quark is

$$H = \vec{\alpha} \cdot \vec{p} + \beta(m + V_S) + V_V + M, \quad (2)$$

where we have set  $\hbar = c = 1$ ,  $\vec{\alpha}$ ,  $\beta$  are the usual Dirac matrices,  $\vec{p}$  is the three momentum,  $m$  is the mass of the light quark, and  $M$  is the mass of the heavy quark.

This one quark Dirac Hamiltonian follows from the two-body Bethe-Salpeter equation in the equal time approximation, the spectator (Gross) equation with a simple kernel, and a two quark Dirac equation, in the limit that  $M$  is large [17–19]. If the vector potential,  $V_V(\vec{r})$ , is equal to the scalar potential plus a constant potential,  $U$ , which is independent of the spatial location of the light quark relative to the heavy one, i.e.,  $V_V(\vec{r}) = V_S(\vec{r}) + U$ , then the Dirac Hamiltonian is invariant under a spin symmetry [20,21],  $[H, \hat{S}_i] = 0$ , where the generators of that symmetry are given by

$$\hat{S}_i = \begin{pmatrix} \hat{s}_i & 0 \\ 0 & \hat{s}_i \end{pmatrix}, \quad (3)$$

where  $\hat{s}_i = \sigma_i/2$  are the usual spin generators,  $\sigma_i$  the Pauli matrices, and  $\hat{s}_i = U_p \hat{s}_i U_p$  with  $U_p = \frac{\vec{\sigma} \cdot \vec{p}}{p}$ . Thus Dirac eigenstates can be labeled by the orientation of the spin, even though the system may be highly relativistic, and the eigenstates with different spin orientation will be degenerate.

For spherically symmetric potentials,  $V_V(\vec{r}) = V_V(r)$ ,  $V_S(\vec{r}) = V_S(r)$ , the Dirac Hamiltonian has an additional invariant algebra, namely, the orbital angular momentum,

$$\hat{L}_i = \begin{pmatrix} \hat{\ell}_i & 0 \\ 0 & \hat{\ell}_i \end{pmatrix}, \quad (4)$$

where  $\hat{\ell}_i = U_p \hat{\ell}_i U_p$  and  $\hat{\ell}_i = (\vec{r} \times \vec{p})_i$ . This means that the Dirac eigenstates can be labeled with orbital angular momentum as well as spin, and the states with the same orbital angular momentum will be degenerate. Thus, for example, the  $n_r p_{1/2}$  and  $n_r p_{3/2}$  states will be degenerate, where  $n_r$  is the radial quantum number.

Thus, we have identified a symmetry in the heavy-light quark system which produces spin-orbit degeneracies independent of the details of the potential. If this potential is strong, the heavy-light quark system will be very relativistic; that is, the lower component for the light quark will be comparable in magnitude to the upper component of the light quark. It is remarkable that nonrelativistic behavior of energy levels can arise for such fully relativistic systems.

This symmetry is similar to the relativistic symmetry [2] identified as being responsible for pseudospin degeneracies observed in nuclei [3,4]. In contrast to spin symmetry, pseudospin symmetry has the pairs of states  $[n_r s_{1/2}, (n_r - 1)d_{3/2}]$ ,  $[n_r p_{3/2}, (n_r - 1)f_{5/2}]$ , etc. degenerate, making the origin of this symmetry less transparent. The pseudospin generators are

$$\hat{S}_i = \begin{pmatrix} \hat{s}_i & 0 \\ 0 & \hat{s}_i \end{pmatrix}. \quad (5)$$

For pseudospin symmetry, the nuclear mean scalar and vector potential must be equal in magnitude and opposite in sign, up to a constant,  $V_V = -V_S + U$ . Relativistic mean field representations of the nuclear potential do have this property; that is,  $V_S \approx -V_V$  [22,23]. We will return later to the question of whether the relation  $V_V = V_S + U$  arises in QCD.

It has previously been observed that pseudospin symmetry improves with increasing energy of the states, for various potentials [2]. A similar behavior may be expected for spin symmetry, consistent with the experimental observations that spin-orbit splittings decrease for higher mass states [1,7].

The Dirac Hamiltonian (2) encompasses the effects of the OGE and Thomas precession spin-dependent terms customarily included in nonrelativistic models [1].

