Intermittent Implosion and Pattern Formation of Trapped Bose-Einstein Condensates with an Attractive Interaction

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The collapsing dynamics of a trapped Bose-Einstein condensate with attractive interaction are revealed to exhibit two previously unknown phenomena. During the collapse, the condensate undergoes a series of rapid implosions that occur *intermittently* within a very small region. When the sign of the interaction is suddenly switched from repulsive to attractive, e.g., by the Feshbach resonance, density fluctuations grow to form various patterns such as a shell structure.

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Bose-Einstein condensation (BEC) of trapped atomic vapor has been realized in ^{87}Rb [1], ^{23}Na [2], ^{1}H [3], and ^{7}Li [4]. The last species is unique in that it has a negative s-wave scattering length, implying that the interactions between atoms are predominantly attractive. It has been believed that, in a spatially uniform system, such atomic vapor would collapse into a denser phase. However, when the system is spatially confined and when the number of BEC atoms is below a certain critical value $N_{\rm c}$, the zero-point motion of the atoms serves as a kinetic obstacle against collapse, allowing a metastable BEC to be formed [5–15]. Just below $N_{\rm c}$, BEC may collapse via macroscopic quantum tunneling [10,13,16–19], and, above N_c, it is predicted [13] that the collapse will occur not globally, but only locally near the center of BEC where the atomic density exceeds a certain critical value. Inelastic collisions also lead to the decay of BEC in regions where the atomic density is very high [9].

In current experiments [4], there are abundant above-condensate atoms that replenish the lost atoms, allowing BEC to grow again. We may therefore expect collapse-and-growth cycles of BEC to occur. Various mechanisms that give rise to these oscillations have been discussed [13,20–22]. And, indeed, recent experiments [23] have suggested the occurrence of dynamic collapse-and-growth cycles of BEC, but the results have neither favored nor excluded any one of these possible mechanisms.

The s-wave scattering length of atoms can be varied using the Feshbach resonance [24], suggesting that not only the strength of the interaction but also its sign can be controlled. By using this technique, the sign of the interaction has recently been successfully switched from positive to negative in BEC of ^{85}Rb [25]. Kagan *et al.* [26] discussed a global collapse of the condensate in such situations.

In this Letter, we predict two new phenomena associated with the collapse of BEC. One is *intermittent implosion,* in which the local collapses occur in rapid sequence at the center of the condensate. This phenomenon is caused by competition between the attraction of atoms towards the trap center and the loss of atoms by inelastic collisions. While our analysis is based on the theory developed by Kagan *et al.* [22], this phenomenon is different from the collapse-and-growth cycles and other fine structures predicted by them. The other prediction is that of pattern formation in the atomic density, following a sudden switch in sign of the interaction from repulsive to attractive. This phenomenon occurs because density fluctuations caused by the change in sign of the interaction grow and self-focus due to the attractive interactions.

We consider a system of Bose-condensed atoms with mass m and s-wave scattering length a, confined in a parabolic potential. When inelastic collisions can be neglected, the dynamics of the condensate wave function is well described by the Gross-Pitaevskii (GP) equation [27] $i\partial_t\psi = -\nabla^2\psi/2 + r^2\psi/2 + g|\psi|^2\psi$, where the length, time, and ψ are normalized in units of $d_0 = (\hbar/m\omega_0)^{1/2}$, ω_0^{-1}, and $(N_0/d_0^3)^{1/2}$, respectively, and $g \equiv 4\pi N_0 a/d_0$ is the dimensionless strength of the interaction. The wave function is then normalized to unity.

The metastability of BEC with attractive interactions may be understood by the Gaussian approximation [6,10,18]. If we approximate the wave function as having a Gaussian form with size R, the effective potential for R is given by $V_{\rm eff} = 3(R^{-2} + R^2)/2 + \gamma R^{-3}/2$ with $\gamma \equiv (4N_0/\sqrt{2\pi})(a/d_0)$. The effective potential $V_{\rm eff}$ has a local minimum when $N_0 < N_c \equiv (\sqrt{2\pi}\, d_0/4a)\gamma_c$; therefore, the metastable BEC is formed. Since the approximation breaks down when a rapid implosion takes place, we numerically solve the GP equation without resorting to the Gaussian approximation.

