Chaos in Superstring Cosmology

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It is shown that the general solution near a spacelike singularity of the Einstein-dilaton-p-form field equations relevant to superstring theories and M theory exhibits an oscillatory behavior of the Belinskii-Khalatnikov-Lifshitz type. String dualities play a significant role in the analysis.

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An outstanding result in theoretical cosmology has been the discovery by Belinskii, Khalatnikov, and Lifshitz (BKL) that the generic solution of the four-dimensional Einstein's vacuum equations near a cosmological singularity exhibits a never ending oscillatory behavior [1]. (See [2] for a summary of the evidence supporting the BKL conjectural picture.) The oscillatory approach toward the singularity has the character of a random process, whose chaotic nature has been intensively studied [3]. However, two results cast a doubt on the physical applicability, to our universe, of the BKL picture. First, it was surprisingly found that the chaotic BKL oscillatory behavior disappears from the generic solution of the vacuum Einstein equations in spacetime dimension $D \geq 11$ and is replaced by a monotonic Kasner-like power-law behavior [4]. Second, it was proved that the generic solution of the four-dimensional Einstein-scalar equations also exhibits a nonoscillatory, power-law behavior [5,6].

Superstring theory [7] suggests that the massless (bosonic) degrees of freedom which can be generically excited near a cosmological singularity correspond to a high-dimension ($D = 10$ or 11) Kaluza-Klein-type model containing, in addition to Einstein's D-dimensional gravity, several other fields (scalars, vectors and/or forms). In view of the results quoted above, it is *a priori* unclear whether the full field content of superstring theory will imply, as a generic cosmological solution, a chaotic BKL-like behavior, or a monotonic Kasner-like one. In this Letter we report the result that the massless bosonic content of all superstring models ($D = 10$ IIA, IIB, I, het_E, het_{SO}), as well as of M theory ($D = 11$ supergravity), generically implies a chaotic BKL-like oscillatory behavior near a cosmological singularity. (Our analysis applies at scales large enough to excite all Kaluza-Klein-type modes, but small enough to be able to neglect the stringy and nonperturbative massive states.) It is the presence of various form fields (e.g., the 3-form in SUGRA_{11}) which provides the crucial source of this generic oscillatory behavior.

Let us consider a model of the general form

$$S = \int d^D x \sqrt{g} \left[R(g) - \partial_\mu \varphi \partial^\mu \varphi - \sum_p \frac{1}{(p+1)!} e^{\lambda_p \varphi} (dA_p)^2 \right]. \qquad (1)$$

Here, the spacetime dimension D is left unspecified. We work (as a convenient common formulation) in the Einstein conformal frame, and we normalize the kinetic term of the "dilaton" φ with a weight 1 with respect to the Ricci scalar. The integer $p \geq 0$ labels the various p forms $A_p \equiv A_{\mu_1 \cdots \mu_p}$ present in the theory, with field strengths $F_{p+1} \equiv dA_p$, i.e., $F_{\mu_0 \mu_1 \cdots \mu_p} = \partial_{\mu_0} A_{\mu_1 \cdots \mu_p} \pm p$ permutations. The real parameter λ_p plays the crucial role of measuring the strength of the coupling of the dilaton to the p form A_p (in the Einstein frame). When $p = 0$, we assume that $\lambda_0 \neq 0$ (this is the case in type-IIB where there is a second scalar). The Einstein metric $g_{\mu\nu}$ is used to lower or raise all indices in Eq. (1) ($g \equiv -\det g_{\mu\nu}$). The model (1) is, as it reads, not quite general enough to represent in detail all the superstring actions. Indeed, it lacks additional terms involving possible couplings between the form fields [e.g., Yang-Mills couplings for $p = 1$ multiplets, Chern-Simons terms, $(dC_2 - C_0 dB_2)^2$-type terms in type IIB]. However, we have verified in all relevant cases that these additional terms do not qualitatively modify the BKL behavior to be discussed below. On the other hand, in the case of M theory (i.e., $D = 11$ SUGRA) the dilaton φ is absent, and one must cancel its contributions to the dynamics.

The leading Kasner-like approximation to the solution of the field equations for $g_{\mu\nu}$ and φ derived from (1) is, as usual [1],

$$g_{\mu\nu} dx^\mu dx^\nu \simeq -dt^2 + \sum_{i=1}^{d} t^{2p_i(x)} (\omega^i)^2, \qquad (2a)$$

$$\varphi \simeq p_\varphi(x) \ln t + \psi(x), \qquad (2b)$$

where $d \equiv D - 1$ denotes the spatial dimension and where $\omega^i(x) = e^i_j(x) dx^j$ is a time-independent d-bein.

