Creep Motion in a Granular Pile Exhibiting Steady Surface Flow

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We investigate experimentally granular piles exhibiting steady surface flow. Below the surface flow, it has been believed that a "frozen" bulk region exists, but our results show no such frozen bulk. We report here that even the particles in layers deep in the bulk exhibit very slow flow and that such motion can be detected at an arbitrary depth. The mean velocity of the creep motion decays exponentially with depth, and the characteristic decay length is approximately equal to the particle size and is independent of the flow rate. It is expected that the creep motion we have seen is observable in all sheared granular systems.

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Granular materials exhibit behavior not seen in ordinary fluids or solids. Considerable effort has been made to understand them, but further studies of various phenomena in granular materials are still required [1–4]. One of the typical behaviors of granular materials is the avalanche behavior of a sandpile [1,5,6]. In contrast to ordinary fluids, granular materials can form piles with a sloped surface. When the angle of the surface exceeds some critical value, the pile cannot sustain the steep surface and an avalanche occurs. Without careful observation, flow of particles in an avalanche appears to be limited to a surface layer, with a "frozen" bulk region below. Many studies have been made under such an assumption without convincing experimental evidence [1,5,6]. Indeed, it is difficult to detect the frozen bulk clearly because an avalanche is a transient behavior. If such a frozen region actually exists and is well separated from the flow layer, it should be possible to observe and definitively identify it even in the case of steady surface flow. With the purpose of making such an observation, we experimentally investigated particle movements in piles exhibiting steady surface flow.

Here we report unexpected experimental findings in such piles: Even the particles in deep layers are not frozen but exhibit very slow flow (creep), and such motion can be detected at an arbitrary depth (Fig. 1). The mean velocity of the creep motion decays exponentially with depth and the characteristic decay length is approximately equal to the particle size and is independent of the flow rate. This velocity profile differs from that for the surface flow, and it is in this sense that the flow of particles is separated into two regions. We believe that creep motion of the type we have seen should be observable in all sheared granular systems.

Our experiments were performed in a quasi-two-dimensional system, as described by Fig. 2. Alumina beads of diameter a were continuously fed into the gap (A) existing between two vertical, parallel plates separated by width W. In this gap, the particles formed a triangularly shaped pile with rapid flow on the surface slope, pouring out of the system from the right side (B). The existence of the short wall on the right side eliminates the slipping of particles on the bottom of the cell. Its presence thus assures that the motion of particles we observe is strictly due to the flow on the slope of the pile. In order to maintain a steady surface flow on the pile, particles were continuously fed onto the pile from the left

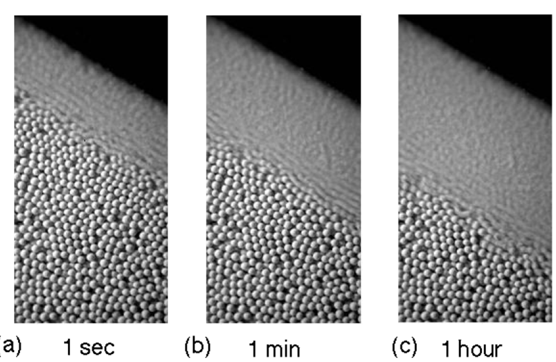

FIG. 1. Snapshots of a granular pile in a steady flow state. Particles were fed constantly from the surface of the left side, out of these figures. The particles used were monodisperse alumina spheres with diameter 1.1 ± 0.1 mm (weight density 3.6 g/cm^3). All photographs were taken under the same conditions, with only the shutter speeds differing: (a) 1 sec, (b) 1 min, (c) 1 h. (a) A boundary between the surface flow and the "frozen" layers appears to exist at a depth of several particles below the surface. (b) The apparent frozen layer of (a) is seen to flow. Similarly, (c) reveals the flow of a thicker layer than that revealed by (b). These observations reveal that the apparently frozen particles under the rapidly flowing surface are not stationary but slowly creep and that the layer in which this creeping motion can be detected grows as observation time increases.

