Josephson Effect without Superconductivity: Realization in Quantum Hall Bilayers

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We show that a quantum Hall bilayer with the total filling $\nu = 1$ should exhibit a dynamical regime similar to the flux flow in large Josephson junctions. This analogy may explain a conspicuous peak in the interlayer tunneling conductance [Phys. Rev. Lett. **84**, 5808 (2000)]. The flux flow is likely to be spatiotemporally chaotic at low-bias voltage, which will manifest itself through broadband noise. The peak position can be controlled by an in-plane magnetic field.

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Motivation.—In the classic realization of the Josephson effect, the dynamical variable is the difference between phases of pairing wave functions in two superconductors. The coupling of this variable to electromagnetism is dictated by general principles of gauge invariance. When pairs tunnel from one superconductor to the other, there is a coherent transport of the charge conjugate to this phase difference, and the Josephson effect follows. One is led to exactly the same equations, except with $2e \to e$, when one has a condensate of excitons—bound states of particles and holes—extending across a junction. This is because an exciton, despite its overall neutrality, does couple to the gauge potential *difference* on the two sides of the junction, as a unit charge e. In the exciton condensate the coherent charge transfer is actually simpler than in a superconductor: it requires a single tunneling act instead of two [1]. Notwithstanding the hidden connotations, the excitonic condensate can be the ground state of the junction [2,3]. One might be optimistic that in suitable materials exciton Josephson effects should be common and could exist at much higher temperatures than conventional superconductivity, since the fundamental particle-hole interaction is attractive.

In this Letter we argue that the first observation of such effects has already occurred, albeit in a low-temperature device: a bilayer quantum Hall system with total filling factor $\nu = 1$. For small interlayer separations this system spontaneously develops an order parameter $\Delta \equiv \langle \psi_1^\dagger \psi_2 \rangle \propto e^{i\phi}$, where $\psi_i^\dagger$ is the electron creation operator in layer i [4]. When expressed in terms of fermionic operators $\chi_\uparrow^\dagger \equiv \psi_1^\dagger$, creating an electron in layer 1, and $\chi_\downarrow^\dagger \equiv \psi_2$, creating a hole in layer 2, Δ takes the familiar Bardeen-Cooper-Schrieffer form $\langle \chi_\uparrow^\dagger \chi_\downarrow^\dagger \rangle$ and enables one to identify ϕ with the phase of the condensate of bound electron-hole pairs. We briefly mention that another fruitful analogy is obtained by treating the layer index as a pseudospin degree of freedom [4]. Along that route, nonzero Δ signifies pseudospin ferromagnetism, while ϕ determines the direction of the in-plane component of the "magnetization."

The Josephson effect in the excitonic superfluid was originally discussed in Ref. [3] and, more recently, in Ref. [5]. Its appearance in the quantum Hall bilayers was also predicted [6,7] but not immediately accepted [8]. Yet a complete formal analogy to the Josephson junction does hold in the low-energy limit, where ϕ obeys a damped-driven sine-Gordon equation of motion, precisely as the phase variable in a large-area Josephson junction. The same equation governs many other physical systems, e.g., sliding charge-density waves and growing crystals, which implies broad relevance of the issues discussed below.

The aforementioned likely candidate for the Josephson phenomenon in a bilayer quantum Hall system is a zero-bias peak in the differential tunneling conductance dI/dV discovered by Spielman *et al.* [9]. We interpret this peak as an analog of the flux-flow resonance in a Josephson junction. The discovery of such a resonance in 1964 by Eck *et al.* [10] was one of the first verifications of the Josephson effect in conventional superconductors. The experiment of Spielman *et al.* may well play a similar role for the quantum Hall bilayers.

Two theoretical papers motivated by this experiment have appeared [11,12]. Our independent analysis has some overlap and some differences with those works. We start with a perturbative calculation of $I(V)$. One of its predictions is the dependence of the peak position on the external in-plane magnetic field. In principle, that can be used to map out the dispersion relation of the collective mode [4] analogous to Josephson plasma oscillations. However, we point out that the perturbation theory solution is unstable for the low damping characteristic of low-temperature quantum Hall systems, which leads to a spatiotemporally chaotic flux flow and large tunneling current fluctuations. Constructing the theory for such a complicated dynamical regime remains a challenge. For its numerical study in a discretized formulation (Frenkel-Kontorova model), see Ref. [13].

