Two-Component Interference Effect: Model of a Spin-Polarized Transport

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The effect of spin-involved interaction on the transport properties of disordered two-dimensional electron systems with ferromagnetic contacts is described using a two-component model. Components representing spin-up and spin-down states are supposed to be coupled at a discrete set of points. We have found that due to the additional interference arising in two-component systems the difference between conductances for the parallel and antiparallel orientations of the contact magnetization changes its sign as a function of the length of the conducting channel.

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Spin-polarized transport in two-dimensional electron systems has been a field of growing interest during the last several years. Typically, the experiments are performed using a two-terminal device with ferromagnetic metal contacts. A spin polarization of the injected current is expected from the different densities of states for spin-up and spin-down electrons in the ferromagnetic source. This leads to a spin dependent interface resistance, which also exists at the interface of the second ferromagnetic contact, the drain. Together with spin-involved scattering processes within the studied electron system this should result in a conductance which depends on the relative magnetization of the two contacts [1].

The quantum mechanical nature of spin places it out of reach of many of the forces in a solid and the orientation of a carrier's spin can be very long-lived. The conductance $G^{\uparrow\uparrow}$ of a two-terminal device with parallel orientation of magnetic moments of the contacts is thus expected to be higher than the conductance $G^{\uparrow\downarrow}$ for the case of antiparallel moment orientation [1,2]. However, just the opposite results have been reported recently [3] for a two-dimensional electron gas confined in an InAs channel with the permalloy source and drain. It has been found that an ensemble average of the conductance difference $G^{\uparrow\uparrow} - G^{\uparrow\downarrow}$ decreases as a function of the channel length reaching negative values in a quasiballistic regime when the electron mean free path l_e becomes comparable with the channel length.

In the absence of magnetic impurities the natural candidate for spin dephasing and precession effects is the spin-orbit coupling to impurity atoms or defects. It is responsible for the so-called antilocalization effect [4]. Later attention was turned to the effects caused by a Rashba term [5,6] in two-dimensional [7–9] and quasi-one-dimensional systems [10–12]. However, a realistic transport theory for fully quantum coherent systems including the spin-orbit coupling to the impurities or defects has not yet been reported. The problem becomes complicated even if the electron motion is restricted to a two-dimensional space. In general, the spin-orbit interaction turns the problem back to three dimensions.

The goal of this paper is to reveal those features of the transport properties which can be caused by the spin-orbit interaction induced by a scattering potential. In the interesting case of a quasiballistic regime, which exhibits chaotic features, it is very difficult to estimate deviation from the exact solution caused by any used approximation. For this reason we have employed a simplifying two-component model with point interactions for which an exact solution, including fully the quantum coherence, can be found. Nonzero spin-orbit coupling is assumed to be associated with short-range scattering potentials only. Although the treatment is far from a realistic transport theory, it might be useful to understand some mysteries of the recent experimental observation [3].

The free electron system is a typical two-component system if the electron spin is taken into account. If there are no spin-involved forces, electron states are represented by plane waves $\exp(i\vec{k}\vec{r})$ with $\vec{k}$ being a wave vector. Orientation of the electron spin is given by the quantum number $s_z = \pm\frac{1}{2}$ and the electron system can be split into two independent subsystems, each of them composed of electrons having the same spin orientation. However, any perturbation of the background potential can cause a coupling between these subsystems due to a nonzero spin-orbit interaction.

Let us first consider a single scattering potential acting on a two-dimensional electron gas within a finite region of a radius r_0. In accord with the standard scattering theory an incoming wave belonging to one particular subsystem, say of the spin-up states, can be scattered into states belonging to both subsystems. In the short-range limit, $kr_0 \ll 1$, only the s part of the incoming wave gives a nonzero contribution to the scattering process. For a given energy $E = \hbar^2 k^2/2m$, $(k = |\vec{k}|)$, the corresponding solution of the radial Schrödinger equation has two components, $\Psi_\uparrow(r)$ and $\Psi_\downarrow(r)$. Outside the scattering region they can be written as follows [13,14]:

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$$\Psi_\uparrow(r) = J_0(kr) + a(k)H_0^{(1)}(kr),$$
$$\Psi_\downarrow(r) = b(k)H_0^{(1)}(kr), \tag{1}$$

where $J_0(z)$ denotes the cylindrical Bessel function and the Hankel function $H_0^{(1)}(z)$ represents scattered outgoing waves.