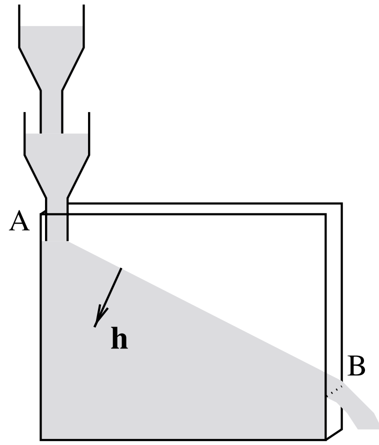

FIG. 2. The geometry of the experiment. We used a transparent material (acrylic plate) for the front plate (400 mm wide) to allow observation of particle motion and aluminum for the back plate to reduce the static electricity effect. The bottom and the left side of the gap were completely bounded by plates, whereas the right side was bounded only in its lower region (a region of height 20 mm).

side using a "double hopper," in which the lower hopper was continuously filled with particles by the upper one. The mouth of the lower hopper was placed in such a manner that it always contacted the top of the pile. This configuration allowed us not only to minimize the effect of the impact on the pile exerted by newly added particles but also to control the feeding rate by changing the height of the mouth of the lower hopper. With this configuration, the feeding rate was essentially controlled by the flow on the pile, and it could thus be closely matched to the outflux from the system. In this way a steady-state system could be established. The flow rate Q for each experimental run was determined by measuring the weight of the particles that poured from the right side of the gap per unit time. Particle velocities were extracted from sequential images taken with a charge-coupled device (CCD) camera at various time intervals δt. In order to measure particle velocities over several orders of magnitude, the time intervals δt were chosen from the range of $1/500$–30 sec [7] depending on the velocities. The spatial resolution of the images is $0.015a$–$0.03a$ for $\delta t < 1/30$ and $0.05a$–$0.07a$ for $\delta t \geq 1/30$. With this system, the important control parameters are the particle diameter a, the flow rate Q, and the width of the gap W. In the present study, we report the results obtained for three different diameters, $a = 1.1 \pm 0.1$, 2.1 ± 0.1, and 3.1 ± 0.1 mm, and the gap width, $W = 10a$. We confirmed that qualitatively the same results are obtained for $W/a = 5$–40.

As is seen in Fig. 1, the surface of the pile is flat in the steady flow state, and its angle ϕ with respect to the horizontal direction is close to that after the flow stops when the supply of particles is cut off, i.e., the angle of repose. In this state, the mean velocity is approximately parallel to the surface, and its functional dependence on the depth h (measured perpendicularly to the surface; see Fig. 2) is the same everywhere, except in the vicinity of left and right boundaries. The mean velocity is determined from an ensemble average of particle displacement per unit time for each depth. Figure 3 displays the velocity profiles as functions of the depth h on a semilog scale, while the inset displays the same data on a normal scale. From the graph in the inset, the mean velocity $\langle v(h) \rangle$ appears to decay as the distance from the surface increases, and it appears to vanish at some finite depth. Although in previous studies [1,5,6] it has been assumed that these deep layers are frozen, we found that the mean velocity is finite at all depths. We call the slow motion of particles in such a deep region *creep motion*.

As is clearly shown in Fig. 3, for deep layers (large h) exhibiting creep motion, the velocity profile assumes the form of simple exponential decay:

$$\langle v(h) \rangle = v_0 \exp(-h/h_c), \tag{1}$$

where h_c is a characteristic length. While v_0 increases with the flow rate Q, h_c is approximately equal to the particle diameter a for all values of Q. The value of h_c suggests that the creep motion is driven by events occurring on a particle-size scale. The exponential form Eq. (1) is not specific to the system of spherical particles. Qualitatively, the same results are obtained for particles with different

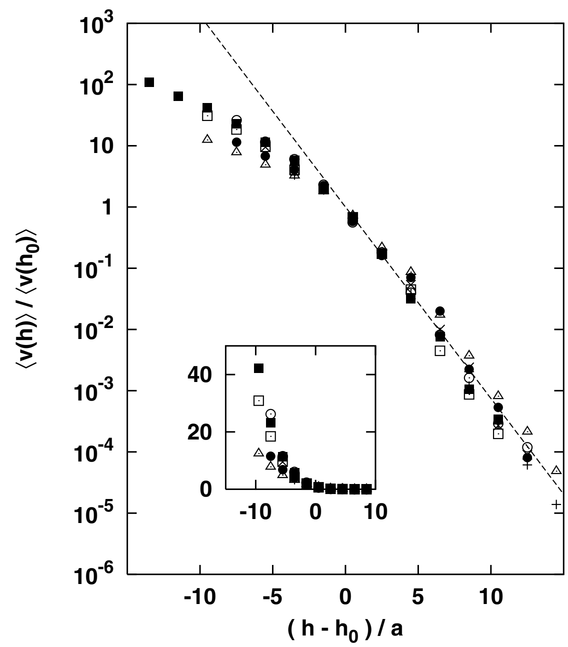

FIG. 3. Mean velocity profiles $\langle v \rangle$ as functions of the depth h from the pile's surface on a semilog scale. The inset presents the same data on a normal scale. The horizontal axes represent the depth from h_0 normalized by the particle diameter a, where h_0 is the distance from the top of the fixed wall existing at the right side of the system to the surface of the particles flowing over it. (See Fig. 5.) The vertical axes represent velocity normalized by the value at h_0. The values of the particle diameter a (mm) and the flow rate Q (g/sec) for the experiments are 1 and 12 (squares), 1 and 43 (solid squares), 2 and 20 (circles), 2 and 50 (solid circles), 2 and 100 (triangles), 3 and 34 (plus signs), and 3 and 58 (crosses). The broken line corresponds to $\exp(-0.72x)$.

