Consequences of Parton Saturation and String Percolation on the Development of Cosmic Ray Showers

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At high gluon or string densities, gluon saturation or the strong interaction among strings, either forming color ropes or giving rise to string percolation, induces a strong suppression in the particle multiplicities produced at high energy. This suppression implies important modifications on cosmic ray shower development. In particular, it is shown that it affects the depth of maximum, the elongation rate, and the behavior of the number of muons at energies about $10^{17}-10^{18}$ eV. The existing cosmic ray data point out in the same direction.

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One of the most crucial astrophysical issues of the highest energy cosmic rays (above $\sim 10^{17}$ eV) is that of their composition. This problem is linked to the identification of the origin and possible sources of these cosmic rays. Current theoretical models expect a transition from galactic to extragalactic or galactic halo sources near the region of the ankle which leads to the usual expectation of the changing of composition from heavy to light elements.

Experimental groups have concentrated on observables which are expected to be more or less independent of the hadronic model used, or which have its dependence under theoretical control. These parameters include the maximum of the cascade, $X_{\max}$, the slope parameter, $\beta = d\log[\rho_\mu(600)]/d\log(E)$, where $\rho_\mu(600)$ is the muon density at 600 m from the core, and the elongation rate, $D_{10} = dX_{\max}/d\log_{10}(E)$. Other parameters have been less frequently used (see Ref. [1] for a general review).

Several measurements have been made of the cosmic ray spectrum and mass composition in the ankle region and beyond, using the above-mentioned parameters [2–6] see also [1]. The results on mass composition are inconclusive. Fly's Eye [2] observes a change on the slope of $X_{\max}$ in the region around 3×10^{17} eV, which is interpreted as a change on the composition from heavy (iron) dominated to light (proton) dominated. However, AGASA [3] measures a muon component and β parameter consistent with iron on that region. Although some part of the discrepancy between AGASA and Fly's Eye may be due to the use of different hadronic models [7], as pointed out by Nagano *et al.* [4] the important issue is that AGASA sees no significant change on the muon component of showers all along the ankle region and beyond, from $10^{16.5}$ to $10^{19.5}$ eV, and thus no strong change on composition is inferred. HIRES and MIA collaborations [5] have jointly measured both the $X_{\max}$ and the β parameter. They observe a strong change of $X_{\max}$ with energy, which implies a large elongation rate, $D_{10} = 95$ g/cm^2, and, on the other hand, they see no change on the slope of the measured muonic component, $\beta = 0.73$, which is broadly compatible with the AGASA observations. HIRES and MIA, however, have measured these parameters in a narrow range of energy, from 10^{17} to 10^{18} eV and with low statistics, only during a limited exposure.

In this paper we show that under rather general conditions a change in the hadronic interactions at the energies of interest is expected, which may have important consequences for the interpretation of cosmic ray data. We cannot tell at present whether this change is enough to produce the observed changes on the cosmic ray data. On the other hand, we can state the necessary conditions for this change to explain the observed data: (i) There should be a change in the hadronic interactions at the observed energy $E_{\text{lab}} \sim 5 \times 10^{17}$ eV for Fe-air collisions. This corresponds to a c.m. energy of $\sim 4200A$ GeV and a density of gluons of ~ 9 fm^{-3}. (ii) At this energy the slope of the growth of the multiplicity with energy should vary from ~ 0.24 to a essentially flat $\lesssim 0.09$. If (i) and (ii) are verified, there is no additional need for a change in composition to explain the data. Here we will consider, as is customary in cosmic ray physics at these energies, a two component model, heavy (or iron) and protons. Lighter nuclei (and protons) will suffer no saturation effects up to the highest energies considered (see below).

It is important to point out that, although the change on the multiplicity may or may not be enough to produce the observed results, some effect should always be present and should be taken into account in any realistic simulation of cosmic ray showers. Currently no Monte Carlo code for cosmic ray showers has yet been implemented with these effects. (Sibyll version 2.0 [8] incorporates some shadowing effect. However, this was done for pp collisions only and does not affect our reasoning below.)

We may consider perturbative, gluons, or nonperturbative, strings, as the fundamental variables of our description. Although we expect the hard, perturbative, component to dominate the particle production at very high energy, none of our results depend on the particular scenario chosen, since they are based uniquely on an effective parameter. It is instructive, however, to consider the physical mechanisms which could apply in both cases.

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At high gluon density, the saturation of gluons [9] and/or a strong jet shadowing [10] are expected. In the case of high string density we expect the fusion of strings [11] or color rope [12], and, probably, above a critical string density the percolation of strings [13] and the formation of quark gluon plasma at the nuclear scale are expected.