Because the peak density grows very high once the collapse begins, we must include in the GP equation the atomic loss due to inelastic collisions. Following the treatment in Ref. [22], we employ the GP equation with loss processes as $i\partial_t\psi = -\nabla^2\psi/2 + r^2\psi/2 + g|\psi|^2\psi - i(L_2|\psi|^2/2 + L_3|\psi|^4/6)\psi/2$, where L_2 and L_3 denote the two-body dipolar and three-body recombination loss-rate coefficients, respectively. The two-body (three-body) loss-rate coefficients must be divided by two (six) because of Bose statistics [28]. We assume that the atoms and

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molecules produced by inelastic collisions escape from the trap without affecting the condensate. Taking $\omega_0 = 2\pi \times 144.5$ Hz [23] and the loss-rate coefficients from Refs. [29,30], we have $L_2 = 3.7 \times 10^{-7} N_0$ and $L_3 = 2.9 \times 10^{-10} N_0^2$. To integrate the GP equation, we employ the finite difference method with the Crank-Nicholson scheme [5]. Since the implosion is extremely rapid, we very carefully controlled the step size to avoid error propagation during the implosion.

Figure 1 shows the time evolution of the peak height of the wave function $|\psi(r = 0, t)|$ (solid curve), the number of BEC atoms $N_0(t)$ (dashed curve), and the absolute squared overlap of the wave function with the initial one $|\int d\mathbf{r}\, \psi^*(\mathbf{r}, 0)\psi(\mathbf{r}, t)|^2$ (dotted curve) for $N_0(0) = 1260$, which is slightly (0.7%) greater than N_c. The last quantity gives an estimate of the "condensate fraction" that is measured by the absorption or phase-contrast imaging (see discussions below). We first prepared BEC in a metastable state that lay just below the critical point, and then increased $|a|$ (or tightened the trap potential) so that $N_0|a|/d_0$ exceeded its critical value. In the early stage of the collapse, the atomic density increases very slowly and the inelastic collisions are unimportant. At $t \simeq 2.87$ a rapid implosion breaks out, which is blown up in the inset of Fig. 1. If the atomic loss were not included, the peak density would grow unlimitedly, and the implosion would occur only once. With the atomic loss, however, the implosion stops in a very short time, and the peak density shows pulselike behavior. This implosion occurs intermittently several times, and with each implosion several tens of atoms are lost from the condensate. As a result, the number of atoms decreases in a stepwise fashion, eventually reaching approximately 78% of its initial value N_0. Since the

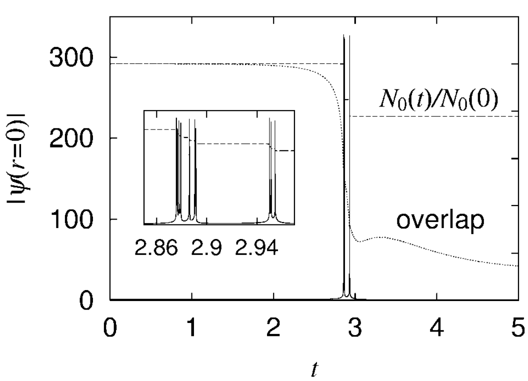

FIG. 1. Time evolution of the wave function $|\psi(r = 0, t)|$ (solid curve referring to the left axis), the number of atoms $N_0(t)/N_0(0)$ (dashed curve referring to the right axis), and the absolute squared overlap of the wave function with the initial one $|\int d\mathbf{r}\, \psi^*(\mathbf{r}, 0)\psi(\mathbf{r}, t)|^2$ (dotted curve referring to the right axis) with two-body dipolar loss and three-body recombination loss. We first prepared BEC in a metastable state slightly below the critical point, and, at $t = 0$, we increased $|a|$ so that $N_0|a|/d_0$ exceeded its critical value. The inset enlarges the view of the intermittent implosion.

collisional loss of atoms takes place where the atomic density is extremely high, the atomic loss primarily occurs within a very localized spatial region $r \lesssim 0.01$.

The duration of the spike shown in Fig. 1 is typically $\Delta t \sim 10^{-3}$. The use of mean-field theory with such rapid dynamics can be justified as follows. The mean-field approximation is applicable for time scales longer than $1/gn$, which is of order 1 in the metastable state of BEC (note that the time is measured in units of ω_0^{-1}). In the region of implosion, the density n becomes more than 10^4 times as high as that of the metastable state, i.e., $1/gn \lesssim 10^{-4}$, and thus the mean-field theory is valid even with the rapid implosion if the relevant time scale is longer than 10^{-4}—which is the case for the situation shown in Fig. 1. The gas parameter na^3 is, on the other hand, $\sim 10^{-2}$ in the region of implosion, and the atoms are still acting in a weakly interacting regime.