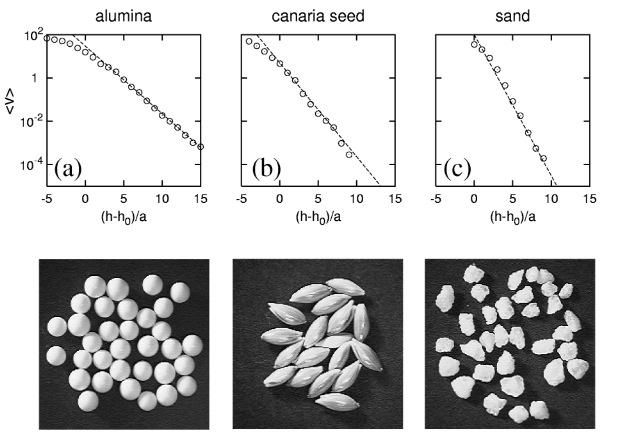

FIG. 4. Mean velocity profiles for three different shaped particles, (a) alumina sphere, (b) seeds, (c) sand, with pictures of each particle at the bottom. For the spindle shaped seeds (b), the length of the seed's minor axis is used as a.

shapes and roughness: spindle shaped seeds and irregularly shaped sand particles (see Fig. 4). This suggests that our results do not depend sensitively on shapes or roughness of particles.

Each velocity profile in Fig. 3 is seen to bend at some depth. At such a depth, there approximately exists a boundary at which the particle motion changes from rapid surface flow to slow creep motion. As shown schematically in Fig. 5, this boundary which is parallel to the surface, lies just above the upper edge of the right side wall. The value h_0 corresponds to the thickness of the surface flow, which increases with the flow rate Q. The particles below the boundary are jammed tightly due to the fixed wall existing below the surface downstream, while the upper surface flow is free from such obstruction. The mean velocity $\langle v \rangle$ was found to have a longer characteristic length for the rapid surface flow above the boundary than for the creep motion; i.e., the creep motion decays more rapidly as a function of h than the surface flow.

In the region of the creep motion, particles are jammed tightly and sheared by the upper flow. For such particles to move, the existence of voids is essential. In our experiments, it is observed that the shear causes plastic global deformation of the network formed by interparticle connections and that such deformation occasionally creates voids. These observations, together with the realization that the characteristic length of the decay is on the order of the particle size, lead to the simple idea that the exponential decay of the velocity profile can be understood on the basis of the void creation process. Actually, a simple probabilistic picture based on the void creation process can reproduce an exponential velocity profile: Let us assume virtual layers parallel to the surface with particle size thickness. If we suppose that a void in the nth layer (n increases from the surface) is generated by shear at the rate P_n proportional to $v_{n-1} - v_n$ and also that the mean velocity v_n is proportional to the void creation rate P_n, then it leads to $v_n = \alpha(v_{n-1} - v_n)$, with some constant α (>0) [8]. We thus expect that v_n is an exponentially decaying function of depth. This rough estimation suggests that void creation plays an important role for the creep motion.

Now we briefly discuss the effect of creep motion on the surface flow. Comparing the surface flow in the presently investigated system with that on a truly frozen bulk (which can be modeled by a monolayer of particles fixed on an inclined board), it is observed that the mean velocity of the flow on the creeping bulk is slower than that on the frozen bulk at a given distance from the lower boundary [9]. This difference should be ascribed to the behavior of particles near each boundary: In the case of the frozen bulk, particles on and under the boundary are inhibited to move, but in the case of the creeping bulk without such inhibition the replacement of the particles can take place between the surface flow and the creeping bulk. Then the flow on a creeping bulk experiences greater resistance [10] due to such fragility of the bulk, which is closely linked to void creation, and, as a reaction to this resistance, creep motion is induced in the bulk under the flow.

Can we find other situations in which creep motion affects macroscopic properties of granular systems? As is seen from Fig. 1, one can detect particle displacement only in a finite thickness of the layer for a given observation time. The observed exponential velocity profile, Eq. (1), implies that such thickness increases with the logarithm of the observation time. This discussion suggests that logarithmically slow evolution of macroscopic properties is likely to occur in the sheared granular systems. This is because the macroscopic properties of the granular materials often depend on the microscopic particle configurations, which can be rearranged slowly through creep motion. In this connection, it is interesting to note recent experiments [11], which show that the mechanical strength of the granular layer increases with the logarithm of a waiting time during which the shear force is applied. They also reported that the creep motion takes place simultaneously with the strengthening.