Taking into account the time reversal symmetry and assuming that the system is invariant with respect to the subsystem interchange, the amplitudes $a(k)$ and $b(k)$ in the short-range limit have to be of the following general form [14]:

$$a(k) = \frac{1 + \frac{2i}{\pi}(\gamma + \ln\frac{k}{2} - A)}{[1 + \frac{2i}{\pi}(\gamma + \ln\frac{k}{2} - A)]^2 + \frac{4}{\pi^2}|C|^2},$$
$$b(k) = \frac{2i}{\pi}C\left[1 + \frac{2i}{\pi}\left(\gamma + \ln\frac{k}{2} - A\right)\right]^{-1}a(k), \tag{2}$$

where A and C are real model parameters. If C is chosen to be zero, the parameter A represents the strength of the scattering process within one particular subsystem and it is related to the radius r_0 of the potential well, $A = \ln r_0$. Nonzero values of the parameter C give rise to a spin-flip process.

The relevant physical parameters characterizing the scattering event are the total scattering cross section σ_0,

$$\sigma_0 = a(k)^2 + b(k)^2, \tag{3}$$

and the spin-flip probability $t^{\uparrow\downarrow}$,

$$t^{\uparrow\downarrow} = \frac{b(k)^2}{a(k)^2 + b(k)^2}, \tag{4}$$

that can be expressed through the parameters A and C. Note that the assumption of the system invariance with respect to the subsystem interchange leads to the independence of σ_0 and $t^{\uparrow\downarrow}$ on the spin orientation of the incoming electron. This assumption has been made for simplicity despite the fact that it need not be satisfied in real systems, e.g., due to a Rashba term [5,6].

The scattering problem for a two-dimensional strip with a finite number of short-range scatterers, as sketched in Fig. 1, can be solved exactly. The scattering matrix for the case of a one-component system without boundaries is well known [13] and the detail analysis for a finite strip has also been reported [15]. The generalization to a two-component system is straightforward and will be published elsewhere. For simplicity we have assumed that all scatterers are identical; i.e., they have the same scattering cross section σ_0 and spin-flip probability $t^{\uparrow\downarrow}$ if they would be placed alone within the two-dimensional space. The scattering matrix has been obtained numerically for a given configuration of point scatterers randomly distributed within a strip region of the length L.

Spin-dependent transport properties are determined by partial transmission coefficients representing transition between left and right subsystems of asymptotic spin-up or

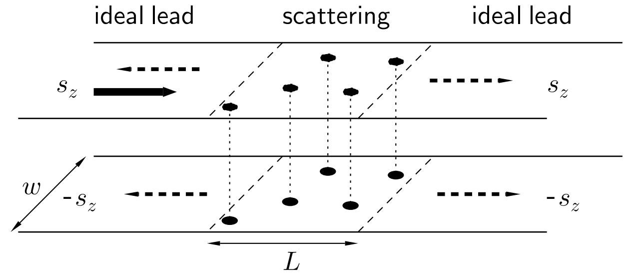

FIG. 1. Scheme of the scattering process in a two-component system. Upper and lower strips of the width w represent the spin subsystems. Scatterers, black points, serve also as connection points between subsystems giving rise to spin-flip processes. Thick full and dashed lines represent incoming and outgoing waves, respectively.

