Theory of Interlayer Tunneling in Bilayer Quantum Hall Ferromagnets

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Spielman *et al.* [Phys. Rev. Lett. **84**, 5808 (2000] recently observed a large and sharp Josephson-like zero-bias peak in the tunnel conductance of a bilayer system in a quantum Hall ferromagnet state. We argue that disorder-induced topological defects in the pseudospin order parameter limit the peak size and destroy the predicted Josephson effect. We predict that the peak would be split and shifted by an in-plane magnetic field in a way that maps the dispersion relation of the ferromagnet's Goldstone mode. We also predict resonant structures in the dc I-V characteristic under bias by an ac electric field.

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Exotic effects induced by interlayer Coulomb interactions have made strongly coupled bilayer quantum Hall systems at the total Landau level filling factor $\nu = 1$ the subject of numerous theoretical and experimental studies [1–8]. When the layers are widely separated they behave as two weakly coupled $\nu = 1/2$ "composite fermion" metals. In this regime interlayer tunneling is strongly suppressed at low-bias voltages by an orthogonality catastrophe associated with the very slow relaxation of charge in each layer [9]. Recently, however, Spielman *et al.* [10] observed a strong and sharp peak in the differential conductance near zero bias for layer separations below a critical value. As we explain below, this dramatic change is associated with the formation of a broken symmetry state with spontaneous interlayer phase coherence and strong interlayer correlations.

In the broken symmetry state, the electrons lower their interlayer exchange energy by going into a state with uncertain layer index (i.e., a coherent superposition with equal probability but fixed relative phase to be found in either layer) [1,5]. Hence it is possible to tunnel an electron between the layers and still leave the system in or near its ground state (much as in the Josephson effect). The orthogonality catastrophe is averted and strong tunneling can occur near zero voltage because the electrons in the destination layer were already avoiding the tunneling electron before it arrived. This is the microscopic physics underlying the charge-e Josephson effect predicted by Wen and Zee and by Ezawa and Iwazaki [4]. The broken symmetry state may be described as a condensate of Chern-Simons bosons [4], as an easy-plane ferromagnet [1,3,5] of pseudospins representing the layer index, or as a superfluid excitonic condensate [11]. We use the pseudospin language below.

In contrast to the true Josephson effect, however, no zero-bias supercurrent (infinite tunneling conductance) was observed. The peak conductance, though enormously enhanced, did not exceed $10^{-2}e^2/h$. In this Letter we explain how density inhomogeneities introduce topological defects (merons) into the SU(2) pseudospin order parameter. These defects carry both charge and vorticity [1] and constitute a dissipative environment which turns the Josephson effect into a finite tunneling peak whose height and width is a measure of the dynamics of the topological defects. We predict dependences of the tunneling current on in-plane magnetic field strength $B_{||}$, bias-voltage frequency, and on the homogeneity of the 2D layers. In particular, we show that a measurement of $I(V, B_{||})$ would test the main premise of our theory, the existence of one low-energy Goldstone mode, and would map its dispersion relation. Finally, we analyze the current distribution for a perfectly homogeneous sample.

The order parameter field of the quantum Hall ferromagnet is a pseudospin unit vector $\vec{m}$. When fluctuations out of the easy plane are small, it can be parametrized by an angle φ and the conjugate "charge" m_z: $\vec{m} = (\cos\varphi, \sin\varphi, m_z)$. In the absence of tunneling, disorder and topological defects, the long-wavelength Hamiltonian density of the $\nu = 1$ bilayer state is [1,5,6]

$$H = \frac{1}{2}\,\rho_s(\nabla\varphi)^2 + \frac{(en_0 m_z/2)^2}{2\Gamma}, \tag{1}$$

where $n_0 = \frac{1}{2\pi\ell^2}$ is the average density. In Hartree-Fock theory, $\rho_s \sim 0.4$ K and the capacitance Γ is increased from its electrostatic value [1,5]. Since the momentum density conjugate to φ is $p_\varphi = \hbar n_0 m_z/2$, the Hamiltonian (1) has a single linearly dispersing collective mode with velocity $u = \sqrt{\rho_s/\Gamma}$. This Goldstone mode signals superfluidity for in-plane currents which are antisymmetric in the layer index [1,4]. Taking proper account of the significant exchange enhancement of Γ yields $u \sim 0.1e^2/\hbar\epsilon$. Equation (1) is valid only in the limit $q \to 0$. For larger q the collective mode frequency ω_q exhibits a roton minimum [2,3]. It is this dispersion curve that may be extracted from a measurement of $I(V, B_{||})$.

