Curing Coupled-Bunch Instabilities with Uneven Fills

S. Prabhakar, J. D. Fox, and D. Teytelman
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
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A new, unified theoretical description of coupled-bunch instabilities in unevenly filled storage rings is presented. Uneven-fill longitudinal dynamics are explained in terms of two physical phenomena: fill-induced tune-spread damping and modulation coupling of strong even-fill eigenmodes. The latter is also present in the transverse plane. The analysis yields simple criteria for optimizing fill shapes to reduce the growth rates of the most unstable modes. Experimental results from the ALS and PEP-II are shown to be in good agreement with the theory.

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Bunches of charged particles in circular accelerators oscillate in potential wells (buckets) created by transversely focusing magnets and longitudinally focusing radiofrequency (RF) accelerating fields. The bunch oscillations are coupled by excitation of long-lasting wake fields in trapped ion/photoelectron clouds, cavitylike structures, and resistive walls in the vicinity of the beam. When the beam current I_0 exceeds some threshold, this coupling overwhelms natural radiation damping, causing unstable growth of oscillations and severe degradation of accelerator performance [1].

Coupled-bunch instabilities are conventionally cured using the following (see [2] and references therein): (a) Minimization of beam impedance. (b) Landau damping, i.e., damping of coherent oscillations via a spread in the resonant frequencies of individual oscillators. (c) Active feedback.

Studies of coupled-bunch instabilities [3,4] have traditionally considered even bunch spacings and equal bunch currents (even fills), since the uneven-fill eigenvalue problem has no general analytic solution. Approaches to the uneven-fill problem have included numerical computation of the eigenvalues of the $N \times N$ bunch coupling matrix in the general N-bunch case [5], and an upper bound on growth rates for an even fill with one gap [6]. The former could be used to search the h-dimensional space of fills for the most stable shape, where h is the number of RF buckets. Unfortunately, this is impractical in accelerators which store hundreds or thousands of bunches.

Analytical complexity notwithstanding, empirically selected uneven fills have successfully raised instability thresholds at the Cornell Electron Storage Ring [7], the SPEAR storage ring [8], and the Advanced Photon Source [9]. Recently, the longitudinally stabilizing effect of interbunch tune spreads, arising from RF cavity transients induced by gaps in the fill, has been noted [10,11]. Though important in large rings, this effect is weak in small rings with revolution frequencies beyond the tunable range of the cavities.

This Letter presents a new, general theoretical framework for coupled-bunch instabilities in unevenly filled rings. Uneven-fill longitudinal dynamics are explained in terms of two physical phenomena: damping from fill-induced tune spreads and modulation coupling of strong even-fill eigenmodes (EEs). These concepts are utilized to devise a simple algorithm for shaping fills to cure instabilities. Experimental results from the ALS [12] and PEP-II [13] are shown to validate the theory.

Consider N identical bunches, equally spaced along a beam orbit of circumference L, moving at a constant speed c. This azimuthally periodic steady-state charge distribution has spatial harmonics at multiples of $2\pi N/L$. In a stationary impedance source, this produces an excitation at temporal frequencies that are multiples of the bunch frequency $N\omega_0$, where $\omega_0 = 2\pi c/L$ is the revolution frequency. If the excitation at some frequency $kN\omega_0$ excites an impedance resonance, then a steady-state wake force (so called because it arises from the steady-state charge distribution) is generated at $kN\omega_0$. Since this wake force has the same periodicity as the bunches, it distorts the potential wells of all bunches equally, and thus produces a constant shift in their oscillation frequencies (constant tune shift). To first order, this has no effect on the stability of coupled-bunch oscillations.

On the other hand, tune shifts that vary from bunch to bunch would reduce interbunch coupling, and thus have a stabilizing effect. If there is a large impedance at some revolution harmonic $l\omega_0$, where l is not a multiple of N, one could shape an uneven fill so as to maximize the Fourier component of the beam spectrum at that frequency. The resulting wake response at $l\omega_0$ would induce a tune spread by distorting the potential wells of different bunches differently. This instability cure is henceforth referred to as fill-induced tune-spread damping.

