Modeling Heart Rate Variability in Healthy Humans: A Turbulence Analogy

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(Received 10 May 2000)

Many complex systems share similar statistical characteristics. In this Letter, a turbulence analogy is proposed for the long-term heart rate variability of healthy humans. Based on such an analogy, the equivalence of an inertial range is found and a cascade model, which captures the statistical properties of the heart rate data, is given.

DOI: 10.1103/PhysRevLett.86.1650 PACS numbers: 87.19.Hh, 87.10.+e

In healthy humans, cardiovascular regulation results in complex sinus rhythm which manifests in the fluctuation of a heart beat interval known as the RR interval (RRi). The RRi fluctuation, also referred to as heart rate variability (HRV), is best known for its $1/f$-like power spectrum [1]. This suggests scale invariance in HRV (power law spectrum is a necessary condition for scale invariance [2]) and the departure of the homeostatic viewpoint of the cardiovascular regulation in healthy humans. The nature of biological signals is known to be highly complex [3,4]. From the RRi increment distribution in healthy humans, evidence of multiple scaling was reported [5,6]. Ivanov *et al.* showed for the first time a multifractal singularity spectrum in normal sinus rhythm [7]. These results suggest interesting similarities between the qualitative feature of RRi fluctuations and other complex phenomena. In this work, ideas of *structure function* and *cascade* from fully developed turbulence (FDT) were used to characterize long-term daytime HRV in healthy humans. The objectives of our study are the modest ones: provide evidence of and model the "turbulence" characteristics of HRV.

Two databases were used to analyze the turbulence analogy. The first (DB1) consists of the 24-hour RRi recording of ten healthy (active) young adults conducting normal daily activities [6]. Because of the relatively high activity level of these subjects, the activation of the sympathetic system (which accelerates heart rate) and the reduction of vagal tone (which decelerates heart rate) are evident from the power contents in the low and high frequencies of the spectrum, respectively. To test if the analogy holds generally, we also looked into a second database (DB2) downloaded from the public domain [8], which, on average, has a larger high frequency content in their data sets (Fig. 1a). DB2 consists of 18 sets of 24-hour ambulatory electrocardiogram (ECG) recordings of normal sinus rhythm from subjects who were referred to the ECG monitoring for symptoms that turned out to be unrelated to cardiac rhythm or that did not recur during the recording. The difference in the high frequency content of the two databases further allows us to draw implications of vagal influence on the turbulence of HRV.

Qualitative similarities between HRV and FDT can be demonstrated from a number of statistics. In addition to the $1/f$-like power spectrum, both complex phenomena exhibit (i) a stretch-exponential-like increment probability density function (PDF) at the small time scale and a near Gaussian increment at the large time scale and (ii) intermittent bursts of large amplitude oscillation at irregular intervals of the high-pass filtered RRi. These characteristics can easily be found (Figs. 1b and 1c); see also [5,6].

To model these similarities, ideas of structure function and cascade were applied. Let $\mathbf{r}(t)$ denote the RRi between the tth and $(t+1)$th heart beats and $\Delta\mathbf{r}(\tau) = \mathbf{r}(t+\tau) - \mathbf{r}(t)$ be the RRi increment. The structure function is defined by $S_q(\tau) = \langle|\Delta\mathbf{r}(\tau)|^q\rangle$, where $\langle\cdot\rangle$ denotes the statistical average and $q > 0$ is a real number. It is of immediate interest to formulate

$$S_q(\tau) \sim \tau^{\zeta(q)}. \tag{1}$$

The Kolmogorov theory of FDT showed that $\zeta(q) = q/3$ over the scales in inertial range, wherein a balance law exists between the energy dissipation and energy injected from the large scale [9,10]. Although there are discrepancies between the experimental data and theory, the power law S_q and the inertial range scales are valid descriptions of the turbulence field. The structure function of the RRi increment also exhibits power law behavior (Fig. 2a). Both databases indicated an "inertial range" lying consistently between ~8 to ~2048 heart beats. The different vagal tones of the two databases did not affect the inertial range since its dynamics lies mainly in the high frequency range (<8 beats). The lower limit of the scaling range agrees with the result in the literature, where a scaling range beginning at around 10 heart beats was reported [11]. For DB2, there appear oscillation superimposed on the power law trend. In some cases, dominant harmonics can be found in the oscillation, which reminds us of the log-periodic behavior of the biological signal [3,4]. But this property does not show up in all the data.