Finally, we note that the exponential velocity profile has been also observed recently in a 2D Couette geometry, where granular layer is sheared with two concentric cylinders [12]. In this case, however, it is reported that a Gaussian correction accompanies the exponential profile unlike

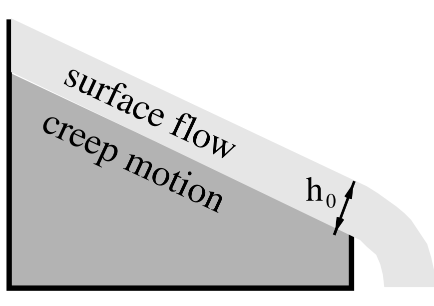

FIG. 5. A schematic picture showing the coexistence of the two different types of motion. The boundary between them is parallel to the free surface, and its position seems to be determined by the height of the fixed wall existing downstream in our system.

in the present system. We think that such Gaussian correction arises from its peculiar geometry, i.e., strong boundary restrictions, which inevitably bring many additional effects, such as a finite-size effect of the layer and a curvature effect of the flows. We believe, therefore, that the exponential decay in the mean velocity reported here is common to all sheared granular systems, regardless of the source of the driving force, and that our results elucidate one of the fundamental properties of sheared granular systems.

In summary, we have experimentally studied granular piles exhibiting steady surface flow. We found that there exist rapid surface flow and slow creep motion below this flow. It is possible to define the boundary between these two types of motion, across which the velocity profile changes continuously. We do not find a frozen layer at any depth. Rather, we found that the velocity of the creep motion decays exponentially according to Eq. (1) and that the characteristic length of this decay is on the order of the particle size and independent of the flow rate. From observations, we conclude that void creation is essential for the creep motion. We also found that similar creep motion exists also for systems consisting of nonspherical particles. This suggests that the behavior we have studied here is quite common in granular systems.

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Note added.—After submission of this Letter, a paper by Mueth *et al.* [13] was published. This paper reports that the Gaussian component appears to be dominant in the case of a 3D Couette geometry. In such geometry, more complicated flow structures can take place than in 2D.

[1] H. M. Jaeger and S. R. Nagel, Science **255**, 1523 (1992); H. M. Jaeger, S. R. Nagel, and R. P. Behringer, Rev. Mod. Phys. **68**, 1259 (1996).
[2] *Physics of Dry Granular Media,* edited by H. J. Herrmann, J-P. Havi, and S. Luding (Kluwer Academic, Dordrecht, Boston, 1998).
[3] *Powders & Grains 97: Proceedings of the Third International Conference on Powders & Grains, Durham, NC, 1997,* edited by R. P. Behringer and J. T. Jenkins (A.A. Balkema, Rotterdam, Brookfield, VT, 1997).
[4] T. Shinbrot and F. J. Muzzio, Phys. Today **53**, No. 3, 25 (2000).
[5] J.-P. Bouchaud, M. E. Cates, J. R. Prakash, and S. F. Edwards, J. Phys. I (France) **4**, 1383 (1994).
[6] T. Boutreux and P. G. de Gennes, J. Phys. I (France) **6**, 1295 (1996); A. Aradian, E. Raphaël, and P. G. de Gennes, Phys. Rev. E **60**, 2009 (1999).
[7] The CCD video camera can capture images at every $1/30$ sec. The shorter time interval than $1/30$ sec was realized with stroboscopic lightning.
[8] A probabilistic cellular automaton model proposed for sheared granular flows, which is based on void picture, also predicts exponential velocity profile. C. Josserand, Europhys. Lett. **48**, 36 (1999).
[9] T. S. Komatsu *et al.* (to be published).
[10] The transverse replacement effectively contributes to energy dissipation of the particles, since particles get into the deeper region with dissipating most of their energy, and particles come up from the deeper region with quite slower speed and decelerate the upper flow.
[11] W. Losert, J.-C. Géminard, S. Nasuno, and J. B. Gollub, Phys. Rev. E **61**, 4060 (2000).
[12] D. Howell, R. P. Behringer, and C. Veje, Phys. Rev. Lett. **82**, 5241 (1999).
[13] D. M. Mueth, G. F. Debregeas, G. S. Karczmar, P. J. Eng, S. R. Nagel, and H. M. Jaeger, Nature (London) **406**, 385 (2000).