Now consider a small multibunch oscillation at a spatial frequency $2\pi l/L$ and a temporal frequency ω_z, such that the bunch amplitudes have the form $a_n(t) = e^{j[2\pi(ln/N)+\omega_z t]}; n = 0, \ldots, N-1$. This excites the impedance sources at $l\omega_0 + \omega_z$. If the impedances are linear and time invariant, the resulting wake force at $l\omega_0 + \omega_z$ maintains the phase relationship of the bunches. The bunches are thus in an eigenmode. In fact, the EEs are nothing but the N Fourier vectors $v_l = [1\, e^{jl\theta}\, e^{2jl\theta} \cdots e^{(N-1)jl\theta}]^T; \theta = 2\pi/N; l = 0, \ldots, N-1$. The in-phase

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component of the wake force produces a tune shift, and the quadrature component acts as a damping or antidamping term, depending on its sign.

We now outline an instability cure based on the second phenomenon mentioned above, namely, modulation coupling. Let us assume that EE l is unstable because the quadrature wake at $l\omega_0 + \omega_z$ acts on each bunch as an antidamping force of the form $F = k\dot{x}$. Let EE m be stabilized by a damping wake force at $m\omega_0 + \omega_z$. If they coexisted, these two EEs would periodically coincide in phase at positions separated by their beat period $L/(l - m)$, i.e., every $n = N/(l - m)$ buckets. Thus, if we filled only every nth bucket, a coupled-bunch oscillation at EE l would be indistinguishable from one at EE m. Any oscillation that excited the antidamping wake at $l\omega_0 + \omega_z$ would necessarily also excite the damping wake at $m\omega_0 + \omega_z$. The two wakes would interfere destructively at each bunch, and the growth rate of the unstable EE would be reduced by an amount equal to the damping rate of the stable EE. This application of modulation coupling would work even if the impedances were broadband, because EE $m + 1$ would damp EE $l + 1$, $m + 2$ would damp $l + 2$, and so on (the beat frequency is the same).

In general, modulation coupling between EEs l and $l + k$ (or $l - k$) increases with the size of the line at $k\omega_0$ in the beam spectrum, i.e., the Fourier component of the fill shape at the beat frequency. Mathematically, this is because impedances are excited by the product of the oscillation-coordinate signal and the beam current profile (fill shape). When two signals are multiplied, their Fourier components shift to the sum frequencies. A special case of modulation coupling is described in [14].

In the absence of wake fields, all longitudinal eigenmodes have the same eigenvalue $-d_r + j\omega_s$, where d_r is the radiation damping rate and ω_s is the natural oscillation frequency. From here on, we shall use the word "eigenvalue" only for the eigenvalue shift produced by wake fields. An eigenvalue λ is unstable if the growth rate $\mathrm{Re}(\lambda - d_r)$ of the eigenmode is positive.

The longitudinal arrival-time error τ_n of the nth bunch centroid is given by

$$\ddot{\tau}_n + 2d_r\dot{\tau}_n + \omega_s^2\tau_n = -\frac{\alpha e}{ET_0} V_n \,, \qquad (1)$$

where the dot denotes differentiation with respect to time, α is the momentum compaction factor, E/e is the nominal beam energy in volts, $T_0 = 2\pi/\omega_0$ is the revolution period, and $V_n(t)$ is the total wake voltage seen by bunch n. Assuming infinitesimally short bunches,

$$V_n(t) = \sum_{p=-\infty}^{\infty} \sum_{k=0}^{N-1} q_k W[t_{n,k}^p + \tau_n(t) - \tau_k(t - t_{n,k}^p)],$$

where q_k is the charge of bunch k, $t_{n,k}^p = (n - k - pN)T_b$, T_b is the bunch spacing (T_0/N), and the longitudinal wake function $W(t)$ equals zero when $t < 0$. The total ring impedance is $Z(\omega) = \int_{-\infty}^{\infty} W(t)e^{-j\omega t}\,dt$.

One could linearize the above equations and derive an $N \times N$ matrix describing the coupling of every bunch to every other. This matrix would be far from sparse. In addition, it would offer no obvious insights into the dynamics of uneven fills. We shall instead use the EE basis: $\boldsymbol{\nu}_m = \sum_{n=0}^{N-1} \tau_n e^{-j2\pi(mn/N)}$, $\tau_n = \frac{1}{N}\sum_{m=0}^{N-1} \boldsymbol{\nu}_m e^{j2\pi(mn/N)}$. As we shall see, this is a natural basis for studying fill shape effects. We assume eigenmodes of the form $\tau_k = B_k e^{j\Omega t}; k = 0, \ldots, N - 1$. Similarly, $\boldsymbol{\nu}_m = D_m e^{j\Omega t}$; $m = 0, \ldots, N - 1$. If $d_r \ll \omega_s$ and $|\Omega - \omega_s| \ll \omega_s$, the projection of τ and V onto the lth EE gives