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The spatially dependent Kasner exponents $p_i(x)$, $p_\varphi(x)$ must satisfy the famous Kasner constraints (modified by the presence of the dilaton):

$$p_\varphi^2 + \sum_{i=1}^{d} p_i^2 - \left(\sum_{i=1}^{d} p_i\right)^2 = 0, \qquad \sum_{i=1}^{d} p_i = 1\,. \quad (3)$$

The set of parameters satisfying Eqs. (3) is topologically a $(d-1)$-dimensional sphere: the "Kasner sphere". When the dilaton is absent, one must set p_φ to zero in Eqs. (3). In that case the dimension of the Kasner sphere is $d - 2 = D - 3$.

The approximate solution (2) is obtained by neglecting in the field equations for $g_{\mu\nu}$ and φ: (i) the effect of the spatial derivatives of $g_{\mu\nu}$ and φ, and (ii) the contributions of the various p-form fields A_p. The condition for the "stability" of the solution (2), i.e., for the absence of BKL oscillations at $t \to 0$, is that the inclusion in the field equations of the discarded contributions (i) and (ii) [computed within the assumption (2)] be fractionally negligible as $t \to 0$. As usual, the fractional effect of the spatial derivatives of φ is found to be negligible, while the fractional effect (with respect to the leading terms, which are $\propto t^{-2}$) of the spatial derivatives of the metric, i.e., the quantities $t^2\overline{R}^i_j$ (where $\overline{R}^i_j$ denotes the d-dimensional Ricci tensor) contains, as only "dangerous terms" when $t \to 0$, a sum of terms $\propto t^{2g_{ijk}}$, where the *gravitational exponents* g_{ijk} $(i \neq j,\ i \neq k,\ j \neq k)$ are the following combinations of the Kasner exponents [4]:

$$g_{ijk}(p) = 2p_i + \sum_{\ell \neq i,j,k} p_\ell = 1 + p_i - p_j - p_k\,. \quad (4)$$

The "gravitational" stability condition is that all the exponents $g_{ijk}(p)$ be positive. In the presence of form fields A_p there are additional stability conditions related to the contributions of the form fields to the Einstein-dilaton equations. They are obtained by solving, *à la* BKL, the p-form field equations in the background (2) and then estimating the corresponding dangerous terms in the Einstein field equations. When neglecting the spatial derivatives in the Maxwell equations in first-order form $dF = 0$ and $\delta(e^{\lambda_p\varphi}F) = 0$, where $\delta \equiv *d*$ is the (Hodge) dual of the Cartan differential d and $F_{p+1} = dA_p$, one finds that both the "electric" components $\sqrt{g}\, e^{\lambda_p\varphi}F^{0i_1\cdots i_p}$ and the "magnetic" components $F_{j_1\cdots j_{p+1}}$ are constant in time. Combining this information with the approximate results (2) one can estimate the fractional effect of the p-form contributions in the right-hand side of the $g_{\mu\nu}$- and φ-field equations, i.e., the quantities $t^2T^0_{(A)0}$ and $t^2T^i_{(A)j}$ where $T^\mu_{(A)\nu}$ denotes the stress-energy tensor of the p form. (As usual [1] the mixed terms $T^0_{(A)i}$, which enter the momentum constraints play a rather different role and do not need to be explicitly considered.) Finally, one gets as dangerous terms when $t \to 0$ a sum of electric contributions $\propto t^{2e^{(p)}_{i_1\cdots i_p}}$ and of magnetic ones $\propto t^{2b^{(p)}_{j_1\cdots j_{d-p-1}}}$.

Here, the *electric exponents* $e^{(p)}_{i_1\cdots i_p}$ (where all the indices i_n are different) are defined as

$$e^{(p)}_{i_1\cdots i_p}(p) = p_{i_1} + p_{i_2} + \cdots + p_{i_p} - \frac{1}{2}\lambda_p p_\varphi\,, \quad (5)$$

while the *magnetic exponents* $b^{(p)}_{j_1\cdots j_{d-p-1}}$ (where all the indices j_n are different) are

$$b^{(p)}_{j_1\cdots j_{d-p-1}}(p) = p_{j_1} + p_{j_2} + \cdots + p_{j_{d-p-1}} + \frac{1}{2}\lambda_p p_\varphi\,. \quad (6)$$

To each p form is associated a (duality-invariant) double family of stability exponents $e^{(p)}$, $b^{(p)}$. The electric (respectively, magnetic) stability condition is that all the exponents $e^{(p)}$ (respectively, $b^{(p)}$) be positive. This result generalizes the results of [8] on the effect of vector fields in $D = 4$. [The fact that, generically, the exponents $e^{(p)}$ and $b^{(p)}$ are either >0 or <0 shows that the addition of a form cannot be described by a modification of the Kasner constraints (3), as can the addition of a dilaton for which $T^\mu_{(\varphi)\nu} \sim t^{-2}$. This was observed in the vector case in [8].]