*IV. Experimental test.*—In the spin symmetry limit, the radial wave functions of the upper components of the Dirac wave function of the two states in the spin doublet will be identical, behaving “nonrelativistically,” whereas the lower components will have different radial wave functions. This follows from the form of the spin generators given in Eq. (3). The (1, 1) entry of the operator matrix is simply the nonrelativistic spin operator which relates the upper component of the Dirac wave function of one state in the doublet to the upper component of the other state in the doublet. Since this operator does not affect the radial wave function, the two radial wave functions must be the same. By contrast, the lower component wave function is

operated on by  $U_p$  which does operate on the radial wave function because of the momentum operator.

As an example, we consider  $D$  mesons, for which we determine scalar and vector potentials by matching the available spectral data, assuming a  $p_{3/2} - p_{1/2}$  splitting at the lower end of the range defined by the experimental value of  $-2 \pm 50$  MeV. This maximizes the wave function differences. In this realistic case,  $V_V \approx V_S + U$ , so the radial wave functions for the upper components are not exactly identical but are very close, whereas the radial wave functions for the lower components are very different, as seen in Fig. 1(a).

The momentum space wave functions for the upper components will also be very similar, again because the spin operator does not affect the wave function. However, since  $U_p$  depends only on the angular part of the momentum,  $\hat{p} = \frac{\vec{p}}{p}$ , it does not affect the radial momentum space wave function. In Fig. 1(b) we see that the radial momentum space wave functions are indeed very similar for the lower components as well. [The differences of the radial wave function upper components in both coordinate

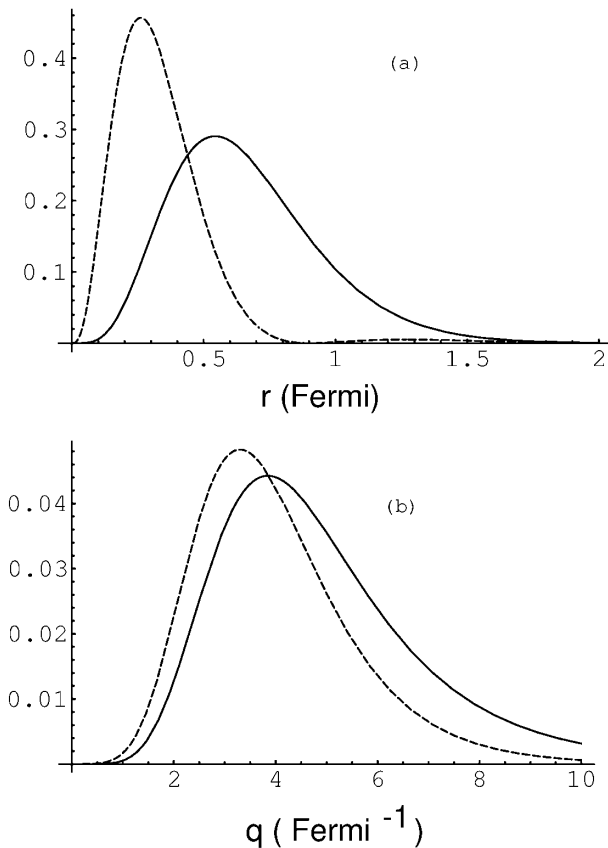


FIG. 1. (a) The square of the Dirac radial wave function of the lower component times  $r^2$ . (b) The square of the Dirac momentum space wave function of the lower component times  $q^2$ .  $p_{3/2}$  is the solid line and  $p_{1/2}$  is the dashed line. The wave functions are solutions of the Dirac equation [see Eq. (2)] with Coulomb potentials  $V_S(r) = \frac{\alpha_S}{r} + U_S$  and  $V_V(r) = \frac{\alpha_V}{r} + U_V$ , where  $\alpha_S = -1.279$ ,  $U_S = 506$  MeV,  $\alpha_V = -0.779$ ,  $U_V = 515$  MeV,  $m = 330$  MeV, and  $M = 1480$  MeV. This corresponds to a  $p_{3/2} - p_{1/2}$  splitting of  $-52$  MeV.

and momentum space are comparable to the differences shown in Fig. 1(b).] This prediction of the symmetry can be tested in the following experiment.