$$\begin{aligned}\dot{\boldsymbol{\nu}}_l + (d_r - j\omega_s)\boldsymbol{\nu}_l &= \frac{\alpha e f_{\mathrm{rf}}}{2EQ_s} \sum_{m=0}^{N-1} I_{l-m} Z_{lm}(\omega_s)\boldsymbol{\nu}_m \\ &= \sum_{m=0}^{N-1} A_{lm}\boldsymbol{\nu}_m \,, \end{aligned} \qquad (2)$$

where f_{rf} is the RF frequency, $Q_s = \omega_s/\omega_0$ is the tune, and the amplitude of the pth revolution harmonic in the beam spectrum is $I_p = \sum_{k=0}^{N-1} i_k e^{-j2\pi(pk/N)}$; $i_k = q_k/T_0$.

$$Z_{lm}(\omega) = Z^{\mathrm{eff}}(l\omega_0 + \omega) - Z^{\mathrm{eff}}[(l - m)\omega_0];$$

$$Z^{\mathrm{eff}}(\omega) = \frac{1}{\omega_{\mathrm{rf}}} \sum_{p=-\infty}^{\infty} (pN\omega_0 + \omega)Z(pN\omega_0 + \omega) \,. \qquad (3)$$

If the fill is even, $I_k = 0$ for $k \neq 0$, and the off-diagonal elements of the coupling matrix A disappear. The diagonal terms yield the well-known equations [4] for even-fill eigenvalues: $\lambda_l = \frac{\alpha e f_{\mathrm{rf}}}{2EQ_s} I_0 Z^{\mathrm{eff}}(l\omega_0 + \omega_s); l = 0, \ldots, N - 1$. Most commonly used fill shapes are close to even ($I_k/I_0 \approx 0$ for most k). Also, Z_{lm} has the same order of sparseness as Z^{eff}. Since the off-diagonal terms are proportional to $I_{l-m}Z_{lm}$, it is apparent from Eq. (2) that the EE basis makes A sparse. In other words, the A-matrix is sparse unless both I_k and Z^{eff} are dense, which is not a very common situation.

Modulation coupling arises from terms of the form $I_{l-m}Z^{\mathrm{eff}}(l\omega_0 + \omega_s)$, which reflect the fact that the longitudinal beam signal is proportional to $i_k\tau_k$.

If $I_k Z^{\mathrm{eff}}(k\omega_0)$ is negligible for all $k \neq 0$, the modulation coupling terms are the only manifestation of fill unevenness. In addition, if $Z(\omega)$ is non-negligible only at n revolution harmonics, where $n \ll N$, we can approximate the most unstable eigenvalues by those of an equivalent A-matrix consisting only of the n corresponding rows and columns. This is a great simplification in large rings with hundreds or thousands of bunches. If we now create a fill so that only I_{l-m} is large, where EE l is the most unstable mode and l is the most stable, we get an equivalent A-matrix that is diagonal except for the coupling between $\boldsymbol{\nu}_m$ and $\boldsymbol{\nu}_l$. This reduces the eigenvalue problem to a quadratic equation with the solution

$$\lambda = \frac{1}{2}(\lambda_l + \lambda_m) \pm \frac{1}{2}\sqrt{(\lambda_l - \lambda_m)^2 + 4C_{l-m}^2\lambda_l\lambda_m} \,, \qquad (4)$$

where C is a modulation parameter defined by $C_p = |I_p|/I_0$. If $C_{l-m} = 0$, the even-fill eigenvalues λ_l and λ_m

are unperturbed. As C_{l-m} approaches unity (it can never exceed 1), one eigenvalue approaches zero and the other approaches $\lambda_l + \lambda_m$. This yields the maximum damping.

It can be shown that the following algorithm maximizes C_p, given the beam current I_0 and the maximum allowable bunch current $i_{\max}$: (1) For each bucket n in the N-bucket pattern, calculate a corresponding "weight" $\cos(2\pi \frac{pn}{N})$. (2) Pick the B buckets with the highest weight, and fill each of them to the same current $i_{\max}$, where $B = I_0/i_{\max}$. Leave the remaining buckets empty.

Figure 1 shows two example fills which maximize C_4 when $h = 1000$, $I_0 = 500$ mA, and $i_{\max} = 1$ mA, 2 mA.

