Two-Dimensional Atom Trapping in Field-Induced Adiabatic Potentials

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We show how to create a novel two-dimensional trap for ultracold atoms from a conventional magnetic trap. We achieve this by utilizing rf-induced adiabatic potentials to enhance the trapping potential in one direction. We demonstrate the loading process and discuss the experimental conditions under which it might be possible to prepare a 2D Bose condensate. A scheme for the preparation of coherent matter-wave bubbles is also discussed.

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Bose-Einstein condensation of dilute atomic gases has been demonstrated for three-dimensional systems, but has as yet to be realized for dilute atomic gases in "low-dimensional" atomic traps where one or more of the motional degrees of freedom can be quantum-mechanically frozen. The availability of such traps would open up the pathway to the experimental study of a host of physical phenomena which presently are under vigorous theoretical debate [1]. Several promising ideas for the realization of two-dimensional atom traps or "planar waveguides" have been proposed recently. Trapping is provided either by optical [2] or by magnetic potentials [3], and the loading is typically achieved through optical pumping. A first realization of such a trap was reported in [4] where about 10^3 atoms were stored in the node of a standing light wave. A very different route to obtain a 2D degenerate quantum gas was pursued in Ref. [5] where a hydrogen quasicondensate was produced on a liquid helium surface.

In this Letter we propose a novel scheme to obtain two-dimensional trapping of ultracold atoms and, possibly, Bose-Einstein condensates (BECs). Our method differs in several important ways from previous proposals: (i) it is based on the use of field-induced adiabatic potentials which are a powerful tool to create *enhanced* trapping potentials from conventional magnetic traps; (ii) it inherently provides a quasiharmonic confinement for the two unfrozen motional degrees of freedom. Under these circumstances the behavior of ultracold Bosonic gases is predicted to be drastically different from the 2D-box case [1] which is typically realized in the planar waveguides; and (iii) loading does not rely on incoherent processes (optical pumping), but is performed by adiabatically deforming a conventional magnetic trap, e.g., an Ioffe-Pritchard (IP) trap. This offers the advantage of working with extremely cold, dense, and, possibly, coherent atomic ensembles throughout the whole process.

The basic scheme will use the field-induced potentials from a magnetic trap to create a shell potential: i.e., a potential where the atoms are confined to the surface of an ellipsoid. Gravity will cause trapped atoms to pool at the bottom of the shell potential where they are in a locally harmonic trap. Then, provided the shell is sufficiently thin, a 2D condensate could be formed. In the following we will first discuss the adiabatic potentials necessary for such a 2D system. We will then show how atoms can be loaded into the 2D trap and we will discuss the parameters needed to achieve a 2D system. Because full 3D quantum calculations of the dynamics of the system are difficult we will appeal to some related, but simpler, model systems to demonstrate both the loading of atoms from a pure magnetic trap and determine the lifetime of the adiabatic trap. One of these models, with gravitational compensation, leads to the possibility of atomic matter-wave bubbles. However, when gravity is not compensated for, and the atoms pool in the shell potential, we may be able to produce a 2D condensate using reasonable experimental parameters.

We will consider a model system of atoms with five Zeeman sublevels (e.g., Rb87 $F = 2$) in a magnetic trap which will be subjected, during loading, to a time dependent external field. We note that the same ideas should work for other systems, e.g., simpler two-level systems are possible [6]. As a result of the coupling to the magnetic field, and for a harmonic 3D trap, there is a trap potential for the $M_F = F$ state which we can write as

$$U_{\mathrm{trap}}(\mathbf{r}) = \frac{1}{2}\, m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2), \qquad (1)$$

where m denotes the atomic mass. Here we keep all three frequencies of the trap (ω_x, ω_y, ω_z) generalized to allow for different orientations of an IP trap. Gravity will act in the z direction and it results in a net potential for atoms in an M_F state of

$$U_{M_F}(\mathbf{r}) = mgz + (M_F/F)U_{\mathrm{trap}}(\mathbf{r}), \qquad (2)$$

where g is the gravitational acceleration.