spin-down states. They are defined as the sum of transmission probabilities over all relevant modes of asymptotic states. To simplify the description by excluding the quantum fluctuations from our consideration we have used configurationally averaged values of the partial coefficients to define 2×2 transmission and reflection matrices $\mathbf{T}$ and $\mathbf{R}$, respectively:

$$\mathbf{T} \equiv \begin{pmatrix} T^{\uparrow\uparrow} & T^{\uparrow\downarrow} \\ T^{\downarrow\uparrow} & T^{\downarrow\downarrow} \end{pmatrix}, \qquad \mathbf{R} \equiv \begin{pmatrix} R^{\uparrow\uparrow} & R^{\uparrow\downarrow} \\ R^{\downarrow\uparrow} & R^{\downarrow\downarrow} \end{pmatrix}. \tag{5}$$

For the considered symmetrical system $T^{\uparrow\uparrow} \equiv T^{\downarrow\downarrow}$, $T^{\uparrow\downarrow} \equiv T^{\downarrow\uparrow}$, $R^{\uparrow\uparrow} \equiv R^{\downarrow\downarrow}$, and $R^{\uparrow\downarrow} \equiv R^{\downarrow\uparrow}$.

In Fig. 2 the dependence of partial transmission coefficients on the length L of the scattering region is shown for different spin-flip probabilities $t^{\uparrow\downarrow}$. Since in the absence of magnetic impurities a weak coupling between subsystems is expected we limit presented numerical examples to the case of small values of $t^{\uparrow\downarrow}$. The used energy corresponds to 31 occupied subbands. Concentration of scatterers ($750/w^2$) and the scattering cross section $\sigma_0 = 0.1217$ were held fixed. These are typical values used to model localization effects in two-dimensional systems.

For some values of the spin-flip probability and lengths L, $T^{\uparrow\uparrow}$ becomes less than $T^{\uparrow\downarrow}$. It means that the polarization of the transmitted current has opposite orientation from the polarization of the injected current. We ascribe this unexpected result to the additional interference appearing in two-component systems. Its origin can be more easily cleared up for the case of a one-dimensional conductor, which leads to the same qualitative results. It can easily be shown that in this case the two-component spin states can be expressed as a linear superposition of states that do not change their orientation during the scattering process. For a single short-range scatterer located at $x = 0$ the transmitted waves have the following form:

$$\Psi_\uparrow(x > 0) = \frac{1}{2}(t_1^{(+)} + t_1^{(-)})e^{ikx},$$
$$\Psi_\downarrow(x > 0) = \frac{1}{2}(t_1^{(+)} - t_1^{(-)})e^{ikx}, \tag{6}$$