The exponent $\zeta(q)$ reveals some interesting properties of the signal. For example, $\zeta(q) = 0$ for a zero-mean, stationary process with independent increments. For a deterministic and oscillatory signal, which is, in a sense, most "nonstationary," one has $\langle|\Delta\mathbf{r}(\tau)|^q\rangle \sim \langle|\Delta\mathbf{r}(\tau)|\rangle^q$ for a fixed q. This expression also holds for a narrowly distributed increment. For example, the Gaussian increment

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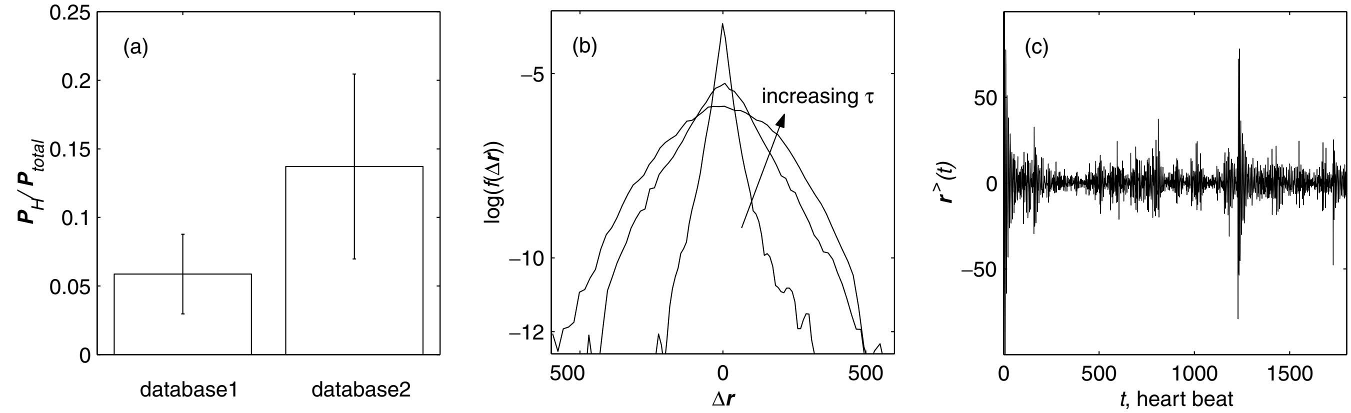

FIG. 1. (a) Group-averaged vagal indicator from DB1 and DB2, $P_H/P_{\rm total}$, where $P_H = \int_{\Omega_H} s(\omega)\, d\omega$, $P_{\rm total} = \int_0^{0.5} s(\omega)\, d\omega$, $\Omega_H = \{0.15 < \omega < 0.5\}$, and $s(\omega) = |\int \mathbf{r}(t) \exp(-i\omega t)\, dt|^2$; (b) PDF of $\Delta\mathbf{r}(\tau)$, $\tau = 1, 128, 2048$ from a typical subject, and (c) the high-passed $\mathbf{r}(t)^>$ at the cutoff frequency 0.35 beat^{-1} from a typical subject.