When we now consider the interaction with a strong external field we obtain spatially dependent dressed states with energies

$$V_{M_F}(\mathbf{r}, t) = mgz + M_F\sqrt{[U_{\mathrm{trap}}(\mathbf{r})/2 - \hbar\Delta(t)]^2 + [\hbar\Omega(t)]^2}, \qquad (3)$$

for $F = 2$ (see Fig. 1). The detuning, $\Delta(t)$, of the external field is defined with respect to the potentials at the center of the trap. The Rabi frequencies between Zeeman sublevels

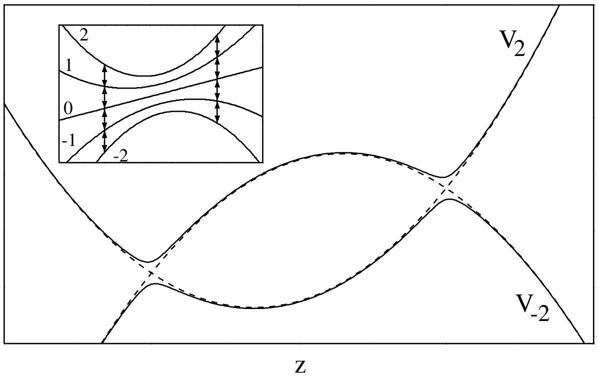

FIG. 1. Illustration of field-induced adiabatic potentials, or dressed states, V_{m_F} [Eq. (3)], $M_F = \pm 2$, as a function of z ($x = y = 0$) and for $\Delta > 0$. Only the two extremes, $V_{\pm 2}$, are shown for simplicity. Dashed curves show the bare potentials $U_{\pm 2}$ [Eq. (2)]. Inset: bare potentials showing rf resonance.

differ, so Ω in Eq. (3) is chosen to be the Rabi frequency between $M_F = 2$ and $M_F = 1$. These dressed states will, for sufficiently strong Ω, provide a trapping potential shell for the atoms. This is illustrated by the potential V_2 shown in Fig. 1. The left-hand side of the potential V_2 is lower in energy because of gravity, resulting in greater numbers of atoms collecting there. We note that application of an rf field is also the basis of evaporative cooling, which leads to a loss of atoms. This, however, mostly corresponds to dynamics on the potential V_{-2}, whereas we shall seek adiabatic transport in the potential V_2.

The effect of gravity on the atoms is seen more clearly in Fig. 2 where the dressed potential V_2 is shown for the x-z plane along with the probability density for an atomic condensate. It is seen that the atoms pool into a curved disk at low z. The disk is curved because the atoms tend to be tightly confined to an ellipsoid surface, or *seam,* where there is resonance between the bare states $|M_F\rangle$: i.e., where $U_{\text{trap}}(\mathbf{r})/2 = \hbar\Delta$. Thus in the three principal directions the seam is located at $r_i = 2a_i\sqrt{\Delta/\omega_i}$, where a_i are the oscillator lengths for the trap ($i = x, y, z$). At the bottom of the potential V_2 the atomic motion is harmonic with the frequencies $\omega_{1,2} = (g/r_z)^{1/2}\omega_{x,y}/\omega_z$ along the surface of the seam and $\omega_{\text{trans}} = (2\Delta/\Omega)^{1/2}\omega_z$ in the transverse, or normal, direction.

The atoms at the bottom of the shell potential will have a finite temperature T, and they are effectively in a 2D trap if $\hbar\omega_{1,2} < k_B T < \hbar\omega_{\text{trans}}$. A good 2D trap has a large ω_{trans} to allow one motional degree of freedom to be frozen out at high temperatures. For an ideal gas, Bose condensation in a 2D trap occurs at $k_B T = \hbar\bar{\omega}(6N)^{1/2}/\pi$ with $\bar{\omega} = (\omega_1\omega_2)^{1/2}$ and N the number of atoms [7]. Then the maximum number of atoms that can undergo a genuine 2D condensation is given by $N \lesssim (\omega_{\text{trans}}/\bar{\omega})^2$; for higher atom numbers there would be transverse excitation and then condensation would take place in a 3D regime. Thus a high ratio $\omega_{\text{trans}}/\bar{\omega}$ is desirable, which will lead to a consideration of large detunings.