Basic equations.—The general form of the equation of motion for ϕ can be established from considerations of symmetry and gauge invariance. To clarify its physical meaning we choose another approach. We start with the charge conservation equation $e\partial_t n + \partial_k j_k + j_{\rm tun} = 0$. The local excess n of the exciton density causes charge imbalance between the layers. We can express it in terms of the local chemical potential difference μ and the capacitance c per unit area: $n = c\mu/e^2$. The Josephson relation entails $\partial_t \mu = \hbar \partial_t^2 \phi$. The in-plane exciton current j consists of the supercurrent $j_s = (e\rho_s/\hbar)\partial_x \phi$ and the converted quasiparticle current $j_{qp} = -\partial_x \mu / e\rho_{xx}$, where ρ_s is the condensate phase stiffness and ρ_{xx} is the resistivity of uncondensed quasiparticles. The Josephson tunneling current is $j_{\text{tun}} = (e/\hbar) n_0 \Delta_{\text{SAS}} \sin(\phi - Qx_1)$, where n_0 is the average electron density per layer, Δ_{SAS} is the tunneling strength, and $Q = eBd/\hbar c$ is the wave vector imposed by the in-plane component B of the external magnetic field along the x_2 axis. (Its presence is necessitated by gauge invariance.) d is the interlayer separation. Assembling all the terms, we obtain the equation for ϕ (cf. Ref. [4]):

$$(\partial_t^2 - \beta \partial_t \partial_x^2 - \partial_x^2)\phi + \sin(\phi + \phi_0 - Qx_1) = 0 . \tag{1}$$

Here we expressed distances in units of "Josephson penetration length" $\lambda_J = (\rho_s / n_0 \Delta_{\text{SAS}})^{1/2}$ and frequencies in units of "Josephson plasma frequency" $\omega_J = v/\lambda_J$. The velocity parameter $v = (e/\hbar)(\rho_s/c)^{1/2} \sim 0.1 e^2/\kappa\hbar$ is the one that enters the dispersion relation [4,6] $\omega^2(q) = \omega_J^2 + v^2 q^2$ of the plasmon, and parameter $\beta = (1/4\pi)(\hbar\omega_J/\rho_s)(\rho_{xx} e^2/h)$ controls dissipation [14]. Finally, we included a random phase shift ϕ_0 to represent disorder (see below). By using the data reported in Ref. [9] and theoretical results reviewed in Ref. [4], we arrived at rough estimates $\lambda_J \sim 5\ \mu\text{m}$, $\omega_J \sim 10^{10}\ \text{s}^{-1}$, and $\beta \sim 0.01$. Boundary conditions for Eq. (1) depend on the sample and measurement geometry. We consider the case where each layer is a square with side L, the contact to layer 1 is along the side $x_1 = -L/2$, $-L/2 < x_2 < L/2$, and the contact to layer 2 is along the side $x_1 = L/2$. This deviates somewhat from the setup used in Ref. [9] but should be inconsequential as long as the bottleneck for the current flow is the interlayer tunneling not the sheet conductivity. This is indeed the case experimentally because the inequality $V \gg I\rho_{xx}$ is satisfied (V is the voltage difference between the contacts). Under such conditions it is also permissible to choose $\phi_L = \phi_R = Vt$, and use

$$I(t) = \int dx_2 [\partial_1 \phi_R(t) - \partial_1 \phi_L(t)]. \tag{2}$$

to calculate the tunneling current. Here $\phi_{L,R} \equiv \phi(x_1 = \pm L/2, x_2)$ and our units of voltage and electric current are $V_0 = \hbar\omega_J/e$ and $I_0 = e\rho_s/\hbar$, respectively.

