Electrostatic Mode Excitation in Electron Holes due to Wave Bounce Resonances

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A kinetic theory of resonant interaction between electrostatic waves and the bounce motion of electrostatically trapped electrons is developed. Precise criteria are derived for the stability of electrostatic potential structures which trap electrons in a highly magnetized plasma. The theory explains the energy transfer from electron phase space holes to waves observed in simulations. It may also account for the destabilization of electrostatic waves propagating obliquely to the geomagnetic field and some characteristics of the holes as observed in the auroral ionosphere.

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Satellites have recently measured bipolar electric field pulses in the current region of the auroral zone, the magnetotail, and the foreshock. These pulses have been linked to electron phase space holes, stable nonlinear solutions of the Vlasov equation, corresponding to areas of lowered electron density and an associated positive potential that maintains a population of trapped electrons [1–4]. Simulations of magnetized holes in two and three dimensions show them persisting for many hundreds of plasma periods before eventually developing kinks and decaying while emitting Langmuir waves propagating obliquely to the ambient magnetic field, **B** [5]. These waves have also been called electrostatic whistler waves [5,6]. In this Letter, we develop a general treatment of the resonant interaction of such electrostatic waves with the bounce motion of electrons electrostatically trapped in phase space holes. This interaction may be important in the auroral ionosphere, linking two widely observed phenomena: electrostatic lower hybrid waves and phase space holes.

This Letter begins with a general discussion of phase space holes and the intense research activity their recent discovery in the Earth's space environment has spawned. Then, we outline the derivation of wave growth rates due to wave-bounce resonances with electrons trapped in a hole. Next we use our theory to explain the simulation results. We conclude with a discussion of the implications of our theory for electron holes observed by satellites.

Electron phase space holes were originally discovered during simulations of the nonlinear stage of the evolution of the two-stream instability [7]. Their stability has been examined in terms of Bernstein-Green-Kruskal (BGK) modes, and nonlinear Landau damping [8–10]. Theoretical studies of phase space holes have acquired renewed importance because of recent measurements of bipolar electric field pulses—a signature of phase space holes—made in the Earth's ionosphere, magnetosphere, and foreshock region [2,4,11–13].

Simulations have contributed to our understanding of the complex nonlinear dynamics associated with electron phase space holes [7,9]. Starting with two cold, counterstreaming, electron beams, parallel to a uniform magnetic field, simulations show the growth of a linear two-stream instability followed by the development of electron holes with a phase space configuration illustrated in Fig. 1. The discovery of holes in the Earth's space environment has triggered a number of additional numerical studies with results applicable to holes in space [5,13,14]. The research presented in this paper is largely motivated by the discovery that large-scale, 2D, electrostatic simulations show that electron holes slowly decay while emitting electrostatic waves [3,5]. These waves obey the dispersion relation $\omega = \omega_p \cos\theta$ where θ is the angle between the wave vector **k** and **B**. The fastest growing modes propagate close to but not perpendicular to **B**, with $|\mathbf{k}|\lambda_D < 0.3$ where λ_D is the Debye length.

To study electron hole stability, we start with a single hole in a 1D equilibrium, and evaluate its stability against 3D perturbations. We are interested in strongly magnetized electron holes, where $\Omega_e \gg \omega_p$ and Ω_e, ω_p are the electron cyclotron and plasma frequencies, respectively. Thus, we are not in the regime treated by Muschietti *et al.* [15]. In our frame of reference the hole is not moving. Since

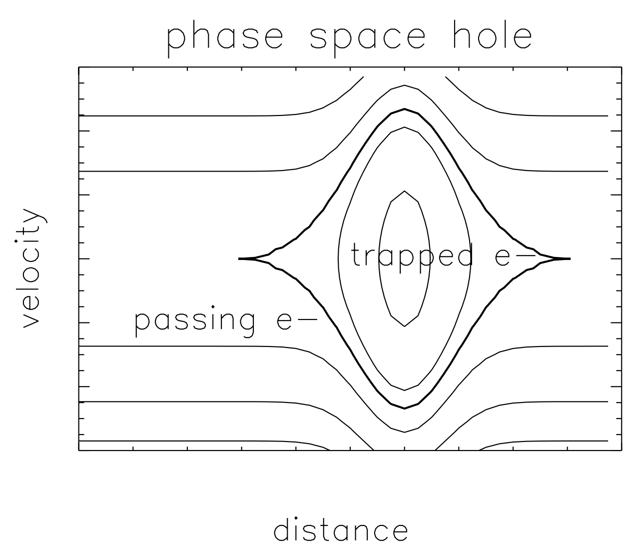

FIG. 1. Sketch of a phase space hole in 1D.