When we start with the atoms in the center of the magnetic trap we have to move to large detunings in an adiabatic process which has to be sufficiently slow to avoid nonadiabatic excitations, but fast enough to avoid losses from leakage. To simulate this loading we have integrated a 2D Gross-Pitaevskii equation for Rb87 using the potentials U_{M_F}, Eq. (2), but neglecting the y degree of freedom. This simplification has been made to make the problem more numerically tractable. The couplings between the M_F states are time dependent, and examples of the evolving atomic density can be seen in Fig. 3. In the first step of a two stage preparation scheme, the rf

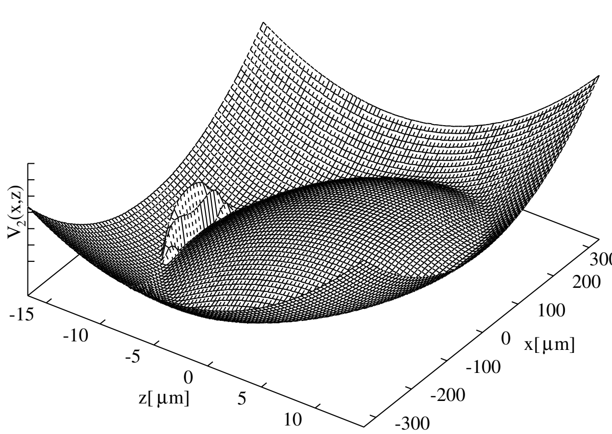

FIG. 2. Upper adiabatic potential $V_2(\mathbf{r})$ (in arbitrary units) at $y = 0$ for $\nu_x = \omega_x/2\pi = 11$ Hz, $\nu_y = \nu_z = 220$ Hz, $\Omega/2\pi = 2.6$ kHz, and $\Delta/2\pi = 13.2$ kHz. These parameters are intermediate in the creation of the 2D system. The ground state of a Rb87 condensate, with about 10^5 atoms, is superimposed on the left.

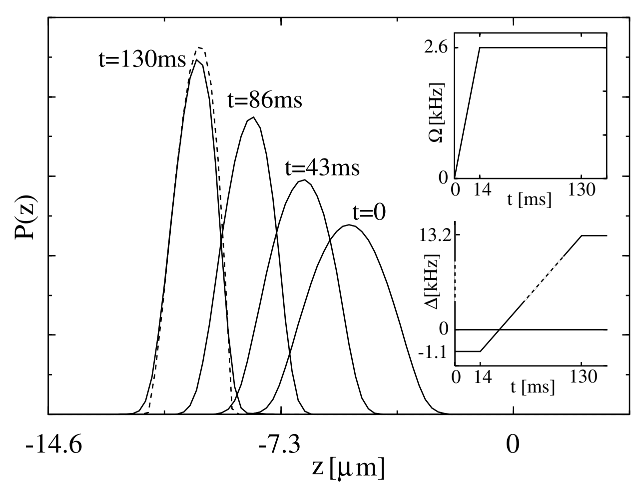

FIG. 3. The initial phase of loading the adiabatic trap is calculated with a 2D model for 10^5 Rb87 atoms and $t = 0$, 43, 86, and 180 ms. The probability density (integrated over x) is shown as a function of z and can be seen to compress as time increases. The transfer process is achieved by starting, at $t = 0$, with no coupling and *negative* detuning (see inset). The dashed curve shows the Thomas-Fermi approximation at $t = 180$ ms.

intensity is linearly ramped up to create the desired final value of Ω at *negative* detuning Δ (see Fig. 3 inset). In the example, in order to reach a final $\Omega = 12\omega_z$ ($\approx$2.64 kHz) the field was switched on at $\Delta = -5\omega_z \equiv -1.1$ kHz within a time $\Delta t = 20/\omega_z = 14.4$ ms. In the second step the rf detuning is simply increased to the final value, but keeping the intensity fixed. In Fig. 3, the detuning Δ was increased to $60\omega_z \equiv 13.2$ kHz within a time $\Delta t = 160/\omega_z = 116$ ms. Because this process is nearly adiabatic, it is not necessary to follow exactly the linear ramp of detuning shown in this example. (We note that this adiabatic transport of the wave packet has some similarity to APLIP [8].) The simulations in Fig. 3 clearly show the compression of the BEC in the z direction. We note that there is a corresponding expansion of the BEC in the x direction. Figure 3 also shows that the Thomas-Fermi approximation gives a very good description.