Before we proceed to the calculations let us explain the origin of the term "flux flow." Let us consider the case $\phi_0 = \partial_t \phi = V = 0$ and focus on the limit $Q \gg 1$, where the stable (ground-state) solution of Eq. (1) corresponds to an almost uniform phase distribution $|\partial_1 \phi| \ll 1$. There is an equivalent alternative formulation in terms of a shifted phase $\theta \equiv \phi - Qx_1$ in which Q disappears from the argument of the sine but reemerges in the boundary conditions for $\partial_1 \theta = \partial_1 \phi - Q$. In this formulation the ground state θ varies rapidly and almost linearly in space, which can be described as the $2\pi/Q$-periodic lattice of 2π solitons. A nonequilibrium state for $V \neq 0$, where the phase increases uniformly with the rate V, can then be visualized as a uniform sliding of the soliton lattice—hence the term flux flow.

Perturbation theory.—The perturbative solution of Eq. (1) for $\phi_0 = 0$ is readily done in terms of the Green's function

$$G(\omega, k) = [\omega^2 + i\alpha(k)\omega - k^2]^{-1}, \qquad \alpha(k) \equiv \beta k^2. \tag{3}$$

The dc current $\bar{I}$ is then obtained by averaging Eq. (2) over time. The full expression is somewhat cumbersome but for L in the range $1 \ll L \ll \bar{I}^{-1}$ it reduces to [10]

$$\bar{I} = \frac{L^2}{2} \frac{\alpha(Q)V}{(V^2 - Q^2)^2 + \alpha^2(Q)V^2} . \tag{4}$$

We estimate $L \sim 50$ and $\bar{I} \sim 10^{-6}$ for the conditions of Ref. [9], so L is in the required range. From Eq. (4) we see that the resonance arises when the velocity V/Q of the soliton train matches the plasmon velocity v ($v = 1$ in the adopted dimensionless units). Under this condition the power dissipation in the system reaches a maximum. More generally, the resonance condition is $eV = \hbar\omega_0(Q)$, where $\omega_0(Q)$ is the $\Delta_{\text{SAS}} \to 0$ limit of the plasmon dispersion relation (in which the plasmon becomes the Goldstone mode). The last conclusion was reached independently in Refs. [11] and [12].

Let us turn to the disordered case, $\phi_0 \neq 0$. From Eq. (1) and the identity $\sin z = \text{Im} e^{iz}$, one can see that the random phase factor $e^{i\phi_0}$ plays the same role for the disordered system as the periodic phase factor e^{iQx_1} for the clean system. The Fourier expansion of $e^{i\phi_0}$ can be thought of as a set of "diffraction gratings" with different wave vectors k, resonating whenever $V = k$. As a result, the resonance is broadened in proportion to the breadth of the Fourier spectrum of the following correlator:

$$\sigma(\mathbf{x}) = \langle e^{-i\phi_0(\mathbf{x})} e^{i\phi_0(0)} \rangle, \tag{5}$$

and the appropriate generalization of Eq. (4) is

$$\bar{I} = -\frac{L^2}{2} \text{Im} \int \frac{d^2k}{(2\pi)^2} \tilde{\sigma}(\mathbf{k} - \mathbf{Q}) G(V, k) . \tag{6}$$

To evaluate the integral we assume that the random phase field ϕ_0 is mainly due to static randomly positioned and randomly oriented bound vortex pairs in the exciton condensate. In the context of the pseudospin ferromagnet analogy mentioned earlier [4], such vortices were recognized as merons, the topological defects of the $O(3)$ nonlinear σ model, and a remarkable fact was established: each meron possesses an overall electric charge $e/2$ concentrated near its core. The competition of the Coulomb repulsion between the like electric charges and attraction between oppositely charged vortices selects the optimal size a of meron pairs; a remains finite below the roughening transition [15] temperature $T_r = 8\pi\rho_s/k_B$. From our point of view, in the context of the Josephson junction analogy, the meron pairs correspond to misaligned Abrikosov vortices trapped in the junction. The I-V characteristic of such junctions was investigated by Fistul and Giuliani [16]. These authors also attempted to go beyond the perturbation theory but that part of their calculation is suspect for reasons that will become clear shortly.