The annihilation  $e^+e^- \rightarrow D_0^*D_0^*$ ,  $D_0^*D_2^*$ , and  $D_2^*D_2^*$  allows for the extraction of the  $D_0^*$  and  $D_2^*$  electromagnetic static form factors and the  $D_0^*$  to  $D_2^*$  electromagnetic transition form factor. The photon interaction ensures that all radial wave functions of the light quark are accessed. When spin symmetry is realized, there are only two independent radial momentum space wave functions, which should enable the prediction of one of the three form factors in terms of the other two. This should enable the verification of the predictions of spin symmetry. On the other hand, non-relativistic models, with no lower components for the wave functions, have only one independent radial wave function, which will lead to the prediction of two of the form factors in terms of the remaining one. This might be too restrictive. The proposed experiment can be carried out at the Beijing Electron Positron Collider at an energy of approximately 1 GeV above the  $\psi(4040)$  peak in the final state  $DD\pi\pi$ .

An equivalent experiment for  $K$  mesons would involve detection of the  $KK\pi\pi$  final state, which has already been measured [24]. The wave functions of  $K$  mesons fitting the experimental spectrum show similar behavior to the  $D$  mesons, with the  $p_{3/2}$  and  $p_{1/2}$  wave functions even more similar than those in Fig. 1(b).

If  $B$  mesons do also exhibit spin symmetry, one can do equivalent experiments around 1 GeV above the  $Y(3S)$  peak at the SLAC, KEK, or CESR  $B$  factories.

*V. QCD origins.*—If such a dynamical symmetry can explain the suppression of spin-orbit splitting in the hadron spectrum, the question remains as to why it might be expected to appear in QCD. To address this, we first recall the ongoing argument as to whether confinement corresponds to a vector or scalar potential [25]. The first natural expectation was that confinement reflected the infrared growth of the QCD coupling constant, enhancing the color-Coulomb interaction at large distances; e.g., Ref. [26]. An involved two- (or multi) gluon effect has been proposed [27] to account for the origin of a scalar confining potential.

The existence of one or the other of these vector and scalar potentials is not necessarily exclusionary—they may both be realized. The arguments in Ref. [28] suggest further that they are related, with the scalar exceeding the vector by an amount which may be approximately constant as one saturates into the linear confining region at large separations. The ratio of the slopes of the two potentials in their common linear (confining) region is given by the square of the ratio of the QCD scale for growth of the coupling constant to the value of the mass gap of the condensate formation. This ratio may be expected to be of order one as both quantities are determined by the underlying QCD scale.

If the two potentials have similar slopes in the region outside that dominated by the color-Coulomb interaction, they would necessarily differ only by an approximately

constant value in that region. Thus, the origin of the dynamical symmetry may not be unreasonable, and may indeed be a natural outcome of nonperturbative QCD.

On the other hand, *identically* equal vector and scalar potentials, except for a constant difference, would appear to be coincidental. An ameliorating effect is that to produce an approximation to the spin symmetry of Eq. (2), this condition need only hold in regions where the wave functions are substantial.

The determination of QCD potentials, from models like the minimal area law, stochastic vacuum model, or dual QCD, and from lattice QCD, is hampered by the problem of rigorously defining the concept of a potential from QCD when one quark is light. It suffices to say that there is no agreement on the mixed Lorentz character of the potential even between two heavy quarks [29], where the potential can be rigorously defined, although lattice QCD results are consistent with simply a vector Coulomb and scalar linear potential [30].

*VI. Summary.*—The observation of “accidental” spin-orbit degeneracies observed in heavy-light quark mesons can be explained by a relativistic symmetry of the Dirac Hamiltonian which occurs when the vector and scalar potentials exerted on the light quark by the heavy antiquark differ approximately by a constant,  $V_V \approx V_S + U$ . Conversely, if future experiments determine that spin-orbit splittings are small not only for the lowest excited states in mesons but are small throughout the meson spectrum, this experimental fact dictates that the effective QCD vector and scalar potentials between a quark and antiquark are approximately equal up to a constant, which would be a significant observation about the nature of nonperturbative QCD. Furthermore, the approximate symmetry predicts that the spatial Dirac wave function for the spin doublets will be approximately equal in momentum space, a feature which can be tested in electron-positron annihilation. We have argued that  $V_V \approx V_S + U$  may occur in QCD, particularly for regions of space dominated by the light quark wave function. Work is in progress to extend this symmetry to purely light quark systems.

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\*Email address: prp@lanl.gov

†Email address: tgoldman@lanl.gov

‡Email address: gino@t5.lanl.gov

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