As the dimensions of the 2D trap become more extreme, numerical simulations become very difficult, and so we must make some general arguments about the necessary conditions to understand whether it is possible to make a 2D condensate in this way. (i) Lifetime: For a given Ω one would like to increase Δ as much as possible to enhance ω_{trans}. However, we will see below, Eq. (9), that for a 2-state system the decay rate γ is less than, say, $0.01\omega_z$ if

$$\Omega^3 \geq \lambda \omega_z^2 \Delta , \tag{4}$$

where the constant $\lambda = 5.7$. As an approximation we adopt this condition for the five-level system and note that it also guarantees harmonic trapping in the transverse direction; (ii) an upper limit for Ω at a given Δ is imposed by the obvious conditions $\omega_{\text{trans}} \gg \omega_{1,2}$ and $r_z \gg a_{\text{trans}}$, where $a_{\text{trans}} = \sqrt{\hbar/m\omega_{\text{trans}}}$. These requirements are typically not in conflict with condition (4). (iii) Harmonicity in x and y: Condensation might also take place in a slightly anharmonic 2D trap, but, in any case, we find that this 2D trap is in a harmonic regime because of constraints on temperature and lifetime; and (iv) loading: while, to avoid leakage, we would want to expand (and flatten) the atom cloud as fast as possible, we need to avoid nonadiabatic effects due to rapid change in the trap frequencies $\omega_{1,2}$. To estimate the minimum duration of this process we stipulate the adiabaticity condition $\dot{\omega}_{1,2} \ll \omega_{1,2}^2$. Relation (4) then leads to the condition

$$t \gg \frac{(\Omega_{\text{final}}/\omega_{x,y})^{3/4}}{\lambda^{1/4}\sqrt{g/2a_{x,y}}} . \tag{5}$$

Note that during loading, Ω can be increased above its intended final value to reduce intermediate adiabatic losses. At present, it is not clear whether the condensate will be preserved during loading or if it could be destroyed due to thermalization processes. In the first case, one would eventually end up with a 2D condensate without observing an actual two-dimensional condensation process (unless additional measures are taken). In the second case, it is interesting to note that the adiabatic potential has a "built-in cooling system" as higher-lying states have a reduced lifetime. Furthermore, it may be possible to apply a certain amount of evaporative cooling with the help of a second, sufficiently detuned rf field.

To summarize, using condition (4) and the other estimates it follows that the trap is effectively 2D for temperatures

$$T < \sqrt{2}\,\hbar\Omega/(\sqrt{\lambda}\,k_B) . \tag{6}$$

Then using the 2D harmonic condensation temperature, and expressing $\bar{\omega}$ in terms of Ω, we find that the number of atoms that can undergo condensation is limited by

$$N < \frac{2\pi^2 a_z \Omega^{7/2}\omega_z^{1/2}}{3g\lambda^{3/2}\omega_x\omega_y} . \tag{7}$$

To give a numerical estimate, we consider an IP trap for Rb87 atoms with $\nu_x = 11$ Hz, and $\nu_y = \nu_z = 220$ Hz. The rf field produces a Rabi frequency $\Omega = 15$ kHz and [from (4)] $\Delta = 12.2$ MHz. The resulting trap frequencies are $\nu_{\text{trans}} = \omega_{\text{trans}}/2\pi = 8.9$ kHz, $\nu_1 = 1.3$ Hz, and $\nu_2 = 27$ Hz. The new trap is vertically shifted by 0.34 mm from the center of the original magnetic potential. The critical temperature is given by 0.43 μK, which would allow condensation of up to 3.6×10^6 atoms. Neglecting atomic interactions the transverse width of the condensate is estimated at 0.08 μm. Equation (5) then suggests a time of the order of a second for the preparation process. In view of these estimates an attempt at the experimental realization of the present proposal seems possible with currently available technology.