The interlayer tunneling operators are

$$T_\pm = -\int d^2r\,\lambda(\vec{r})e^{\pm i\varphi(\vec{r})}e^{\pm iQ_B x}, \tag{2}$$

where the $\pm$ sign refers to the direction of tunneling, $Q_B = \frac{edB_\parallel}{\hbar c}$ is a characteristic wave vector introduced [1] by the magnetic field $B_\parallel$ (we choose the gauge $\vec{A}_\parallel = xB_\parallel \hat{z}$). The quantity $\lambda = \frac{1}{8\pi\ell^2}\Delta_{\rm SAS}$ is proportional to the tunneling amplitude and may vary with position due to disorder in the tunnel barrier. Here we do not discuss this source of disorder since, on its own, it cannot destroy the Josephson effect. As in a Josephson junction, the tunneling term in the Hamiltonian is $T_+ + T_-$, while the tunneling current operator is $ie(T_+ - T_-)/\hbar$. The striking similarity of the expressions above to their counterparts in superconducting Josephson junctions make it clear that a calculation of the tunneling conductance under Eqs. (1) and (2) leads to a Josephson effect, in contrast to the experiment. We now explain the way that disorder destroys the Josephson effect in the present system.

In the $\nu = 1$ bilayer system, a deviation of the total density from $\nu = 1$ introduces topological defects (merons) into the order parameter vector $\vec{m}$. In terms of bosonic Chern-Simons theory, this statement is a consequence of the residual magnetic field left, away from $\nu = 1$, after the external and Hartree-Chern-Simons magnetic fields almost cancel each other. This residual field introduces vortices into the bosonic order parameters of the two layers. In the language of a quantum Hall ferromagnet, this observation is a consequence [1,6] of the coupling of the symmetric density to the order parameter $\vec{m}$. The symmetric part of the density is constrained to satisfy $n(\vec{r}) - n_0 = \frac{1}{8\pi}\epsilon_{ab}\epsilon_{\mu\nu\kappa}m_\mu\partial_a m_\nu\partial_b m_\kappa = \nabla\cdot(\frac{m_z}{8\pi}\hat{z}\times\nabla\varphi) - \frac{m_z}{8\pi}\nabla\times\nabla\varphi$. The deviation from n_0 is then composed of a charge density carried by an electric dipole field, $\frac{m_z}{8\pi}\hat{z}\times\nabla\varphi$, and by a charge density attached to topological defects in $\vec{m}$. The latter are merons of four types, carrying a charge of $\pm\frac{e}{2}$, and characterized by their vorticity (the sign of $\nabla\times\nabla\varphi$ at the core) and the layer in which their charge resides (m_z at the core). Merons interact Coulombically due to their charge, and by a logarithmic interaction due to their vorticity. Below the Kosterlitz-Thouless (KT) temperature $T_{\rm KT} \sim \rho_s$, merons are bound in pairs of opposite vorticity to avoid the logarithmically diverging energy penalty.

In realistic samples there are long-range density fluctuations whose relative magnitude is estimated [12] to be 4%. Thus, the typical distance between the disorder-induced meron pairs is $\sim 12\ell$. The separation between the two merons that constitute a pair is estimated to be $\sim 6\ell$ [1], comparable to the spacing among different pairs. This estimate is obtained by balancing the Coulomb repulsion and logarithmic attraction. Thus, the $\nu = 1$ bilayer sample studied in Ref. [10] is analogous to a superconducting junction with random magnetic flux that introduces many vortices in the two superconductors. Meron pairs may carry a charge $\pm e$ (distributed between the two layers) or be charge neutral. The charged pairs affect the longitudinal resistivity to the flow of symmetric current. In the sample of Spielman *et al.,* this resistivity is large ($\sim 1\ \text{k}\Omega$), indicating that the charged vortex pairs are highly mobile. Furthermore, the dissipation is not frozen out at the lowest attainable temperatures, indicating that these objects are disorder-induced rather than thermally induced. Tunneling in this system is then strongly influenced by these merons, in a way discussed below. The meron pairs do not, however, destroy the antisymmetric superfluid mode unless they become unbound.