of the fractional Brownian motion (fBm) with a Hurst exponent $\alpha/2$ yields $\langle|\Delta\mathbf{r}(\tau)|^q\rangle \sim \langle|\Delta\mathbf{r}(\tau)|\rangle^q \sim \tau^{q\alpha/2}$. Hence, in the monoscale situation such as fBm one has $\zeta(q) = q\zeta(1)$. For the more complicated multifractal, $\zeta(q)$ is described by a concave function ($\zeta(q)'' < 0$). Daytime HRV showed such a property (Fig. 2b). A concave $\zeta(q)$ is a result of the power law (1), in which case, $\zeta(q)\log(\tau)$ can be written as the second characteristic function of $q\log[|\Delta\mathbf{r}(\tau)|]$ and the prescribed property followed [12]. From the property of $\zeta(q)$, multifractal of a continuous spectrum of exponents is assumed. The support of the scaling exponent can be studied by the Legendre transform of $\zeta(q)$. This yields two new functions: $f(h(q))$ and $h(q) = d\zeta(q)/dq$, where $f(h)$ is the Hausdorff dimension of the support which scales locally as τ^h [13]. Numerically, $f(h)$ is estimated from $f(h) = \inf_q(qh - \zeta(q) + 1)$. The group-averaged $f(h)$ of DB1 and DB2 are shown in Fig. 3. The $f(h)$ of DB2 matches well the known result [7] (for $q > 0$). Also, despite the difference in vagal influence, multifractal nature remains in the two databases. The h values for DB1 are on average higher than DB2's. This is consistent with the orthostatic test results: the activation of sympathetic system and the withdrawal of vagal tone increase the absolute value of the spectral exponent [14].

Careful experimentations and theoretical studies in FDT in the past resulted in generally accepted values for $\zeta(q)$'s of the velocity increment; e.g., $\zeta(q) = 0.37, 0.70, 1, 1.28, \ldots$ for $q = 1, 2, 3, 4, \ldots$ [9,15,16]. For HRV, the group-averaged $\zeta(q)$ are $0.134, 0.235, 0.311, 0.368, \ldots$ for DB1 and $0.097, 0.173, 0.231, 0.274, \ldots$ for DB2. Compared with the first few $\zeta(q)$'s of FDT, the difference is given roughly by a constant factor ($\sim$3.11 for DB1 and $\sim$4.22 for DB2 for $1 \le q \le 4$). It suggests

$$\left.\frac{\zeta(q)}{\zeta(p)}\right|_{\rm FDT} \sim \left.\frac{\zeta(q)}{\zeta(p)}\right|_{\rm HRV} \tag{2}$$

for some fixed integer p.

Equation (2) has an interesting implication in a more general setting. Benzi *et al.* showed anomalous scaling in FDT down to the Kolmogorov scale due to what they called the extended self-similarity (ESS) [16]. The ratio $z_{q,p} = \zeta(q)/\zeta(p)$ characterizes ESS by describing the power law relationship between different parts of the PDF's at different τ. For example, rare events contributing the tail

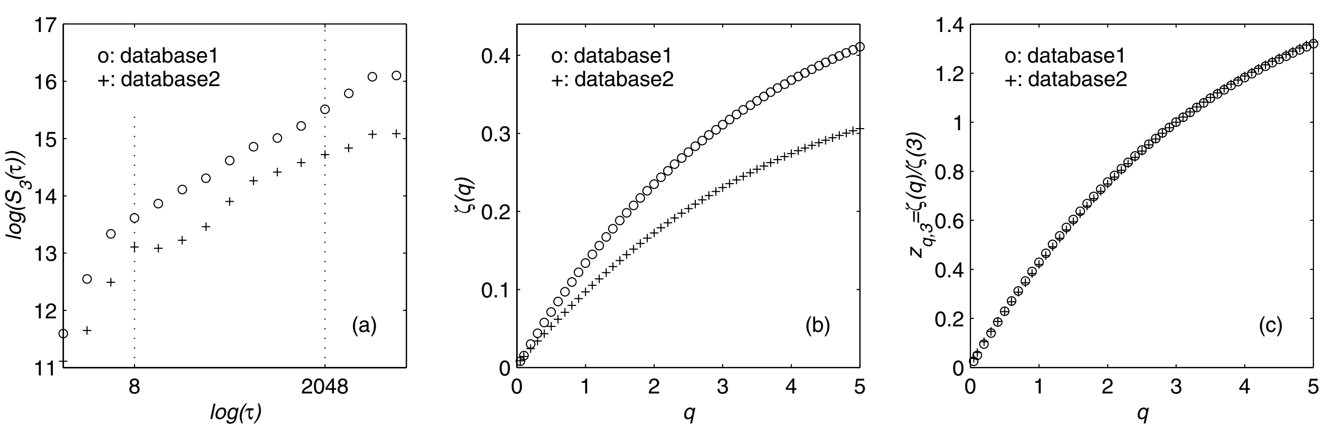

FIG. 2. (a) Structure function from typical subjects [notice the log-periodic modulation in the second data set (+)]; (b) group-averaged $\zeta(q)$; and (c) group-averaged $z_{q,3}$.