The pulselike behavior of the peak atomic density may be interpreted as follows. Initially, the condensate has a negative pressure due to attractive interactions and shrinks towards the central region. When the peak density becomes $g|\psi|^2 \sim L_3|\psi|^4/12$, i.e., $|\psi|^2 \sim 12g/L_3$, the collisional loss rate of the atoms becomes comparable to the accumulation rate of atoms at the center. Since the kinetic and interaction energies depend on the atomic density and its square, respectively, the total energy increases upon the loss of atoms [22]. The atoms near the center of BEC thus acquire outward momentum, and the pressure becomes positive. The change in sign of the pressure may be qualitatively explained by the Gaussian approximation. Initially $-\partial V_{\rm eff}/\partial R \simeq 3R^{-3} - \frac{3}{2}|\gamma|R^{-4} < 0$, which corresponds to negative pressure; later, however, the sign of pressure becomes positive when the number of atoms (or $|\gamma|$) decreases. After the implosion, inward flow outside the region of the implosion replenishes the peak density, turning the sign of the pressure again to negative, which induces the subsequent implosion.

When the inward flow is insufficient to reverse the sign of the pressure to negative, implosion ceases and the atoms are pushed outwards. This phenomenon was predicted in Ref. [22], and has recently been observed at JILA [25] as an atom burst emanating from a remnant condensate. According to the estimation in the previous paragraph and our numerical analysis [15], the energy scale of implosion and that of subsequent explosion are both proportional to g^2/L_3. In the case of ^{7}Li, the mean energy of the expanding atom cloud is $\simeq 80$ μK [15]. The atoms and molecules are also scattered by three-body recombination, in which the release energy is of the order 1 mK. The experimental signature of the intermittent implosion should be a series of bursts of atoms and molecules produced by the above two mechanisms.

In the Rice experiments [23], it has been observed that the number of BEC atoms reduces to 10% ~ 20% of the critical number after the collapse, which is smaller than our theoretical evaluation (dashed curve in Fig. 1). In

Ref. [23], however, only atoms around the peak of the bimodal distribution are counted as the number of BEC atoms, while the part of BEC atoms that expands broadly following the implosion is hidden in the thermal cloud. The number of BEC atoms that was measured experimentally is roughly estimated from the absolute squared overlap of the wave function with the initial metastable one (dotted curve in Fig. 1) which decreases to below 0.2. Thus our result is consistent with the observation of Ref. [23].

The intermittent implosion should be distinguished from the two types of oscillations discussed in Ref. [22], i.e., the collapse-and-growth cycles and the piecemeal collapses originating from oscillations of the entire condensate. The mechanism that causes the intermittent implosion is quite different from that of those oscillations, as discussed above. No intermittent implosion is discussed in Ref. [22], as the collisional loss rate used there is much larger than that used in this Letter.

The scenario for the collapse of BEC in the presence of condensate growth is, therefore, as follows. The BEC grows by being fed by the above-condensate atoms, and, when N_0 exceeds N_c, an implosion occurs that has the intermittent structure shown in Fig. 1. Some of the atoms are lost in the implosion, but they are subsequently replenished, giving rise to the collapse-and-growth cycles. During these cycles, small collapses occur with period $\sim\omega_0^{-1}$ due to oscillations of the entire condensate. These small collapses also have intermittent structures, which we have confirmed by large-scale numerical simulations.

We next consider the situation in which BEC with repulsive interaction is prepared, and the sign of the interaction is then suddenly switched to attractive. Such a situation has recently been realized at JILA [25] using the Feshbach resonance. Figure 2(a) shows the time evolution of the wave function, where we assume that, at $t = 0$, 10^6 ^{23}Na atoms with $a = 2.75$ nm are condensed in the trap with frequency $\omega_0 = 100 \times 2\pi$ s^{-1}. We then change a to a negative value of -1 nm. We follow Ref. [31] to use $L_2 = 1.7 \times 10^{-8} N_0$ and $L_3 = 1.1 \times 10^{-10} N_0^2$. When the interaction is switched to attractive, the atoms begin to compress, as the initial wave function has been expanded due to the repulsive interaction. The inward flow gives rise to a ripple, which then grows up to be a series of pulses, as shown in Fig. 2(a) (the wave function at $t = 0.79$ is multiplied by 0.1). The growth of the density fluctuations can be attributed to the attractive interaction, as the atoms tend to accumulate where the density is high. The ripple is caused by the momentum acquired by the atoms sliding down the trap potential. In fact, it can be seen in Fig. 2(a) that the spaces between adjacent pulses near the trap center are smaller than the outer spaces. Since the potential energy becomes the kinetic energy as $\hbar^2 k^2/2m \sim m\omega_0^2 R^2/2$, where R is the initial dimension of the wave function, the wavelength of the ripple is estimated to be $\lambda/d_0 \sim 2\pi d_0/R$.