By appealing to the experimental observation that there is no dc Josephson effect (i.e., current linear in the tunneling amplitude), we may use Fermi's Golden Rule to calculate the tunneling current perturbatively. For a sample of size L^2,

$$I(V) = \frac{2\pi e\lambda^2 L^2}{\hbar}[S(Q_B, eV) - S(-Q_B, -eV)], \tag{3}$$

where $S(q, \hbar\omega)$, the spectral density for the fluctuations of the operator $e^{i\varphi}$ at wave vector q and frequency ω, is proportional to the Fourier transform of $\langle e^{i\varphi(r,t)}e^{-i\varphi(0,0)}\rangle$ (where the angle brackets denote thermal average). Our prediction regarding the dependence of the tunneling current on $B_\parallel$ can now be easily understood. For weak disorder, the spectral density $S(Q_B, eV)$ is sharply peaked at

$$eV = \hbar\omega_{Q_B}. \tag{4}$$

Thus, as the parallel field is varied, the peak in the tunneling conductance is shifted in a way that reflects the dispersion of the low-energy excitation mode. This is closely analogous to the Carlson-Goldman experiment measuring the collective oscillations of the pair field in a superconductor [13]. An observation of this dispersing peak would also confirm an essential ingredient of the picture we use, namely, the existence of a single branch of low-energy excitations. The parallel field allows only tunneling between states that differ by a momentum Q_B. Energy conservation requires the energy of these states to differ by eV. When there is just one low-energy excitation branch, there is only one value of the voltage where both of these conditions are fulfilled. This is not the case for a Fermi liquid (for $Q_B \neq 0$).

To begin our analysis of the effect of merons, we make the usual separation of the order parameter phase into a singular vortex part φ_m and a smooth spin-wave part φ. Just as in KT physics, the meron vortices destroy the quasi-long-range spatial order. Similarly, it is known that the motion of these objects destroys the quasi-long-range temporal order and consequently introduces dissipation into a 2D superfluid. In the present context this motion will be shown to lead to the destruction of the Josephson effect. This loss of long-range order is captured in the phenomenological ansatz $G_m(r,t) \equiv \langle e^{i\varphi_m(\vec{r},t)}e^{-i\varphi_m(\vec{0},0)}\rangle = \exp(-\frac{r^2}{2\xi^2} - \frac{t}{\tau_\varphi})$. Applying this ansatz to Eqs. (1)–(3) yields

$$I(V,B_{\|}) = \frac{4e\lambda^2L^2}{\hbar^2}\int_0^\infty dt\int d^2r\, G_m(r,t)e^{-(1/2)D(r,t)} \times \sin\frac{C(r,t)}{2}\cos Q_{\mathrm{B}}x\sin\frac{eVt}{\hbar} \quad (5)$$

with a "Debye-Waller factor" $\exp[-D(r,t)/2]$, where

$$D(r,t) \equiv \langle[\varphi(\vec{r},t) - \varphi(\vec{0},0)]^2\rangle = \sum_q \frac{\hbar u}{L^2\rho_s q}[1 - \cos(\vec{q}\cdot\vec{r})\cos(uqt)]\coth\frac{\hbar uq}{2T} \quad (6)$$

(we set $k_B = 1$ throughout) and a commutator term

$$C(r,t) \equiv i[\varphi(\vec{r},t),\varphi(\vec{0},0)] \approx \frac{\hbar}{2\pi\rho_s}\theta(ut - r)\left[t^2 - \left(\frac{r}{u}\right)^2\right]^{-1/2}, \quad (7)$$

which is independent of temperature, and limits the r integral in (5) to a "light cone" of $r < ut$. All correlators are evaluated in the absence of tunneling. We rely on the global $U(1)$ symmetry and the freedom to renormalize ξ and τ_φ to partially justify the simplifying approximation of neglecting all disorder in the spin-wave Hamiltonian.

As long as $2\pi\rho_s \gg \hbar/\tau_\varphi \gg T$ we can expand Eq. (5) to first order [14] in C and approximate D by its zero-temperature value. In this limit the current becomes

$$I(V,B_{\|}) = \frac{e}{h}\frac{\xi^2\lambda^2L^2}{4\Gamma}e^{-D/2}\int d^2p\, e^{-|\vec{p}-\vec{Q}_B|^2\xi^2/2}\frac{\hbar}{\omega_p} \times \left\{\frac{\delta_\varphi}{(eV - \hbar\omega_p)^2 + (\delta_\varphi)^2} - \frac{\delta_\varphi}{(eV + \hbar\omega_p)^2 + (\delta_\varphi)^2}\right\}, \quad (8)$$

where $\delta_\varphi \equiv \hbar/\tau_\varphi$. For large τ_φ, ξ, Eq. (8) shows a peak in the current at the voltage corresponding to the Goldstone mode energy in accordance with Eq. (4). The effect of τ_φ, ξ is to smear this peak over a range of $\hbar/\xi$ in momentum and $\hbar/\tau_\varphi$ in voltage. As long as $Q_B\xi \gg 1$ and $uQ_B\tau_\varphi \gg 1$, this smearing is insignificant.