The main result reported here is that, for all superstring models, there exists no open region of the Kasner sphere where all the stability exponents $g(p)$, $e(p)$, and $b(p)$ are strictly positive. To define the set of stability conditions for the various superstring models, let us review their field content and give the values of the crucial dilaton couplings λ_p. The simplest case is the massless bosonic sector of M theory, i.e., of SUGRA in $D = 11$. In that case, there is a 3-form and no dilaton. The parameters p^M_α, $\alpha = 1, \ldots, 10$, run over the 8-dimensional sphere S^8_M defined by $\sum_\alpha (p^M_\alpha)^2 = 1 = \sum_\alpha p^M_\alpha$. The presence of a 3-form A_3 uncoupled to any dilaton implies that the electric and magnetic stability exponents are, respectively, given by (5) and (6) with $p = 3$, $\lambda_p = 0$, and $d = 10$, i.e., $e^{M(3)}_{\alpha_1\alpha_2\alpha_3} = p^M_{\alpha_1} + p^M_{\alpha_2} + p^M_{\alpha_3}$ and $b^{M(3)}_{\alpha_1\cdots\alpha_6} = p^M_{\alpha_1} + \cdots + p^M_{\alpha_6}$.

The $D = 10$ type-IIA string theory involves, besides $g_{\mu\nu}$ and a dilaton $\varphi = \Phi/\sqrt{2}$ (with $g_s = e^\Phi$ being the string coupling) a 1-form, a 2-form, and a 3-form. The (Einstein-frame) dilaton coupling parameters of the forms are $\lambda^A_1 = 3\sqrt{2}/2$, $\lambda^A_2 = -\sqrt{2}$, and $\lambda^A_3 = \sqrt{2}/2$, respectively. Besides the dilaton Kasner exponent p^A_φ, there are nine metric exponents p^A_i, $i = 1, \ldots, 9$. They run over S^8_A defined by Eqs. (3).

The $D = 10$ type-IIB string theory involves (besides $g_{\mu\nu}$) two scalars: the dilaton $\varphi = \Phi/\sqrt{2}$ and the $R-R$ 0-form C_0, two 2-forms $B_2(NS-NS)$ and $C_2(R-R)$, and one "self-dual" $R-R$ 4-form C_4. The dilaton coupling strengths of the forms are $\lambda^B_{C_0} = 2\sqrt{2}$, $\lambda^B_{B_2} = -\sqrt{2}$, $\lambda^B_{C_2} = +\sqrt{2}$, and $\lambda^B_{C_4} = 0$. [$\lambda^B_{C_2}$ refers to the more complicated mixed coupling $e^\Phi(dC_2 - C_0 dB_2)^2$]. The Kasner exponents p^B_φ, p^B_i $(i = 1, \ldots, 9)$ run over S^8_B defined by Eqs. (3).

The $D = 10$ type-I string theory involves (besides $g_{\mu\nu}$ and φ) an SO(32) vector potential and a 2-form. The dilaton couplings are $\lambda_1^I = \sqrt{2}/2$ and $\lambda_2^I = +\sqrt{2}$. The Kasner exponents p_φ^I, p_i^I ($i = 1, \dots, 9$) run, as for IIA and IIB, on the S_I^8 defined by Eqs. (3).

Finally, the $D = 10$ heterotic string theories involve (besides $g_{\mu\nu}$ and φ) an SO(32) or $E_8 \times E_8$ vector potential and a 2-form. Their respective (Einstein-frame) dilaton couplings are $\lambda_1^h = -\sqrt{2}/2$, $\lambda_2^h = -\sqrt{2}$. The Kasner sphere S_h^8 for p_φ^h, p_i^h ($i = 1, \dots, 9$) is the same as for IIA, IIB, or I.