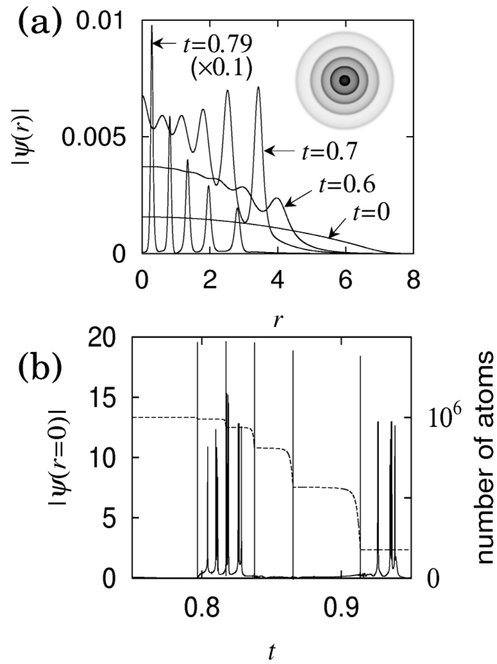

FIG. 2. (a) Time evolution of the wave function $|\psi(r,t)|$. A condensate of 10^6 ^{23}Na atoms was prepared, and, at $t = 0$, the s-wave scattering length was changed from 2.75 nm to -1 nm. The trap frequency is $\omega_0 = 100 \times 2\pi$ s^{-1}, and the loss-rate coefficients are described in the text. The wave function at $t = 0.79$ is multiplied by 0.1. The inset shows the gray-scale image of the column density at $t = 0.79$. (b) Time evolution of the peak height of the wave function $|\psi(r = 0, t)|$ (solid curve referring to the left axis) and the number of the remaining BEC atoms (dashed curve referring to the right axis).

The inset of Fig. 2(a) shows a gray-scale image of the column density integrated along the z axis, $\int_{-\infty}^{\infty} |\psi(x,y,z)|^2\, dz$, at $t = 0.79$. This quantity is proportional to the optical thickness in both absorption and phase-contrast imaging, where the laser light propagates along the z axis. The concentric circles represent the formation of a shell structure in the atomic density.

The shell-structure formation is due to the instability of the initial atomic distribution and the self-focusing effect, while the collisional loss is unimportant for that formation since the atomic density is not as high at the early stage of collapse when this phenomenon appears. When implosion begins, however, the collisional loss of atoms begins to play an important role. Figure 2(b) shows the time evolution of the peak height of the wave function $|\psi(r = 0, t)|$ (solid curve) and the number of BEC atoms (dashed curve). The peak density *decreases* at about $t = 0.79$ because the atoms near the center are attracted towards the innermost shell, as shown in Fig. 2(a). The shells move inward, and the first implosion occurs when the innermost shell arrives at the center of the trap. In Fig. 2(b), five implosions caused by the arrivals of the shells at the center of the trap are shown. The number of lost atoms associated with such implosions becomes larger for the outer shell, since the number of atoms contained in each shell is proportional to the square of its original radius. The velocity of shells moving inward is roughly determined by the free motion of atoms, and the collapse time is on the order of ω_0^{-1}. Between these implosions, the smaller intermittent implosions discussed above occur. During each intermittent implosion, several tens of atoms are lost, a loss that cannot be discerned in Fig. 2(b) because the total number of atoms is by far greater.

In the case of an axially symmetric trap, the pattern of the atomic density arising from the change of the interaction is sensitive to the asymmetry of the trap. We performed numerical simulations and found that various patterns can be formed. For a pancake-shaped trap, the atomic motion associated with the change in the interaction is larger in the axial direction than in the radial direction. As a result, the ripple arises in an axial direction, leading to a layered structure. For a cigar-shaped trap, the ripple arises in a radial direction, leading to a cylindrical shell structure. These structures undergo a complicated evolution and show various patterns such as rings and clusters. Intermittent implosions also occur for an axially symmetric trap. The details of these phenomena will be reported elsewhere.

While we have analyzed the specific examples of ^{7}Li and ^{23}Na, the results should be valid for other atomic species in which the s-wave scattering length can be varied, since the parameters g and L_3 can be controlled by choosing a and N_0. With other values of g and L_3, the behaviors may be qualitatively different. When L_3 is much larger than the value used here, no intermittent implosion occurs. Formation of a shell structure depends on the value of g that is proportional to N_0. For instance, in the situation of Fig. 2, when the initial number of atoms is $N_0 = 10^5$, the number of shells is reduced to two, and, when $N_0 = 10^4$, no shell structure appears.

In conclusion, we have predicted two new phenomena concerning the collapsing dynamics of BEC with attractive interactions. That is, the intermittent implosion in the atomic density occurs very rapidly compared with the inverse trap frequency, and in a very localized region compared with the trap size. When the sign of the interaction is suddenly switched from repulsive to attractive, the atomic density forms a shell structure in a spherically symmetric trap, and various patterns are formed for an axially symmetric trap.

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