One general feature of all these hadronic phenomena is the strong suppression of particle multiplicity compared to the multiplicity expected in their absence (see Ref. [14] for several predictions). As a framework we will use the quark gluon string (QGS) model [15], a modified version of the dual parton model (DPM) [16]. The model is based on the large N expansion of quantum chromodynamics (QCD) but it is largely phenomenological and describes most of the soft hadronic physics rather well. In the quark-gluon string model, multiparticle production is related to the interchange of multiple strings which break and subsequently hadronize.

In this model, one can most easily understand the expected changes in the behavior of hadronic collisions at high energies. In the transverse plane, strings are seen as small circles of fixed radius, r. As the energy increases, the number of strings interchanged increases and the total area occupied by strings increases. At high energy, strings start to overlap and fuse together. For high enough string density, n_c, strings may percolate in a second order phase transition, i.e., continuous paths of strings are formed in the collision area. Since the number of independent strings is reduced after the fusion, one expects a depletion on the number of particles produced, i.e., a reduction on the multiplicity.

In the QGS the multiplicity grows with energy as $n(s) \sim s^{\Delta}$, where Δ is related to the intercept of the soft Pomeron [15]. (Here we consider minimum bias events, which are relevant for cosmic ray experiments. The parameter Δ may depend on the centrality of the collision.) In the case of percolation, the reduction of multiplicity is given by [14,17]

$$n'(s) = n(s)\sqrt{\left(\frac{1 - e^{-\eta}}{\eta}\right)}, \tag{1}$$

and the parameter η is the fraction of the total area occupied by strings $\eta = \frac{\pi r^2 N_s}{\pi R^2}$. Here N_s is the number of strings produced in the collision, r is the string's transverse size, and R is the total collision area. N_s grows with energy as $N_s \sim s^{\Delta'}$, where Δ' is the intercept of the soft Pomeron [15]. Therefore at large η the total multiplicity grows with energy as

$$n'(s) \sim s^{\Delta - \Delta'/2}. \tag{2}$$

In perturbative QCD a reduction in the number of jets produced as the energy increases is also expected [10]. At high energy the number of jets produced grows with energy as $n(s, p_t^2) \sim (s/4p_t^2)^{\Delta_H}$, where Δ_H is the intercept of the hard Pomeron and p_t is the transverse momentum of the jet. A (mini)jet occupies a transverse area of order π/p_t^2 saturation occurs when the area occupied by the jets equals the total transverse area [10,14], $\frac{n(s,p_t^2)\pi/p_t^2}{\pi R^2} = 1$, which gives

$$n(s, p_t^2) \sim s^{\frac{(\Delta_H)}{(1+\Delta_H)}}. \tag{3}$$

Both Eqs. (2) and (3) have been checked directly in Monte Carlo simulation [10,11]. Surprisingly, for nucleus-nucleus collisions the reduction in the power of multiplicity growth with energy is of the same order for the cases of both string fusion and shadowing. The power changes from ~0.24 to ~0.19. In general parton saturation, shadowing, string fusion, or percolation will produce the effect of reduction of the multiplicity although we expect the degree of this reduction to be model dependent.

For cosmic ray showers, the rate of change of the multiplicity with energy is directly related to the change of the elongation rate. This has been known for a long time as the elongation rate theorem [18]. The elongation rate theorem can be deduced easily. Let $X_{\max}(E)$ be the maximum depth of the shower produced by a primary of energy E. On average, the first interaction occurs at depth λ, the mean-free path of the initial particle. In this first interaction the initial particle splits into $n(E)$ particles, each carrying an average energy $E/n(E)$. Therefore, we have

$$X_{\max}(E) = \lambda + X_{\max}[E/n(E)]. \tag{4}$$

Assuming that $X_{\max}(E)$ depends logarithmically on energy we get $X_{\max}(E) = A \log_{10}[E/n(E)] + B$, where $A = X_0 \ln 10$ and B are constants. $X_0 = 37\ \mathrm{g/cm^2}$ is the electromagnetic radiation length. If we now assume that $n(E) \propto E^{\Delta}$, we get

$$X_{\max}(E) = A(1 - \Delta)\log_{10}(E) + B'. \tag{5}$$

This is the elongation rate theorem. We can now directly read the elongation rate from the above equation $D_{10} = A(1 - \Delta)$. As stated previously, a change in the multiplicity growth with energy implies a change in the elongation rate.