Tune-spread damping arises from terms of the form $I_{l-m}Z^{\text{eff}}[(l - m)\omega_0]$ in Eq. (2). The tune shift of bunch k relative to the mean tune is

$$\delta\omega_s^k = j\,\frac{\alpha e f_{\text{rf}} I_0}{EQ_s} \sum_{l=1}^{N-1}\left[\left(\frac{I_l}{I_0}\right) Z^{\text{eff}}(l\omega_0) e^{j2\pi kl/N}\right]. \quad (5)$$

$\delta\omega_s$ is purely real, since the real part of the summand is an odd function of l, with period N. All unstable modes are damped by the interbunch tune spread.

If n is the most unstable EE, Eq. (5) indicates that a good strategy would be to design a fill that optimizes $C_n = |I_n|/I_0$. The best value of C_n for damping EE n is different from the optimum for other EEs: (A) Tune-spread damping of EEs other than n can be calculated in the usual way [15], if they are not coupled to other prominent EEs by modulation coupling or by tune-spread terms on the nth diagonal of A. (B) The damping of EE n is larger than that of other modes, since the combination of tune spread and fill unevenness introduces coupling between $\boldsymbol{\nu}_n$ and $\boldsymbol{\nu}_{N-n}$. If tune-spread damping and coupling to $\boldsymbol{\nu}_{N-n}$ are the only significant effects and $\lambda_{-n} \approx -\lambda_n^*$ [16], then the variation of λ_n with fill fraction is shown in Fig. 2 (numerical computation, assuming use of the above-mentioned fill-optimizing algorithm). Dashed lines show the evolution of λ_n from a few even-fill starting points. This figure is symmetric about both axes. EE n is best damped by maximizing C_n, i.e., by minimizing the fill fraction.

The theoretical predictions of tune-spread damping were first tested at the ALS, where the diagnostic capabilities of a digital longitudinal feedback system [17,18] were utilized to measure the eigenvalues (growth rates and coherent tune shifts) of all unstable EEs simultaneously. The measurement technique is described in [19,20]. In most cases, only two of the 328 ALS modes were unstable: modes 204 and 233. The effective impedance at $233 f_0$ was used to create a tune spread by maximizing C_{233} [see Eq. (5)].

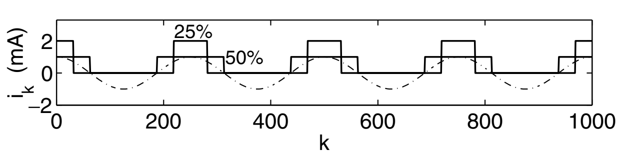

FIG. 1. Illustration of fill optimization. $h = 1000$, $I_0 = 500$ mA. Solid lines: 50% fill and 25% fill maximize C_4 for $i_{\max} = 1$ mA, 2 mA. Dash-dotted line: Reference sinusoid at $4\omega_0$.

A baseline even-fill instability measurement was first made at $I_0 = 172$ mA. This gave the following eigenvalues [21]: $\lambda_{204} = (0.47 \pm 0.02) - (0.05 \pm 0.03)j$ ms^{-1} and $\lambda_{233} = (0.61 \pm 0.02) - (1.16 \pm 0.03)j$ ms^{-1} (assuming that $d_r = 0.074$ ms^{-1}). It is evident from Fig. 2 that "tune-spread fills" with fill fractions less than 60% almost completely damp the primary target mode, which is EE 233 in this case. Thus, any residual instability in the tune-spread fill must correspond to the tune-spread-damped mode 204.

Although many methods exist for calculating the instability growth rate once the bunch tune distribution is calculated [15,22], we use numerical computation of the eigenstructure of the mode coupling matrix, since it is the most exact. For this we need to know the shunt impedance R_s, the resonant frequency f_r, and the quality factor Q of the two cavity modes responsible for the measured values of λ_{204} and λ_{233}. If the effective impedance corresponding to an even-fill eigenvalue is $R + jX$, then the shunt impedance of the cavity mode is given by $(\frac{f_r}{f_{\text{rf}}}R_s - R)^2 + X^2 = [\frac{f_r}{f_{\text{rf}}}R_s]^2$ [22]. By correlating this result with data on ALS cavity modes [23], we get (nominally) $R_s = 11.36$ kΩ, $f_r = 1809.69$ MHz, and $Q = 2900$ for EE 204 and $R_s = 43$ kΩ, $f_r = 2852.92$ MHz, and $Q = 9149$ for EE 233. The numerical calculation then gives us an eigenvalue of $(0.1 \pm 0.04) + (1.62 \pm 0.06)j$ ms^{-1} for the Landau-damped mode 204. Error bars are calculated by assuming that errors in measured eigenvalues arise from fluctuations in f_r. Note that the real part of the most unstable eigenvalue is 6 times smaller than in the even-fill case. The measured eigenvalue for a 175-mA beam with $C_{233} = 0.67$ is $(0.09 \pm 0.003) + (1.63 \pm 0.005)j$ ms^{-1}, in agreement with the theoretical prediction.