Let us denote for each given theory "th" (where th $= M, A, B, I, h$ labels the theory) the full (finite) sequence of stability exponents as $w_J^{\text{th}}(p)$, where J labels all the possible exponents within each theory. For example, when th $= M$ the label J takes 690 values corresponding to the set $\{w_J^M\} = \{g_{\alpha\beta\gamma}^M, e_{\alpha_1\alpha_2\alpha_3}^{M(3)}, b_{\beta_1\cdots\beta_6}^{M(3)}\}$. The condition of "Kasner stability" of each theory is that there exist an open region of the corresponding Kasner sphere S_{th}^8 where $w_J^{\text{th}}(p) > 0$ for *all* the labels J. However, we have proven that, for all theories, $\inf_J w_J^{\text{th}}(p)$ is strictly negative for all values of $p \in S_{\text{th}}^8$, except at a finite number of isolated points where it vanishes.

Let us first consider M theory. We have proven a stronger result, namely, that the electric stability conditions alone are never fulfilled. If, at any point on S_M^8, we order the Kasner exponents as $p_1^M \le p_2^M \le \cdots \le p_{10}^M$, the most stringent electric stability criterion involves $f_0(p) \equiv p_1^M + p_2^M + p_3^M$. To show that this function is nonpositive on the cell $p_1^M \le \cdots \le p_{10}^M$ of the Kasner sphere, we maximize it subject to the constraints $\sum (p_\alpha^M)^2 = 1, \sum p_\alpha^M = 1$. These constraints can be taken into account by introducing two Lagrange multipliers. After a straightforward (but rather long) exhaustive analysis, we have found that $f_0^{\max} = 0$, this maximum being reached only at $p_1 = \cdots = p_9 = 0$, $p_{10} = 1$.

To deal with the type-IIA theory, we use the fact that IIA is the Kaluza-Klein (KK) reduction of M on a circle. This fact dictates the link between the field variables of the two models. If we label by the letter y the compactified dimension this link implies the following relation between the (Einstein-frame) Kasner exponents of the two theories ($i = 1, \dots, 9$):

$$p_\varphi^A = \frac{6\sqrt{2}\, p_y^M}{8 + p_y^M}, \qquad p_i^A = \frac{8p_i^M + p_y^M}{8 + p_y^M}. \qquad (7)$$

Forgetting about this KK motivation [9], we can consider that Eqs. (7) define a one-to-one map π_{AM} from S_M^8 to S_A^8: $p_\alpha^A = \pi_{AM}(p_\beta^M)$. Using this map, we have then shown that the complete set of IIA stability conditions is logically equivalent to the complete set of M stability conditions. The instability of the Kasner behavior of M theory proven above then implies that the Kasner behavior of IIA is also unstable.

To deal with the type-IIB theory, we use the fact that IIA and IIB are related by T duality. The link between the field variables of the two models dictated by T duality [10] enables one to derive a certain fractionally linear map π_{BA} between their (Einstein-frame) Kasner exponents, which can be used, as above, to prove the Kasner-stability equivalence of the types IIA and IIB theories. Since type-IIA is unstable, type-IIB is also unstable.

At this stage, we know that M, IIA, and IIB are *equivalent* with respect to Kasner stability, and are all unstable. It remains to tackle the type-I and heterotic theories, which are equivalent because their stability conditions are algebraically mapped onto each other by the S-duality transformation $p_\varphi^I = -p_\varphi^h, p_i^I = p_i^h$. To study the Kasner stability of the heterotic theory, we found it very convenient to replace the Einstein-frame Kasner exponents (p_φ^h, p_i^h) by their *string-frame* counterparts (α_i^h). The link between the two is (in $d + 1$ spacetime dimensions, see, e.g., [11])

$$p_\varphi = \frac{\sqrt{d-1}\,\sigma}{d - 1 - \sigma}, \qquad p_i = \frac{(d-1)\alpha_i - \sigma}{d - 1 - \sigma}, \qquad (8)$$

with $\sigma \equiv [\sum_i \alpha_i] - 1$ and $i = 1, \dots, d$. In terms of the α's the Kasner sphere S^{d-1} is simply the usual unit sphere, $\sum_i (\alpha_i)^2 = 1$. In our case, $d = 9$ and one should add a label "h" to both the p's and the α's. In terms of the string-frame exponents it is found that the h-stability conditions are equivalent to the simpler inequalities $\alpha_i^h > 0$ and $\alpha_i^h + \alpha_j^h + \alpha_k^h < 1$ (where i, j, k are all different) subjected to the constraints $\sum_i (\alpha_i^h)^2 = 1$. It is easy to verify that these inequalities can never hold when the space dimension is $d = 9$. In that case, the closest one comes to satisfying the inequalities is at the isotropic point $\alpha_i = 1/3$ for which the second inequality is saturated. This concludes our proof that the heterotic model (and therefore also the type-I one) is Kasner unstable. Finally the two blocks of theories (M, A, B) and (I, h) are both Kasner unstable, though for different algebraic reasons.