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electrons are highly magnetized we assume they move only along x which is the direction parallel to $\mathbf{B}$. Also, we ignore ion dynamics. The hole potential, $\Phi_0(x)$, and the equilibrium distribution, f_0, satisfy

$$v\partial_x f_0 - \frac{q}{m}\partial_x\Phi_0\partial_v f_0 = 0 \qquad (1)$$

with possible BGK-type solutions. Equation (1) implies that $f_0(x,v)$ is a function of only the electron energy, $\epsilon = mv^2/2 + q\Phi_0(x)$. Hence at equilibrium all electrons conserve energy, and their equations of motion are $\partial_t v = -q\partial_x\Phi_0/m$ and $\partial_t x = v$. In order to test the equilibrium for stability against 3D electrostatic waves we linearize the total distribution, $f = f_0 + \delta f(x,v)\exp i(\mathbf{k}_\perp \cdot \mathbf{x}_\perp - \omega t)$ and the potential, $\Phi = \Phi_0 + \delta\phi(x)\exp i(\mathbf{k}_\perp \cdot \mathbf{x}_\perp - \omega t)$. The perturbed Vlasov's equation becomes

$$-i\omega\delta f + \left(v\partial_x - \frac{q}{m}\partial_x\Phi_0\partial_v\right)\delta f = \frac{q}{m}\partial_x\delta\phi\partial_v f_0, \qquad (2)$$

while Poisson's equation becomes $(\partial_x^2 - k_\perp^2)\delta\phi = -q/\epsilon_0\int dv\,\delta f$.

The linearized equation Eq. (2) can be solved using the method of characteristics, following the phase space trajectories of $f_0(x,v,t)$ constructed from the equations of motion of the particles. We change variables from x, v to $\epsilon(x,v)$, $s(x)$, and $\hat\sigma(v)$, where s parametrizes the position of a given electron of energy, ϵ, along its trajectory. $\hat\sigma$ is the sign of v, equal to 1 or -1 for right or left moving electrons, respectively. Then, $\partial_x = q\partial_x\Phi_0(x)\partial_\epsilon + \partial_s$ and $\partial_v = \hat\sigma m v\partial_\epsilon$.

In the new variables $v\partial_s f_0(s,\epsilon,\hat\sigma) = 0$. Note that v is now not an independent variable, but just $\sqrt{2[\epsilon - q\Phi_0(s)]/m}$, a function of energy, and s. In the new variables, Eq. (2) becomes $-i\omega\delta f + \hat\sigma v(\epsilon,s)\partial_s\delta f = q\partial_s\delta\phi\hat\sigma v\partial_\epsilon f_0$. Removing the adiabatic response from δf, with the ansatz $\delta g = \delta f - q\partial_\epsilon f_0\delta\phi$ we obtain

$$-i\omega\delta g + \hat\sigma v(\epsilon,s)\partial_s\delta g = qi\omega\delta\phi\partial_\epsilon f_0. \qquad (3)$$

Equation (3) is a first-order ordinary linear differential equation with a solution consisting of homogeneous and inhomogeneous parts, written $\delta g(\epsilon,s,\hat\sigma) = \delta g_I + \delta g_H$ where $\delta g_H(\epsilon,s,\hat\sigma) = \delta g_1(\epsilon,\hat\sigma)\exp i\omega\hat\sigma I_{s_1}^s$ and

$$\delta g_I(\epsilon,s,\hat\sigma) = qi\omega\hat\sigma\partial_\epsilon f_0\int_{s_1}^{s}\frac{ds'}{v(\epsilon,s')}\,\delta\phi(s')e^{i\omega\hat\sigma I_{s'}^{s}}. \qquad (4)$$

In these expressions $I_a^b(\epsilon) = \int_a^b dl/v(\epsilon,l)$ which is the transit time of a particle with energy ϵ moving between points a and b along its trajectory defined by the equilibrium Φ_0. $\delta g_1(\epsilon,\hat\sigma)$ and $s_1(\epsilon)$ are integration constants along the characteristic, which are determined from boundary conditions, and parametrized by energy.