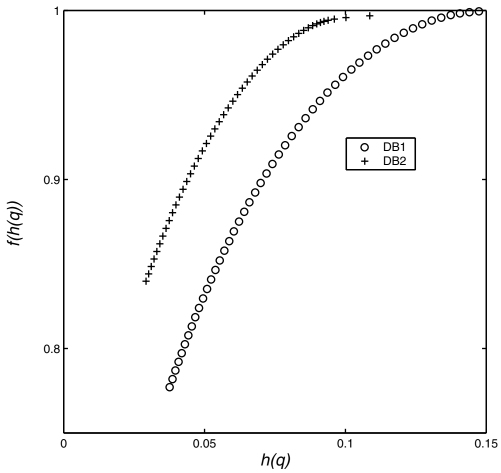

FIG. 3. $f(h)$ vs h.

of the PDF dominate the power law of S_q for large q, and typical events at the origin of the PDF dominate that of S_q for small q. For statistical reasons, it is desirable to consider small p, q. The $z_{q,p}$ based on the group-averaged $\zeta(q)$ converge to a close set of values: letting $p = 3$ and $q = 1 \sim 4$, $z_{q,3} = 0.43, 0.75, 1, 1.18$ for DB1 and $z_{q,3} = 0.42, 0.75, 1, 1.19$ for DB2 (Fig. 2c). In FDT, $z_{q,3}$ of the velocity increment is the same as $\zeta(q)$ given above since $\zeta(3) = 1$. These results suggest that not only HRV and FDT are qualitatively similar, but they are also quantitatively "close" in the sense of ESS.

The similar statistics do not imply details of the complex systems. But it does suggest that similar signal fluctuation generating mechanisms may be in play. To generate a nonstationary, multifractal field, Benzi *et al.* proposed using wavelets to model the velocity in FDT based on a dyadic cascade and power law wavelet coefficients [17]. Hausdorff and Peng proposed an $1/f$ model by adding Gaussian components at different time scales [18]. Both models are able to reproduce the desired statistics and generate multiscaled time series [6,17]. In general, intermittence and nonstationarity are major components in complex signals [19,20]. While the former is best simulated multiplicatively, the latter is additive in nature. In FDT, Marshak *et al.* proposed a bounded cascade to account for multiplication in the large scale and additivity in the small scale [20]. Based on these results, a procedure simulating the "cascade" in daytime HRV is proposed: The first cascade level is a flat field $r_0(t) = c_0$. A two-step procedure is then followed and repeated: (a) divide the time domain into random subintervals, and (b) multiply a random factor to the field at the subinterval. Let the field after J cascades be denoted as $r_J(t)$; then

$$r_J(t) = c_0 \prod_{j=1}^{J} \mathbf{w}_j(t), \tag{3}$$

where $\mathbf{w}_j$ is called the time scale component with $\langle \mathbf{w}_j \rangle = 1$ and $\sigma_j^2 = \langle (\mathbf{w}_j - 1)^2 \rangle$. $r_J(t)$ only updates its value at random time variables $t_k^{(J)}$'s which in turn form the random subintervals in (b). The number of such time intervals $\mathcal{N}_j$ at the jth cascade level also describes the characteristic time scale of $\mathbf{w}_j(t)$. For a self-similar process, it is necessary that $R_t \langle \mathcal{N}_j \rangle = \langle \mathcal{N}_{j+1} \rangle$ where $R_t > 1$ is a constant. In [20], $\mathbf{w}_j = 1 \pm f_j, f_j \to 0$ as $j \to \infty$. For the present case, σ_j decreases its value in j. Choosing random weights $\mathbf{w}_j$ and $\{t_k^{(j)}\}$ are attempts to model the time varying nature of the cardiovascular system.