The expression for the differential conductance simplifies considerably in the limit $Q_B = 0$, $\xi \ll u\tau_\varphi$, and $eV \ll \frac{\hbar u}{\xi}$:

$$\frac{dI}{dV} = \frac{1}{8}\frac{e^2}{h}\frac{\xi^2}{\ell^2}\frac{n_0L^2\Delta_{\mathrm{SAS}}^2}{\rho_s}e^{-D/2}\frac{\delta_\varphi}{(eV)^2 + (\delta_\varphi)^2}. \quad (9)$$

Interestingly, we see that when $\tau_\varphi = \infty$, i.e., when the merons provide a random *static* background phase field, a Josephson-like singularity of $\frac{dI}{dV}$ is still present (as is the antisymmetric superfluid property). As shown below, the singularity is also present at finite temperature $T \ll \rho_s$. Static topological defects break translational invariance and thus open more phase space for excitation of spin waves in the tunneling process. However, they do not expand the degrees of freedom involved beyond the single spin-wave mode, and thus do not dephase the process enough to destroy the zero-voltage singularity.

The temperature dependence of (5) originates from the temperature dependence of D and the temperature dependence of ρ_s and τ_φ. Here we calculate the temperature dependence of D. At zero temperature it gives the space- and time-independent result $D_0 \equiv \int_{q\ell<\sqrt{2}} d^2q\,\frac{\hbar}{\Gamma uq} \sim 4.8$. At finite temperature we approximate $\coth x \approx 1 + \frac{1}{x}e^{-x}$, define dimensionless length and time variables, $\tilde{r} \equiv \frac{rT}{\hbar u}$ and $\tilde{t} \equiv \frac{tT}{\hbar}$, and obtain (for large r, t, and $r < ut$),

$$D(\tilde{r},\tilde{t}) \approx D_0 + \frac{T}{2\pi\rho_s}\log|(\tilde{t} + i/2) + \sqrt{(\tilde{t} + i/2)^2 - \tilde{r}^2}|^2 \quad (10)$$

The temperature dependence of D affects $I(V)$ then only at high temperature ($T \gg eV$), where we can approximate $\tilde{t} + i/2 \approx \tilde{t}$. For $u\tau_\varphi \gg \xi$ and $B_{\|} = 0$, Eq. (5) reduces to

$$I(V) \sim \frac{e\lambda^2L^2}{\pi\rho_s\hbar}\int_0^\infty dt\int_{r<ut} d^2r\exp\left[-\frac{1}{2}\left(\frac{r}{\xi}\right)^2 - \frac{t}{\tau_\varphi}\right] \times \frac{|\frac{tT}{\hbar} + \sqrt{(\frac{tT}{\hbar})^2 - (\frac{rT}{\hbar u})^2}|^{-T/2\pi\rho_s}}{\sqrt{t^2 - (r/u)^2}}\sin\left(\frac{eVt}{\hbar}\right). \quad (11)$$

Most of the contribution is then from long times, while r is limited to be smaller than ξ. For a static meron background ($\tau_\varphi = \infty$) we find, for $eV \ll T$,

$$\frac{dI}{dV} \propto \frac{\lambda^2\xi^2L^2}{\rho_s^2}\left(\frac{T}{V}\right)^{1-T/2\pi\rho_s}, \quad (12)$$

which is consistent with the more complete scaling form which can be derived in the classical limit from the expression of Nelson and Fisher for the dynamical structure factor of the XY model [15]. In the presence of a finite τ_φ, the temperature dependence of D affects the tunneling current significantly only in the window $\frac{\hbar}{\tau_\varphi}$, $eV < T < T_{\mathrm{KT}}$. In the experiment of Spielman *et al.,* the peak width is much larger than the temperature. Thus, the observed temperature dependence probably results from the temperature dependence of ρ_s and τ_φ rather than D.