Two classes of electrons exist. First, those with energy $0 \geq \epsilon \geq q\Phi_{\max}$ which are trapped with turning points $s_{1,2}(\epsilon)$ given by $\epsilon = q\Phi_0(s_{1,2})$, and a bounce period $\tau_b(\epsilon) = 2I_{s_1}^{s_2}$. Second, there exist passing electrons with $\epsilon \geq 0$. To complete the solution of Eq. (3) we need to specify boundary conditions for both classes of particles. We apply periodic boundary conditions to the entire system. This allows us to compare to simulations and corresponds to the observed sequences of bipolar structures. For the passing particles we require $\delta g_p(\hat\sigma = \pm 1, s, \epsilon) = \delta g_p(\hat\sigma = \pm 1, s + L, \epsilon)$ and $\delta\phi(x) = \delta\phi(x + L)$ where L is the extent of the system in the x direction and we can set $\delta\Phi(s) = \sum_l \Phi_l \exp ils/L$. To lowest order in $k_x D\delta n/n_0$, where δn is the density perturbation due to the holes, n_0 is the bulk density, and D is the hole width, the passing particle contribution, δg_p, becomes

$$\delta g_p = qi\omega\hat\sigma\partial_\epsilon f_0\sum_{l=-\infty}^{\infty}\Phi_l\frac{\exp(2il\pi s/L)}{2il\pi v(\epsilon,s)/L - i\hat\sigma\omega}. \qquad (5)$$

For the trapped particles we require that the number of particles of energy ϵ approaching a turning point $s_{1,2}$ will be the same as the number of particles leaving the turning point after being reflected from it; formally $\delta g_t(\hat\sigma = 1, s_i, \epsilon) = \delta g_t(\hat\sigma = -1, s_i, \epsilon) = \delta g_i$ for $i = 1,2$ resulting in

$$\delta g_1 = -\frac{q\omega\partial_\epsilon f_0}{\sin\omega I_{s_1}^{s_2}}\int_{s_1}^{s_2}\frac{ds'}{v(\epsilon,s')}\,\delta\phi(s')\cos(\omega I_{s'}^{s_2}). \qquad (6)$$

The problem is closed by Poisson's law, which includes passing and trapped electrons

$$-k^2\delta\phi + \frac{q}{\epsilon_0}\sum_{\hat\sigma=\pm1}\int_0^\infty\frac{d\epsilon}{mv(\epsilon,x)}\,\delta f_p = -\frac{q}{\epsilon_0}\sum_{\hat\sigma=\pm1}\int_{q\Phi_{\max}}^{0}\frac{d\epsilon}{mv(\epsilon,x)}\,\delta f_t. \qquad (7)$$

The passing electron terms contain poles as seen from Eq. (5), which give rise to the usual parallel Landau resonance if $\omega L = 2\pi l v$ for some integer l. Their contribution to the growth rate is $\gamma_\parallel = -\omega_p \times \cos\theta\sqrt{\pi}(k\lambda_D)^{-3}\exp[-(k\lambda_D)^{-2}]$. Additionally, if $\sin\omega I_{s_1}^{s_2}(\epsilon_R) = 0$ for some energy $0 > \epsilon_R > q\Phi_{\max}$, there is a pole in the trapped electron term of Poisson's law, as can be seen from Eq. (6) [16]. This pole expresses the bounce resonance of the trapped electrons with an electrostatic wave. Defining $\omega_b(\epsilon) = 2\pi/\tau_b(\epsilon)$ to be the bounce frequency, the resonance condition is

$$\omega\tau_b(\epsilon) = 2n\pi \quad \text{or} \quad \omega = n\omega_b(\epsilon). \qquad (8)$$