Within the chosen model of disorder, we obtain

$$\tilde{\sigma}(k) = Aa^2(ka)^{\gamma-2}, \qquad \gamma \equiv \pi n_m a^2, \tag{7}$$

$$A \simeq 2\pi\gamma, \qquad \gamma \ll 1, \tag{8}$$

$$A \simeq 2\pi \ln(ka)^{-1}, \qquad \gamma = 2, \tag{9}$$

where n_m is the average density of the meron pairs. Substituting the above formulas into Eq. (6) yields, for $Q = 0$ and $V_* \ll V \ll \beta^{-1}$,

$$\bar{I} = I_*(V_*/V)^{2-\gamma}, \qquad I_* = L^2\alpha(V_*)V_*. \tag{10}$$

Here $V_* = [A(V_*)a^\gamma/8\beta]^{1/(5-\gamma)}$ is determined from the condition that the variance of ϕ, which is given by

$$\langle\phi^2\rangle = \frac{1}{2}\int \frac{d^2k}{(2\pi)^2}\,\tilde{\sigma}(\mathbf{k} - \mathbf{Q})\,|G(V,k)|^2, \tag{11}$$

becomes of the order of 1. At $V < V_*$ the premise of the perturbation theory ($\phi \ll 1$) is violated and formula (10) is invalid. However, $V = V_*$ is not the true boundary of the perturbative domain. Below we will show that the perturbation theory actually breaks down in a wider range of voltages, $V < V_{**}$, where $V_{**} \gg V_*$ for small β.

In the following we focus on the case $\gamma \approx 2$ (not an unreasonable value [17]), where the Fourier spectrum of $e^{i\phi_0}$ is extremely broad [see Eq. (7)]. The flux-flow resonance is equally broad so that it shows up as a plateau in $\bar{I}(V)$ (see Fig. 1). With the chosen value of γ we get the expression (physical units temporarily restored)

$$I_* \sim (e\rho_s/\hbar)\,(L^2a^2/\lambda_J^4) \tag{12}$$

for the height of the plateau. Similar expressions were also derived in Refs. [11] and [12].

Even though the response at low V is beyond the reach of the perturbation theory, the "Josephson critical current," which is a zero-bias parameter, can nevertheless be calculated. The collective-pinning theory of Vinokur and

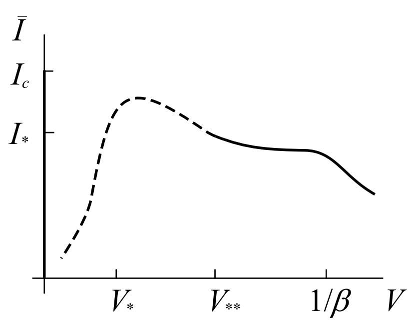

FIG. 1. I-V characteristic given by the perturbation theory with $\gamma \approx 2$ (solid line). The dashed line is a conjecture.

Koshelev [18] yields (for $\gamma = 2$)

$$I_c = 2(e\rho_s/\hbar)\,(L^2a^2/\lambda_J^4)\ln^2(\lambda_J/a). \tag{13}$$

Reminiscent of the inequality between the coefficients of static and kinetic friction, I_c exceeds I_*. Note that I_c is essentially a static quantity. It need not coincide with the $V \to 0$ limit of the dynamical response formulas similar to Eq. (6). The overall dependence of $\bar{I}$ on V is likely to appear the way it is shown in Fig. 1.

Choosing $\gamma \approx 2$ enables us to reproduce the plateau feature in the experimental data [9] (the peak in dI/dV corresponds to a plateau in $\bar{I}$ vs V), but the theoretical estimate of I_* is off by 2 orders of magnitude. However, this estimate is highly unreliable in view of large uncertainties in the parameters of the model and of their strong renormalization by thermal fluctuations, known from the theory of the roughening transition [15].

Instability of a tachyonic flux flow.—It is known, although not widely appreciated, that the perturbation theory solution for the case $\phi_0 = 0$—in the form of a moving soliton lattice—can become unstable when the lattice velocity exceeds the velocity of Josephson plasmon, $V/Q > 1$. The instability can be understood as a parametric resonance driven by the external source of frequency V and wave vector Q. Indeed, if $V > Q$, then kinematics allows a simultaneous excitation of two counterpropagating plasma waves, with wave vectors $k_\pm = (Q \pm V)/2$.

For large Q the instability appears when (cf. Ref. [19])

$$G^{-1}(k_+, k_+)G^{-1}(-k_-, k_-) < 1/4, \qquad V > Q, \tag{14}$$

and so the perturbation theory solution on the "tachyonic" side $V > Q$ is stable only if $V > V_{**} = (4/\beta)^{1/3}$ ($Q \ll V_{**}$ is assumed). This condition is much more restrictive than the naive $V - Q \gg Q^{-1}$ derived from the criterion $|\phi| \ll 1$.