In Fig. 1 we show $X_{\max}$ as a function of energy for the Fly's Eye and HIRES-MIA experiments. Data have been taken from Refs. [2,5]. The errors shown are only statistical. An additional systematic error of ~20 $\mathrm{g/cm^2}$ must be included in the data. The dashed line represents our calculation for the slope parameter, $D_{10} = 65\ \mathrm{g/cm^2}$ ($\Delta = 0.24$ from our simulations) for Fe-air collision without fusion, normalized with the data at 6×10^{17} eV. The dotted line has a slope parameter $D_{10} = 78\ \mathrm{g/cm^2}$, which would imply a maximum reduction in the slope of growing

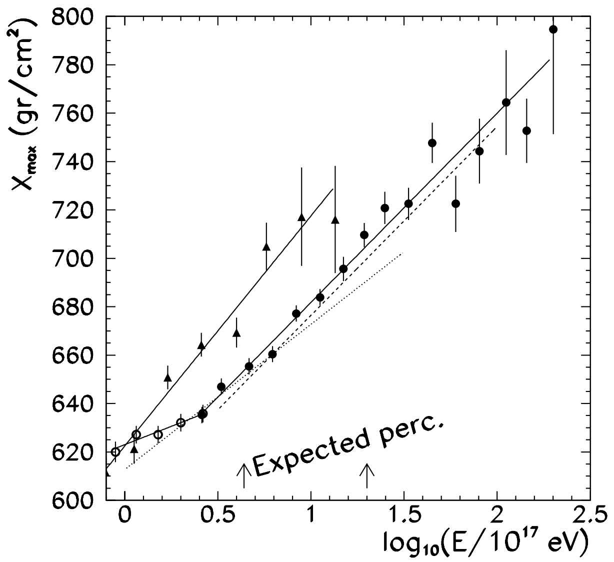

FIG. 1. Depth of shower maximum as a function of the logarithm of the primary energy as measured by Fly's Eye (circles) and HIRES-MIA (triangles). Solid lines are fits to the data. Dashed and dotted lines are our prediction for a strong reduction on the multiplicity at an energy of $\sim 6 \times 10^{17}$ eV. The arrows mark the expected region where percolation occurs for Fe-air collisions in the string fusion model.

multiplicities: from $\Delta \sim 0.24$ to $\Delta \sim 0.09$. This qualitative result has been confirmed with a detailed Monte Carlo simulation (see Ref. [19]). However, our results are done with a cross section independent of the energy. This could change the absolute values of the elongation rate [18]. The data from HIRES-MIA is not completely consistent with the Fly's Eye data. Notice that the elongation rate theorem predicts an elongation rate always less than 85 g/cm^2; a larger elongation rate would imply multiplicities *decreasing* with energy. Therefore, if the HIRES-MIA result is confirmed, with $D_{10} = 95$ g/cm^2, a change of the composition is necessary to explain the data. In the figure we also show the energy region in which a phase transition is expected in Fe-air collision using the string fusion model [19], corresponding to the range obtained from percolation theory, $1.1 \leq \eta \leq 1.2$. A multicomponent composition would produce several changes of this type at the energies where the saturation takes place for each nucleus. For instance, for Si-air collision, percolation takes place at $E_{\rm lab}(\text{Si-air}) \sim 100{-}1000 E_{\rm lab}(\text{Fe-air})$, i.e., at the highest energies for cosmic rays. By using the results of Eskola *et al.* [10], we obtain saturation of partons in central Fe-air collisions at the same range of energies and with similar energy dependence. Indeed the multiplicity calculated in these models in a broad range of energies and primary nuclei is very close to the multiplicities calculated in the string fusion model [10]. Results based on partonic models which do not include saturation predict a much larger multiplicity at LHC energies [20].

The experimental situation with the lateral distribution of muons $\rho_\mu(r)$ is clearer. Both in simulations and experiments it is found that the shape of the lateral distribution function for muons is rather independent of the primary energy and composition. Therefore, at a fixed distance to the core, r_0, $\rho_\mu(r_0)$ is proportional to the total number of produced muons in the shower. Under rather general arguments this number scales with energy $\rho_\mu(r_0) \propto N_\mu = AE^\beta$, where A is a normalization constant and β is the slope parameter. β is found to be constant over a wide range of energies [4,21].