We outline now the calculation of the trapped electron contribution to the growth rate. We solve Eq. (7) perturbatively, using the fluid theory as the zeroth order approximation and the kinetic terms as first-order corrections. We write Eq. (7) symbolically as

$$L_0(\omega)\delta\phi + L_P(\omega)\delta\phi = L_T(\omega)\delta\phi\,, \qquad (9)$$

where L_P is the parallel Landau resonant term due to the passing particles, and $L_0(\omega)\delta\phi$ is the fluid contribution, $(\partial_x^2 - k_\perp^2 - \omega_p^2\partial_x^2/\omega^2)\delta\phi$ for the particular case of the oblique Langmuir waves. $L_T(\omega)\delta\phi$ represents the bounce-resonant trapped electrons, and

$$L_T(\omega)\delta\phi = -\frac{q}{\epsilon_0}\sum_{\hat\sigma=\pm1}\int_{q\Phi_{\max}}^{0}\frac{d\epsilon}{mv(\epsilon,x)} \times \delta g_1(\epsilon,\hat\sigma)e^{i\omega\hat\sigma I_{s_1}^{s}}\,. \qquad (10)$$

For the growth rate calculation we kept only the resonant contribution to δf_t in Eq. (7). Left multiplying Eq. (9) by $\delta\phi^*$ and integrating over x we obtain the quadratic $\langle\delta\phi^* L_0(\omega)\delta\phi\rangle + \langle\delta\phi^* L_P(\omega)\delta\phi\rangle = \langle\delta\phi^* \times L_T(\omega)\delta\phi\rangle$. Defining ω_0 and $\delta\phi_0$ to be an eigenvalue and its corresponding eigenfunction for $L_0(\omega)\delta\phi = 0$, we set $\omega = \omega_0 + \lambda\delta\omega$ and $\delta\phi = \delta\phi_0 + \lambda\delta\phi_1$ where $\lambda \ll 1$. We expand the quadratic in powers of λ, assume that the kinetic terms are of first order, and, using the self-adjointness of $L_0(\omega_0)$ for real ω_0, we obtain $\delta\omega\,\partial_\omega\langle\delta\phi_0^* L_0(\omega)\delta\phi_0\rangle|_{\omega=\omega_0} + \langle\delta\phi_0^* L_P(\omega_0)\delta\phi_0\rangle = \langle\delta\phi_0^* L_T(\omega_0)\delta\phi_0\rangle$. Note that $\partial_\omega L_0(\omega) = 2\omega_p^2(x)\omega^{-3}\partial_x^2$. For convenience define $Q = \langle\delta\phi_0^* L_T(\omega_0)\delta\phi_0\rangle$. Using δg_1 from Eq. (6) we obtain, after some reorganization,

$$Q = \frac{q^2\omega}{\epsilon_0 m}\int_{q\Phi_{\max}}^{0} d\epsilon\,\frac{2\partial_\epsilon f_0}{\sin\omega I_{s_1}^{s_2}}\,G(\epsilon)\,, \qquad (11)$$

where

$$G(\epsilon) = \int_{s_1}^{s_2} dx\,\frac{\delta\phi_0^*(x)}{v(\epsilon,x)}\cos\omega I_{s_1}^{x} \times \int_{s_1}^{s_2}\frac{ds'}{v(\epsilon,s')}\,\delta\phi_0(s')\cos(\omega I_{s'}^{s_2})\,. \qquad (12)$$

For the pole corresponding to energy ϵ_R such that $\omega I_{s_1}^{s_2}(\epsilon_R) = n\pi$ we set $\epsilon = \epsilon_R + \delta\epsilon$ and expand $\sin\omega I_{s_1}^{s_2}(\epsilon) \approx \delta\epsilon(-1)^n\omega\,\partial_\epsilon\tau_b(\epsilon_R)/2$. The resonant part of Q becomes

$$Q_R = -i\pi\frac{q^2\omega}{\epsilon_0 m}\sum_n\frac{2\partial_\epsilon f_0(\epsilon_n)}{(-1)^n|\partial_\epsilon\tau_b(\epsilon_n)|\omega/2}\,G(\epsilon_n)\,, \qquad (13)$$

where n runs over all possible bounce resonances, and ϵ_n satisfies $\omega = n\omega_b(\epsilon_n)$. Since the fractional electron density change in the trapping region is very small, we will consider the case of a single $\mathbf{k}$ harmonic, $\delta\phi_0(x) = A\exp(ik_x x)$. We can evaluate $G(\epsilon_n)$ to lowest order in $k_x\Delta_T(\epsilon_n)\omega_b^2(\epsilon_n)/\omega^2$ where $\Delta_T(\epsilon) \equiv s_2(\epsilon) - s_1(\epsilon)$ is the width of the well at a particular energy ϵ. The simulations show that the excited waves have parallel wavelengths significantly larger than the width of the potential wells so this approximation is easily satisfied. Finally, the growth rate γ due to the wave-bounce resonances is