where

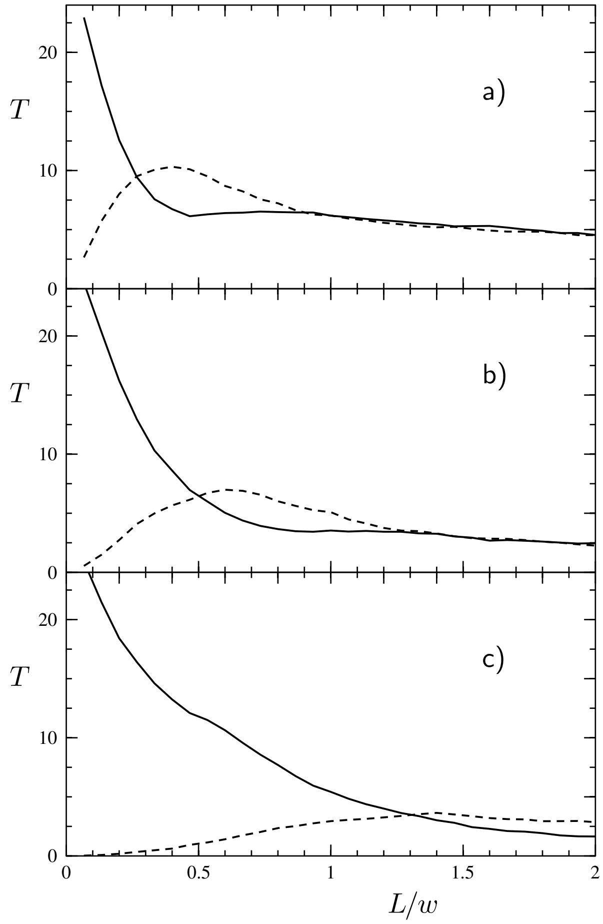

FIG. 2. The partial transmission coefficients $T^{\uparrow\uparrow}$ (full line) and $T^{\uparrow\downarrow}$ (dashed line) as a function of the scattering region length L for several values of the spin-flip probability: (a) $t^{\uparrow\downarrow} = 8.1 \times 10^{-3}$; (b) $t^{\uparrow\downarrow} = 0.2 \times 10^{-3}$; (c) $t^{\uparrow\downarrow} = 0.0005 \times 10^{-3}$.

$$t_1^{(\pm)} = \frac{1}{1 + i\frac{A}{2k}\frac{1}{1 \pm AC}}, \tag{7}$$

and two real parameters A and C representing the scattering have a similar meaning as that introduced above. The amplitudes of transmitted waves have a similar form for the case of n scatterers, and partial transmission coefficients are given as follows:

$$T_{1\mathrm{D}}^{\uparrow\uparrow} = \frac{1}{4}\,|t_n^{(+)} + t_n^{(-)}|^2; \; T_{1\mathrm{D}}^{\uparrow\downarrow} = \frac{1}{4}\,|t_n^{(+)} - t_n^{(-)}|^2, \tag{8}$$

where $t_n^{(+)}$ and $t_n^{(-)}$ are amplitudes of transmitted waves through the one-component one-dimensional system with n scatterers represented by short-range potentials $A(1 + AC)^{-1}\delta(x - X_i)$ or $A(1 - AC)^{-1}\delta(x - X_i)$, respectively. There are no transitions between these one-dimensional subsystems and each of them has its own localization length. Transmitted waves $t_n^{(+)}\exp(ikx)$ and $t_n^{(-)}\exp(ikx)$ acquire different phase shifts and the resulting interference between them is the origin of strongly damped oscillations of the difference $T_{1\mathrm{D}}^{\uparrow\uparrow} - T_{1\mathrm{D}}^{\uparrow\downarrow}$ with increasing number of scatterers, i.e., with scattering region length L. A detailed analysis of this one-dimensional case will be published elsewhere.

While the partial transmission coefficients are affected by the above discussed additional interference, the total transmission coefficient $T = 2(T^{\uparrow\uparrow} + T^{\uparrow\downarrow})$ is not affected and it shows the standard antilocalization effect [4]. The value of T increases with increasing spin-flip probability, as shown in Fig. 3.

The device conductance of a two-component quantum system is determined by a matrix of partial transmission coefficients, $\mathbf{T}_{\mathrm{dev}}$,

$$G = \frac{e^2}{h}\,(1,1)\mathbf{T}_{\mathrm{dev}}\begin{pmatrix}1\\1\end{pmatrix}; \qquad \mathbf{T}_{\mathrm{dev}} \equiv \begin{pmatrix} T_{\mathrm{dev}}^{\uparrow\uparrow} & T_{\mathrm{dev}}^{\uparrow\downarrow} \\ T_{\mathrm{dev}}^{\downarrow\uparrow} & T_{\mathrm{dev}}^{\downarrow\downarrow}\end{pmatrix}, \tag{9}$$

which depend on the properties of ferromagnetic contacts and their interfaces with the two-dimensional electron gas. To estimate their effect we have used the idea of polarization filters [12]. The source and the drain are considered to be standard reservoirs and spin-dependent effects are modeled by filters placed within the asymptotic region of ideal leads.