Our results so far show that the generic solution of the low-energy string models can never reach a monotonic Kasner-like behavior. Following the BKL approach [1] one can go further and study the evolution near a cosmological singularity as a sequence of Kasner-like "free flights" interrupted by "collisions" against the "potential walls" corresponding to the various stability-violating exponents g, e, or b. We have studied this problem [12] and found the following universal "collision law" giving the Kasner exponents $\bar{p}'^\mu$ of the Kasner epoch following a collision in terms of the old ones:

$$\bar{p}'^\mu = \left(1 - 2\frac{(w \cdot \bar{p})(w \cdot u)}{(w \cdot w)}\right)^{-1} \times \left[\bar{p}^\mu - 2\frac{(w \cdot \bar{p})w^\mu}{(w \cdot w)}\right]. \qquad (9)$$

Here, $\bar{p}^\mu$ stands for $\bar{p}^0 \equiv p_\varphi$ and $\bar{p}^i \equiv p_i$. The scalar products are computed with the metric $G_{\mu\nu}$ occurring in the quadratic form entering the first Eq. (3), namely, $G_{00} = 1$, $G_{0i} = 0$, $G_{ij} = \delta_{ij} - 1$, while the vector u

[entering the second Eq. (3)] has "covariant" components $u_0 = 0$, $u_i = 1$. Finally, the "contravariant" vector w^μ characterizes the "wall" responsible for the collision and is defined in such a way that the corresponding exponent [$g(p)$, $e(p)$, or $b(p)$] reads $w(p) = w_\mu \bar{p}^\mu \equiv G_{\mu\nu} w^\mu \bar{p}^\nu$. [For example, for the wall associated with the electric exponent $e_{123}(p) \equiv p_1 + p_2 + p_3$, w_μ reads $w_0 = 0$, $w_i = 1$ for $i = 1, 2, 3$ and $w_i = 0$ for $i > 3$.] The result (9) (which is a rescaled geometrical reflection in the hyperplane $w_\mu \bar{p}^\mu = 0$) applies uniformly to all possible walls: gravitational, electric, or magnetic. It generalizes particular results derived by many authors [13].

To summarize, in all string models, the generic solution near a cosmological singularity for the massless bosonic degrees of freedom exhibits BKL-type oscillations, i.e., a (formally infinite) alternation of Kasner epochs. The primary sources of this BKL behavior are (i) the presence of p forms in the field spectrum of the theories and, (ii) the strength of their dilaton couplings. In the absence of p forms, or if the λ_p's were somewhat smaller, the monotonic Kasner behavior would be stable and generic. (When turning off the p forms, there is no electric or magnetic stability condition to take into account and one recovers the nonchaotic behavior of [4] and [5,6].) The general rule defining the change of Kasner exponents from one epoch to the next is given by Eq. (9), where w is the wall (among the various gravitational, electric, or magnetic ones) for which $w(p) = w_\mu \bar{p}^\mu$ is most negative. We anticipate that the discrete dynamics (9) will define (in all string models) a chaotic motion on the Kasner sphere. At this stage, the physical consequences of such a chaotic motion are unclear. It might constitute a problem for the pre-big-bang scenario [14] which strongly relies on the existence, near a (future) cosmological singularity, of relatively large, quasiuniform patches of space following a monotonic, dilaton-driven Kasner behavior. By contrast our findings suggest that the *spatial inhomogeneity* continuously *increases* toward a singularity, as all quasiuniform patches of space get broken up into smaller and smaller ones by the chaotic oscillatory evolution. In other words, the spacetime structure tends to develop a kind of "turbulence" [15]. This process can be meaningfully described by the classical model (1) until either the curvature becomes of the order of the string scale, or the string coupling becomes large. (As in the original BKL case, we expect that the total number of classically describable oscillations is rather small [16].)

We are aware of the limitations of our result (tree-level bosonic massless modes only) but we think that our finding suggests that the full quantum, string-theory behavior might be at least as complicated, near a cosmological singularity, as our simplified analysis shows.

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