Before concluding this Letter we consider a simplified, 2-state, spherical model in which we may neglect gravitational effects, i.e., we consider atoms that are free to move about the surface of the seam. This model will realize atomic *bubbles,* i.e., a 2D system, but the condensate dynamics is more tractable numerically and analytically as the wave equation is effectively 1D. We use this system to provide information on the decay rate at the seam, which leads to Eq. (4), but the atomic bubbles that are realized when gravity is compensated may be of interest in their own right. For a spherical system (e.g., an IP trap with an appropriately large bias field) we assume a radial $l = 0$ form of the wave function components $\Phi_i(\mathbf{r}) = \phi_i(r)/\sqrt{4\pi}\,r$. Then working in an interaction picture with respect to the applied fields, and neglecting atomic interactions, the time development is determined by the radial Schrödinger equation

$$\begin{aligned} i\hbar\dot{\phi}_1 &= \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + \frac{1}{2}m\omega^2 r^2 - \frac{\hbar\Delta(t)}{2}\right)\phi_1 \\ &\quad + \hbar\Omega(t)\phi_2 , \\ i\hbar\dot{\phi}_2 &= \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} - \frac{1}{2}m\omega^2 r^2 + \frac{\hbar\Delta(t)}{2}\right)\phi_2 \\ &\quad + \hbar\Omega(t)\phi_1 , \end{aligned} \tag{8}$$

where ω is the (spherical) magnetic trap frequency. The upper dressed state of this system supports a vibrational "ground" state $|0\rangle$ in the same way as the five-level system. This state can also be loaded and deformed by changing Ω and Δ slowly in time. However, because of nonadiabatic transitions, there is leakage from the state $|0\rangle$, i.e., it decays. The decay rate for the harmonic system in Eq. (8) can be determined, at fixed Ω and Δ, with the help of semiclassical methods developed in connection with molecular predissociation [9,10]. Applying these techniques we find that the decay rate γ shows a tendency to decrease exponentially with increasing Ω, although there are also "resonances." In the desirable limit of $\gamma \ll \omega$, the exponential suppression may be approximated as [11]

$$\gamma \sim 2\omega \exp(-\pi\Omega^{3/2}/\sqrt{2}\,\omega\Delta^{1/2}), \qquad (9)$$

which shows that while high Rabi frequencies are desirable to prevent losses, the detuning should not be too large. Resonances, where the actual γ is significantly smaller than this approximation, take place when the state $|0\rangle$ is in resonance with an eigenstate of the bare harmonic trapping potential. This stabilization effect may be used to obtain extremely long-lived states at rather moderate coupling strengths.

Matter-wave bubbles can, with gravitational compensation, be prepared and used to observe other phenomena. For example, if a bubble is formed and then the rf coupling is turned off, the bubble will divide into the components on the separate, bare, sublevels. For a two-level system, one component will collapse, collide with itself at the origin, and then expand outwards again. Anything inside the bubble will be efficiently exposed to an incoming s-wave packet. A similar effect can be gained by turning off both the coupling and the magnetic trap: in this case the spherical wave packet spreads, filling in its center. However, unless the bubble is very small [i.e., smaller than $\sim(\hbar^2/gm^2)^{1/3}$ or 5×10^{-7} m for Rb87 [11]], or the nonlinear interactions are very strong, none of these effects can be seen without gravitational compensation. This can be achieved over a finite region of space by exposing the trapped atoms to an additional optical dipole potential U_d. Utilizing a horizontal Gaussian laser beam we can choose the laser intensity such that at the turning point z_t of U_d, which is defined by $\partial^2 U_d(z_t)/\partial z^2 = 0$, the slope $\partial U_d/\partial z$ balances the gravitational acceleration. In this way, the combined optical and gravitational potential is almost constant around z_t; the lowest-order corrections being cubic in $z - z_t$. The magnetic trap is then placed inside this area. To test this scheme we made a 2D simulation for a BEC of about 10^5 Rb87 atoms trapped in a magnetic potential with $\nu_z = \nu_y = 220$ Hz, and $\nu_x = 30$ Hz. We found that when $\Delta = 6.6$ kHz and $\Omega = 2.64$ kHz we could form a bubble of radial and axial diameters 15 and 110 μm if we use a dipole potential with a beam waist radius of 73 μm and power of 1 W.

In conclusion, we have proposed a novel scheme to create two-dimensional atom traps by using rf-induced adiabatic potentials in a shell form. In the presence of gravity the atoms at the bottom of the shell potential are in a two-dimensional harmonic trap which may sustain an atomic BEC. Gravitational compensation with an optical potential also allows us to create stable atomic bubble states which would allow the study of atomic dynamics on a closed two-dimensional surface and the creation of novel matter-wave states. The prospects for the experimental realization of the scheme seem to be promising. Nevertheless, more work is needed to obtain a detailed understanding of all aspects of engineering and loading such traps.

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