For the sake of simplicity we assume that electrons reflected by the filter will be equally distributed between all available quantum channels without any change of their spin orientation and that their coherence is completely destroyed. In this case the filtering effect can be described by the 2×2 diagonal matrix of reflection probabilities α and β:

$$\mathbf{F}_{s,d} \equiv \begin{pmatrix} \alpha_{s,d} & 0 \\ 0 & \beta_{s,d}\end{pmatrix}, \tag{10}$$

where indices s and d represent the source filter and the drain filter, respectively.

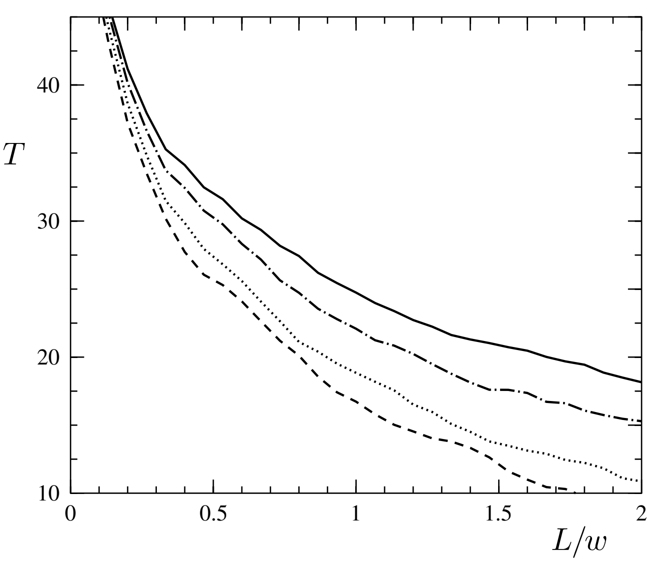

FIG. 3. The total transmission coefficient T as a function of the scattering region length L for several values of the spin-flip probability: $t^{\uparrow\downarrow} = 8.1 \times 10^{-3}$ (full line), $t^{\uparrow\downarrow} = 4.2 \times 10^{-3}$ (dash-dotted line), $t^{\uparrow\downarrow} = 1.1 \times 10^{-3}$ (dotted line), and $t^{\uparrow\downarrow} = 0.0005 \times 10^{-3}$ (dashed line).

The conductance of the above described model device can be expressed as a function of the already defined coefficients $T^{\uparrow\uparrow}$, $T^{\uparrow\downarrow}$, $R^{\uparrow\uparrow}$, and $R^{\uparrow\downarrow}$ describing the scattering process of the same device without filters. For the case of N available quantum channels (subbands) within each subsystem we get the following expression for the transmission matrix $\mathbf{T}_{\text{dev}}$ entering Eq. (9):

$$\mathbf{T}_{\text{dev}} = (\mathbf{1} - \mathbf{F}_d)\mathbf{N}\mathbf{M}_d\mathbf{T}\mathbf{K}_{d,s}\mathbf{N}\mathbf{M}_s(\mathbf{1} - \mathbf{F}_s), \qquad (11)$$

where $\mathbf{N}$ stands for the product of N and unit matrix $\mathbf{1}$. The effect of multiple reflections between filters and the scattering region is represented by matrices $\mathbf{M}_s$ and $\mathbf{M}_d$:

$$\mathbf{M}_s = (\mathbf{N} - \mathbf{F}_s\mathbf{R})^{-1}; \qquad \mathbf{M}_d = (\mathbf{N} - \mathbf{R}\mathbf{F}_d)^{-1}, \quad (12)$$

and

$$\mathbf{K}_{d,s} = [\mathbf{1} - \mathbf{M}_s\mathbf{F}_s\mathbf{T}\mathbf{F}_d\mathbf{M}_d\mathbf{T}]^{-1}. \qquad (13)$$