Very close to the threshold V_{**}, the parametric resonance described above produces a small modulation of the uniform soliton train [19], but as V moves closer to Q other unstable modes proliferate and the system dynamics quickly enters the regime of spatiotemporal chaos. In Fig. 2 we show the comparison between the results of the naive perturbation theory and our numerical simulations of a one-dimensional system. As expected, they agree when either $V \gtrsim V_{**}$ or $Q - V \gtrsim Q^{-1}$, but at intermediate V's they differ substantially and a strong broadband noise in I sets in.

Let us now show that a similar instability must occur in the disordered case as well. To estimate the corresponding V_{**}, we approximate the unstable modes by wave packets of plasma waves with a Gaussian spread of wave vectors, $\psi_\pm(\mathbf{k}) = (2\pi\Delta k^2)^{-1}\exp[-|\mathbf{k} - \mathbf{k}_\pm|^2/2(\Delta k)^2]$. If Δk is smaller than $\alpha(k_\pm)$, then $G(k, \mathbf{k})$ is approximately the same for dominant $\mathbf{k}$'s within the wave packets, and so the left-hand side of the instability criterion (14) need not be modified. The right-hand side, however, is changed from

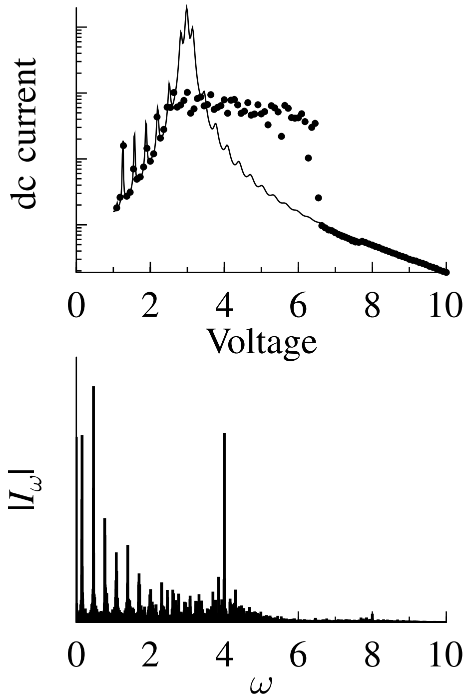

FIG. 2. Top: I-V characteristic. Dots are from numerics, and the solid line is from a full perturbation theory formula for a finite-length system. It contains both the main resonance of Eq. (4) and smaller "Fiske resonances." The scatter of the data points at $V > 2.5$ is due to statistical noise. The parameters of simulation are $L = 20$, $Q = 3$, $\beta = 0.01$, and $\phi_0 = 0$. Bottom: an example of the frequency spectrum of $I(t)$ in the above simulation. Note the narrow peak at ω equal to the applied voltage $V = 4$ and the strong broadband noise at smaller ω's.

1/4 to $(1/2\pi)\tilde{\sigma}(V - Q)(\Delta k)^2$ and the threshold voltage becomes $V_{**} \sim \beta^{-1/2}\tilde{\sigma}^{-1/8}$. Thus, for small β the lowest voltage at which the perturbation theory is expected to apply is $V_{**} \gg V_*$. Our conjecture on the behavior of $\bar{I}$ in the interval $V_* < V < V_{**}$ (depicted in Fig. 1) is based on a notion that chaos leads to a higher effective dissipation [see Fig. 2 (top) and Refs. [13] and [19]]. The suppression of $\bar{I}$ below V_* is an educated guess motivated by the work of Fistul and Giuliani [16].

The probable observation of Josephson effects without bulk superconductivity in a bilayer quantum Hall ferromagnet paves the way for exploring a vast variety of other Josephson phenomena in this system. It should also inspire attempts to realize them in other systems. The complexity and appeal of the emerging theoretical issues (not fully appreciated in the current literature), as well as their recurrence in other contexts warrant further study. From this perspective the lack of quantitative agreement between the perturbation theory and experiment is stimulating.

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