By construction, it is noted that the large (small) amplitude fluctuation is contributed by the large (small) time scale components. The large scale introduced in the first few cascades are generated multiplicatively, whereas the small scale fluctuation is introduced to $r_J(t)$ additively since higher order terms are small due to the decreasing σ_j. Also, the simulated data are bounded due to the decreasing σ_j. This implies boundedness of the increment, which in turn sets an upper bound for $\zeta(q)$ in large q. Thus, $\zeta(q)$ cannot be a linear function from a bounded cascade.

The model simulation (3) is based on three parameters J, σ_j, and R_t and the probability laws of $\mathbf{w}_j$ and $\{t_k^{(j)}\}$. In this study, $J = 15$ and $R_t = 2$ were used. The Gaussian and uniform distributions were assumed for $\mathbf{w}_j$ and $t_k^{(j)}$,

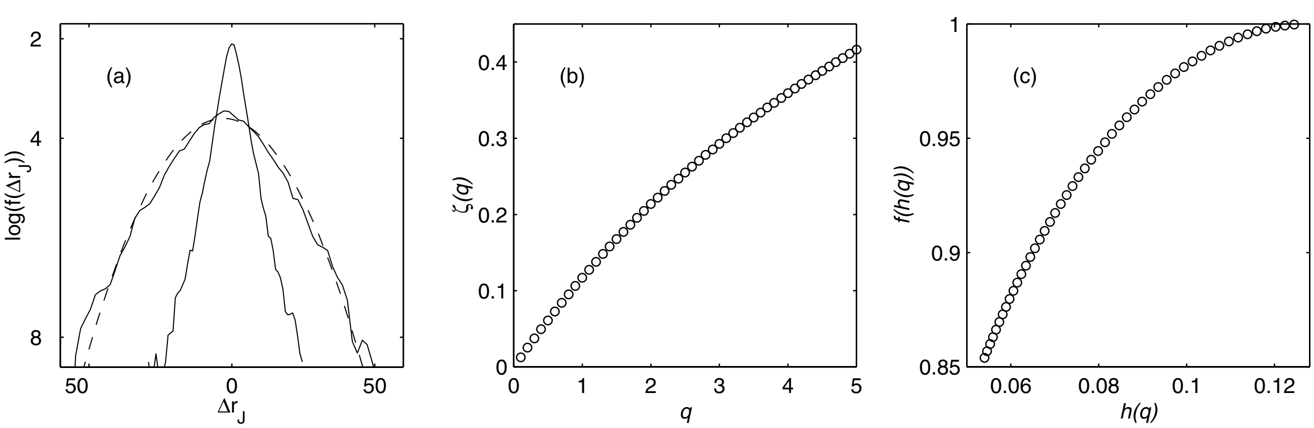

FIG. 4. (a) Increment PDF of $r_J(\tau)$ at $\tau = 1$ and $\tau = 4096$. The dashed line is the fit of a Gaussian PDF at $\tau = 4096$. (b) $\zeta(q)$ of $r_J(t)$, and (c) the multifractal spectrum $f(h)$ of $r_J(t)$.

respectively. For very small σ_j, $r_J(t)$ can be approximated by a sum of Gaussian variables. It is thus monoscaled. For large σ_j, the time series appears too intermittent to be considered in the normal range of HRV. Using $\sigma_j = R_t^{-2.5-j/J}$ made it possible to simulate some of the important RRi statistics, including power law spectrum, stretch-exponential-like increment PDF (Fig. 4a), $\zeta(q)$ (Fig. 4b), and $f(h)$ spectra (Fig. 4c).

In summary, we showed that there are similarities in the statistics and the generating mechanism of signal fluctuation between daytime RRi of healthy humans and the velocity in fluid turbulence. The similar group-averaged $\zeta(q)$ ratio based on ESS is interesting. Further tests are needed to examine the robustness of using this ratio to characterize HRV. Our preliminary study on the model lends hope for using ideas from FDT to study RRi fluctuation. In particular, Fig. 4 shows strong evidence that a cascade mechanism can generate some of the crucial statistics of daytime RRi fluctuation. Further studies are necessary to characterize detailed behaviors of the model and its implications. The juxtaposition proposed here should yield fruitful interactions between disciplines and better understanding of the similitudes of the two complex phenomena.

We thank NSERC of Canada and HSFO of Ontario for supports.

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