The device conductance, Eq. (9), depends on the reflection probabilities $\alpha_{s,d}$ and $\beta_{s,d}$ modeling the effect of ferromagnetic contacts. For the case of the parallel orientation of the contact magnetization the conductance $G^{\uparrow\uparrow}$ can be obtained by setting $\alpha_s \equiv \alpha_d$ and $\beta_s \equiv \beta_d$. To get $G^{\uparrow\downarrow}$ for the antiparallel contact magnetization $\alpha_s \equiv \beta_d$ and $\beta_s \equiv \alpha_d$ have to be used. While the conductance values strongly depend on the reflection probabilities, the sign of the conductance difference $\Delta G \equiv G^{\uparrow\uparrow} - G^{\uparrow\downarrow}$ is not affected. In Fig. 4 the relative conductance change

$$\frac{\Delta G}{G_0} \equiv 2\,\frac{G^{\uparrow\uparrow} - G^{\uparrow\downarrow}}{G^{\uparrow\uparrow} + G^{\uparrow\downarrow}} \qquad (14)$$

as a function of the scattering-region length L is shown for the case of ideal filters, $\alpha_s = 1$ and $\beta_s = 0$. It corresponds to injection of fully polarized current and vanishing interface resistance. Other used parameters are the same as that for transmission coefficients presented in Fig. 2.

Spin-injection experiments usually show a large interface resistance. Also, the real devices are of the larger dimensions, usually hundreds of occupied subbands, and evaluation of the corresponding scattering matrix is more time-consuming. Nevertheless, the obtained results have the same qualitative features as that described above. Using the model parameters relevant for the device studied by Hu *et al.* [3], nearly quantitative agreement with the measured data can be reached if the value $t^{\uparrow\downarrow} = 4.8 \times 10^{-3}$ is chosen for spin-flip probability, as shown in the inset of Fig. 4.

The main result of the described model is that in mesoscopic disordered systems the additional quantum coherence arising in two-component systems can lead to a higher conductance of two-terminal devices with antiparallel contact magnetization than that for parallel configuration.

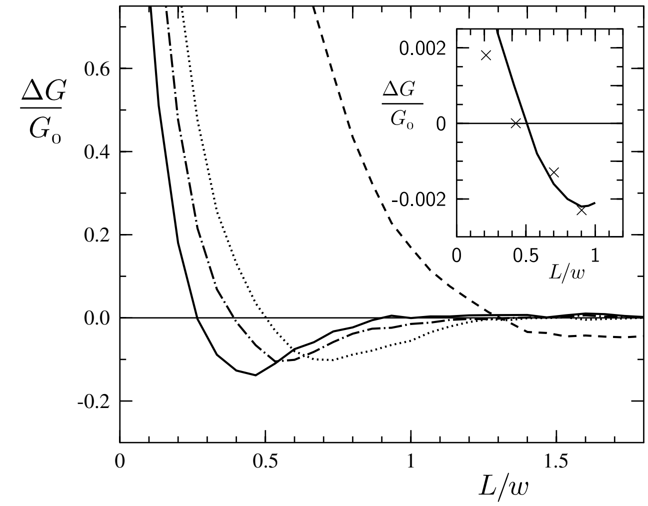

FIG. 4. Relative conductance change $\Delta G/G_0$ as a function of the scattering region length L for several values of the spin-flip probability: $t^{\uparrow\downarrow} = 8.1 \times 10^{-3}$ (full line), $t^{\uparrow\downarrow} = 1.1 \times 10^{-3}$ (dash-dotted line), $t^{\uparrow\downarrow} = 0.2 \times 10^{-3}$ (dotted line), and $t^{\uparrow\downarrow} = 0.0005 \times 10^{-3}$ (dashed line). In the inset crosses represent experimental data obtained by Hu *et al.* and the full line is the result of the model calculation for the following parameters: $N = 173$, $\alpha_s = 0.01$, $\beta_s = 0$, scatterer concentration $1500/w^2$, $\sigma_0 = 0.1341$, and $t^{\uparrow\downarrow} = 4.8 \times 10^{-3}$.

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