$$\gamma = \frac{4}{n_0 L\pi}\sum_n\frac{\partial_\epsilon f_0(\epsilon_n)}{|\partial_\epsilon\tau_b(\epsilon_n)|}\,\Delta_T(\epsilon_n)^2 \times \omega_b(\epsilon_n)\,\frac{n^3[1 - (-1)^n\cos k_x\Delta_T(\epsilon_n)]}{[n^2 - 4k_x^2\Delta_T(\epsilon_n)^2/\pi^2]^2}\,, \qquad (14)$$

where L is the size of the entire system. Therefore, to drive waves we need $\partial_\epsilon f_0(\epsilon_n) > 0$ at the resonant energies. The even n contribution tends to zero as $k_\parallel\Delta_T(\epsilon_n) \to 0$.

The existence of resonances is a requirement distinct from $\partial_\epsilon f_0(\epsilon) > 0$. Resonances exist if τ_b exceeds a certain limit, as seen from Eq. (8). No resonances exist if $\omega_b(\epsilon) > \omega_p$ for any trapped electron energy ϵ corresponding to a plasma not dense enough, or hole potential wells that are too narrow and deep. We can have resonances for multiple n, corresponding to various angles of wave propagation. For higher θ, the corresponding resonant energy moves up in the well and the maximum θ corresponds to a resonant energy equal to the energy threshold separating passing and trapped particles. On the other hand, higher n implies higher k_x. The present calculation is valid for small $k_x\Delta_T(\epsilon_n)$. For large enough k_x, or equivalently small enough θ, we do not expect wave growth. This sets an upper limit to k_x and equivalently a lower limit to θ.

The theory outlined above explains the wave excitation in the simulation results [5]. The distribution of electrons trapped in the holes appearing in these simulations satisfies $\partial_\epsilon f_0(\epsilon) > 0$. At the same time $\partial_\epsilon\tau_b(\epsilon) > 0$ as shown in Fig. 2 for the potential well of an electron hole prior to onset of the electrostatic whistlers. The resonant condition, Eq. (8), makes $\omega_p\tau_b(\epsilon)\cos(\theta)/2\pi$ an integer, for electrostatic whistlers. In Fig. 3 we show a contour plot of this function versus ϵ and θ, for the τ_b profile shown in Fig. 2. The large contribution of the $n = 1$ resonance as compared to $n = 2$ and the requirement $k_x\Delta_T < 1$ for growth make $n = 1$ the dominant term. Hence, the concentration of wave activity in a single band around $\theta = 80°$. The wave growth rate in the simulations is in order of magnitude agreement with Eq. (14). Finally, parallel Landau damping explains the absence of $|\mathbf{k}|\lambda_D > 0.3$ in the simulations.

Ergun *et al.* [2] shows the remarkable fit of a Gaussian to the potential variation within observed electron holes and the statistics of electron solitary structure occurrence rates

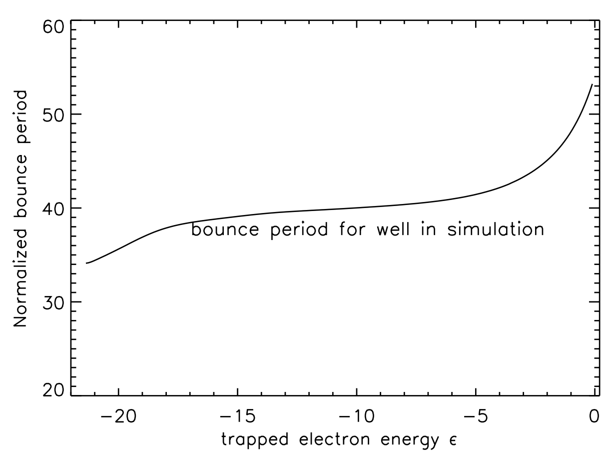

FIG. 2. Normalized bounce period $\omega_p\tau_b$ as a function of energy for the potential well corresponding to Fig. 3.