It is rather simple to calculate the slope parameter, β, for a pionic cascade from a scaling argument similar to the elongation rate theorem. The number of muons is proportional to the number of charged pions at the maximum. The number of pions, at maximum, produced by a primary of energy E_0 is given by

$$N_\pi(E_0) = f_\pi n \int_0^1 dx\, P(x) N_\pi(xE_0)\,, \qquad (6)$$

where $f_\pi = 2/3$ is the charged pion fraction, n is the total pion multiplicity, and $P(x)$ is the probability of producing a pion with a fraction of energy x of the primary energy. By assuming a scaling form, $N_\pi = AE^\beta$, we get

$$1 = f_\pi n \int_0^1 dx\, P(x) x^\beta = f_\pi Z(\beta)\,, \qquad (7)$$

where $Z(\beta)$ is the spectrum-weighted moment. For a given $P(x) = 1/n dn/dx$, the above equation gives an implicit equation for β. It reduces to the textbook expression if we assume energy equipartition, i.e., $P(x) = \delta(x - 1/n)$, which gives $\beta = [1 + \log(f_\pi)]/\log(n) \sim 0.82$ for $n \sim 10$. For realistic models the slope parameter β ranges between 0.7–0.9. The DPM model gives a value of $\beta \sim 0.89$ which agrees with that of the QGSJET model [4]. In the presence of fusion the slope parameter is reduced and we get $\beta \sim 0.72{-}0.77$, depending on the specific implementation of the fusion model. This number was calculated using both Eq. (7) and a direct calculation with a Monte Carlo code.

In Fig. 2, we show the density of muons at 600 m, $\rho_\mu(600)$, as a function of the energy for the AGASA measurements, given as a parametrization, and the HIRES-MIA measurements, shown as triangles. Also shown are the QGSJET results for pure iron and proton. Notice that the slope parameter measured by HIRES-MIA agrees with our slope parameter for the case of fusion. Our result, a change of slope parameter from that of pure iron to 0.77 at the energy where percolation is expected, is shown for comparison only. We can see that again it is consistent with the data. The slope measured by AGASA is different from the one calculated for either proton or iron for the QGSJET model. A rapid change in composition, as suggested by the HIRES-MIA data

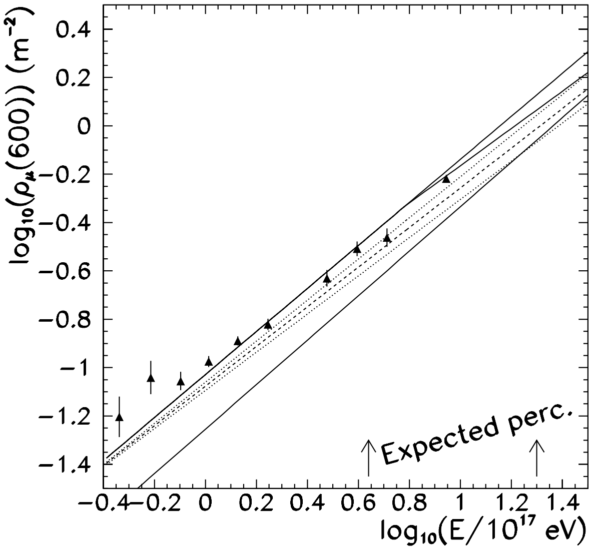

FIG. 2. Logarithm of the muon density at 600 m as a function of the logarithm of the energy as measured by HIRES-MIA (triangles) and AGASA (dashed line). Dotted lines are the errors of the AGASA parametrization. Also is shown the prediction of the QGSJET model for pure iron (upper solid line) and proton (lower solid line) and our prediction for a change of slope as given in the text. The arrows mark the position where percolation occurs for Fe-air collisions in the string fusion model.

on $X_{\max}$, would imply a kink in the data for $\rho_\mu(600)$ at the same energy which is not seen. Instead, our result points towards a mild change on the slope parameter, from 0.9–0.8 to 0.77 which would be hardly seen, given the large error bars in the data. Notice that this corresponds to a relative change of $\Delta\beta/\beta \sim 6\%$. The relative change in D_{10} is, on the other hand, $\Delta D_{10}/D_{10} \sim 20\%$.

Given the current systematical and statistical errors, we cannot conclude that indeed cosmic ray experiments observe saturation of gluons or percolation of strings in hadronic interactions. High quality data with large statistics, as expected from HIRES and the Pierre Auger observatories, are needed. However it is suggestive that all existing cosmic ray data are consistent with such interpretation. BNL RHIC and LHC measure the total multiplicity in the relevant energy region and ascertain whether a strong reduction of multiplicity takes place or not. The first analyzed RHIC data, reported by the PHOBOS collaboration [22] for central Au-Au collisions at $\sqrt{s} = 65$, $130A$ GeV, are in agreement with the predictions of the saturation parton model [20] and with the string fusion model [23]. For instance, at $\sqrt{s} = 130A$ GeV we obtain $dN_{\rm ch}/d\eta = 616$ for $|\eta| < 1$ compared to the experimental $555 \pm 12 \pm 35$. In any case, even if the change of composition is real, these effects must be taken into account in a complete simulation of cosmic ray showers.

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