Figure 3(a) shows the estimated cavity-induced growth rates in the PEP-II Low Energy Ring (LER) at the nominal bunch spacing of 8.4 ns ($N = 873$), when $I_0 = 1$ A. The estimate is based on off-line cavity measurements [24]. The two largest cavity resonances are expected to drive bands of modes centered at 93.1 MHz (EE 683) and 105 MHz (EE 770) unstable. They also stabilize corresponding bands at 25.9 MHz (EE 190) and 14 MHz (EE 103). Here the best modulation-coupling cure would

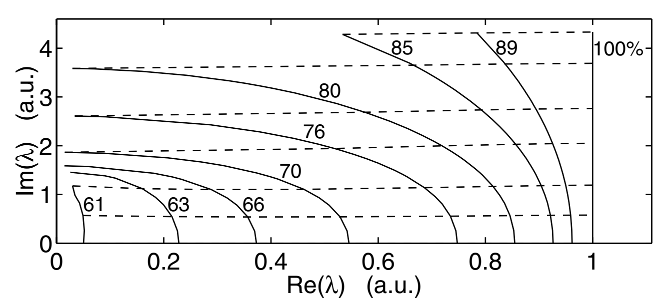

FIG. 2. Graphical look-up table for fill-induced damping of eigenvalue of unstable longitudinal EE n as C_n is increased from 0 (100% of ring filled) to 0.5 (61% filled). Dashed lines: Evolution of λ_n from a few even-fill starting points.

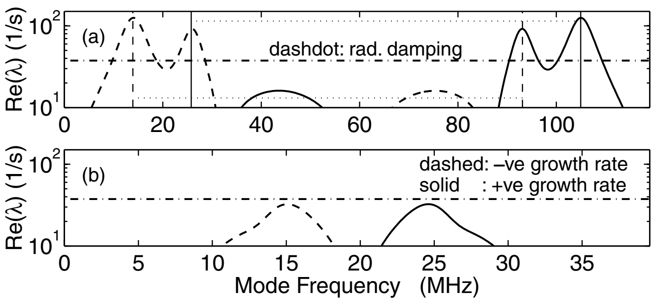

FIG. 3. PEP-II LER expected modal growth rates vs mode frequency ($l\omega_0 + \omega_s$) at $I_0 = 1$ A for the following: (a) Even fill at nominal 8.4 ns spacing (feedback required). (b) Even fill at 3×8.4 ns spacing (stabilized by modulation coupling).

be to couple the modes around 105 MHz to those near 25.9 MHz by maximizing C_{580}, i.e., C_{293} ($C_p = C_{N-p}$). This automatically couples 93.1 MHz to 14 MHz. In general, if $\boldsymbol{\nu}_a$ couples to $\boldsymbol{\nu}_{N-b}$, then $\boldsymbol{\nu}_b$ couples to $\boldsymbol{\nu}_{N-a}$. Maximizing C_{291} should work as well, since ω_0 is small compared to the bandwidths of the resonances. This is easily achieved by filling every third nominally spaced bucket, since $291 = 873/3$. The calculation illustrated in Fig. 3(b) shows that such a fill should be stable at 1 A.

Modulation coupling was expected to raise the instability threshold from 305 mA (nominal spacing) to 1.16 A (3× nominal spacing). The measured thresholds are 350 and 660 mA, respectively. The improvement is significant, though smaller than expected, probably because the impedance resonances are located 3–5 MHz away from their expected positions.

Since mixing of oscillation-coordinate and fill-shape signals occurs in all planes, modulation coupling also affects transverse oscillations. Similarly, higher bunch-shape oscillations can also be damped by modulation coupling. Although bunches that are axially centered in the beam pipe induce no transverse steady-state wake, transverse tune-spread damping might be achieved by shifting the beam orbit (or a resonant structure) transversely.

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