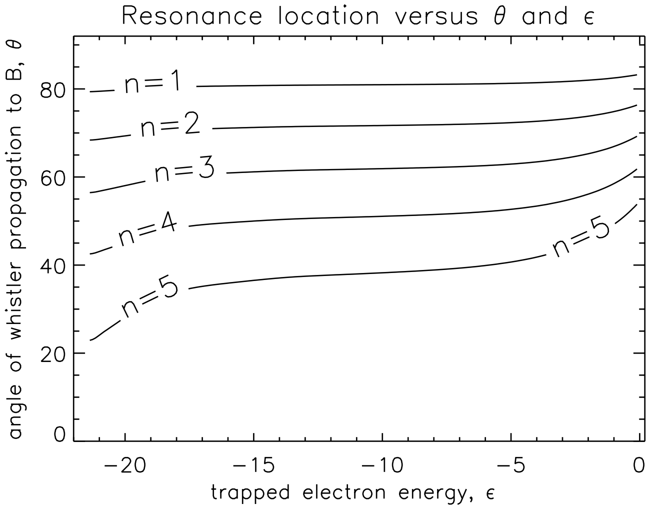

FIG. 3. Contour plot of $n = \omega_p \tau_b(\epsilon)\cos(\theta)/2\pi$ showing bounce resonances versus ϵ and θ for the potential well of a simulated long-lived electron hole. The bottom of the potential well is at -21.37 units.

versus their half-width and peak potential. To evaluate the stability of these solitary structures against oblique Langmuir waves we consider the family of potentials

$$\Phi_0(x) = \Phi \frac{m\omega_p^2 L^2}{q} \max\left(\exp - \frac{x^2}{2L^2} - \exp - \frac{s^2}{8L^2}, 0\right) \tag{15}$$

parametrized by the ratio of the structure size, s, over the Gaussian half-width, L, and the normalized amplitude, Φ. Figure 4 shows the maximum propagation angle at which waves would be bounce resonant with electrons trapped within the structures, as determined by Eq. (8). The observed solitary structures fall within $0.065 < \Phi < 0.2$ and $0 < L < 5\lambda_D$, while $s/L \approx 3.6$ follows from the criterion for identifying them. Figure 4 shows that the resonance condition is satisfied for the observed structures. Given that $\partial_\epsilon \tau_b > 0$ for a Gaussian potential, instability requires $\partial_\epsilon f_0 > 0$ for the trapped electron distribution and $k_\parallel s = |\mathbf{k}| s \cos\theta < 1$ as discussed previously. These conditions should not be difficult to satisfy for the observed solitary structures, though precise information on the trapped electron distribution is necessary to determine growth rates.

In conclusion, we analyzed the resonant interaction of electrostatic waves with the bounce motion of electrons trapped in electron phase space holes. This mechanism explains the growth of obliquely propagating electrostatic waves in simulations of electron holes. The instability criteria are not restrictive, and thus we believe that auroral electron holes drive electrostatic waves in the ionosphere. Wave growth comes at the expense of electron hole

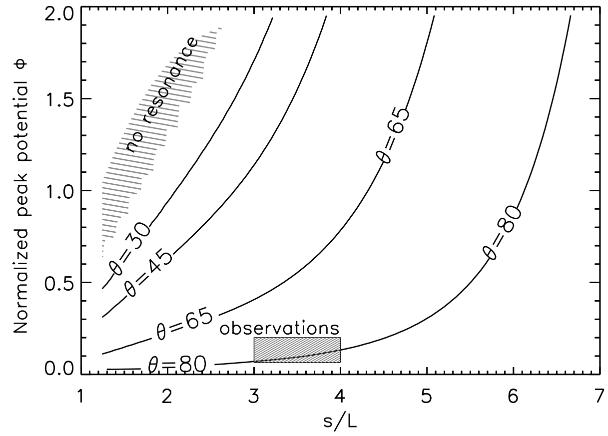

FIG. 4. Maximum propagation angle, θ, of bounce resonant waves for a two-parameter family of Gaussian potentials as described in the text. The observations mentioned in Ergun *et al.* [2] are within the shaded rectangle.

stability and a nonlinear theory should provide an estimate of hole lifetime. Although we concentrated on the case of obliquely propagating Langmuir waves, this bounce resonance mechanism should also drive lower hybrid waves under appropriate conditions.

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