By using Eq. (8), we can fit the width of the conductance peak in the experiment with a phenomenological value $\delta_\varphi \approx 0.75$ K. This value gives $u\tau_\varphi \approx 11\ell$, which is remarkably close to our estimate of ξ based on the meron pair spacing. [Sufficiently close that the Lorentzian approximation in Eq. (9) for the peak width will be somewhat inaccurate.]

Our naive estimate for the peak height in the experiment is too large by some 2 orders of magnitude, but is highly uncertain due to the exponential sensitivity to the ultraviolet cutoff and the acoustic approximation used in computing the Debye-Waller factor. In addition, the estimate

$\Delta_{SAS} \approx 90\ \mu$K is exponentially sensitive to the parameters in the modeling of the barrier potential (in particular, the poorly understood effective mass appropriate for the high Al concentration in the barrier) and so might be off by a significant factor [12]. It might also be possible that the superfluidity and tunneling occur predominantly in isolated regions close to filling factor $\nu = 1$ containing few vortices. The parasitic series transport resistance $\sim 1/\sigma_{xx}$ in this Corbino-like geometry could significantly reduce the peak height.

We now consider interlayer tunneling under the combined effect of a time-independent dc voltage and a time-dependent ac electric field $E \sin\omega t$, directed perpendicular to the two layers. As long as the system is not heated, this field can be incorporated into our calculation by writing $T_{\pm} = -\int d\vec{r}\, \lambda e^{\pm i[\varphi(\vec{r}) + (eEd/\hbar\omega)\cos\omega t]}$. By repeating the calculation carried out above, we find that the tunneling differential conductance $\frac{dI}{dV}(V)$ exhibits peaks at $eV = n\hbar\omega$, with n an integer. This feature is common to all tunneling systems, where the dc differential conductance is strongly peaked around zero voltage (for example, a bilayer system at zero magnetic field). We note, however, that the quantum Hall ferromagnet is relatively less prone to heating, due to the small longitudinal conductivity.

Finally, we discuss the current distribution in an idealized zero-disorder and vortex-free system. Equations (1) and (2) then do indeed lead to a Sine-Gordon equation for the phase, as in a long Josephson junction. However, due to the two dimensionality of the problem, the critical current is not proportional to the area of the sample. Consider a setup where the current is fed into one layer from, say, $x = -\infty$, and taken out from the other layer at $x = \infty$, and where tunneling is limited to the region $-\frac{L}{2} < x < \frac{L}{2}$. Since the symmetric part of the current ($I_{\rm sym}$) is conserved, the boundary conditions for the Sine-Gordon equation require $\frac{\partial\varphi}{\partial x}|_{x=L/2} = -\frac{\partial\varphi}{\partial x}|_{x=-L/2} = I_{\rm sym}$. For $L \gg \xi_J \equiv \sqrt{4\pi\ell^2\rho_s/\Delta_{SAS}} \sim 4\ \mu$m, the time-independent solution to the Sine-Gordon equation in the tunneling region is $\varphi(x) \approx 2 \arccos \tanh \frac{\frac{L}{2} - |x|}{\xi_J}$; tunneling takes place only within a distance of order ξ_J of the $x = \pm\frac{L}{2}$ lines, and the maximal current that can tunnel is L independent, and is given by $(2e/\hbar)\rho_s W/\xi$ (here W is the width of the current contact). For the parameters we use, this current is ~ 4 nA$/\mu$m $\cdot W \sim 80$ nA. The experimental measurement current was much smaller than this value. Thus, the absence of a Josephson effect cannot be attributed to a large measurement current. For the sample geometry described above and used in the experiment, the tunnel resistance is effectively in series with the Hall resistance. The observed tunnel resistance is, however, much larger, $\sim 10^2 h/e^2$, again indicating that there is no Josephson effect.

To conclude, we have attributed the lack of a Josephson effect in tunneling measurements in a bilayer quantum Hall $\nu = 1$ state to density inhomogeneities that introduce dynamical topological defects into the order parameter. The observed peak width is quantitatively consistent with this picture. We showed that a measurement of the tunneling $I(V)$ dependence on $B_{\|}$ would map the dispersion relation of the low-energy mode of the system, and that tunneling in the presence of an ac electric field would result in resonances at voltages corresponding to the ac frequency. Finally, we showed that, even for a perfect sample where the Josephson effect takes place, the critical current would not scale with the size of the sample.

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