Phase Transitions from Preheating in Gauge Theories

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We show by studying the Abelian Higgs model with numerical lattice simulations that nonthermal phase transitions arising out of preheating after inflation are possible in gauge-Higgs models under rather general circumstances. This may lead to the formation of gauged topological defects and, if the scale at which inflation ends is low enough, to electroweak baryogenesis after preheating.

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Over the past few years there has been somewhat of a revolution in our understanding of the dynamics of the end of inflation. The traditional picture of reheating arising out of the perturbative decay of the inflaton field as it oscillates about the minima of its potential has been replaced by the possibility of an explosive particle production during an earlier period, known as preheating. During preheating, parametric resonance of the inflaton field generates very large fluctuations of the scalar fields coupled to the inflaton, leading to the production of large numbers of particles [1]. Because of the weakness of the interactions, the short-wavelength modes do not thermalize, and the effective temperature of the long-wavelength modes is much higher than in the standard reheating scenario. This may lead to symmetry restoration and, when the Universe cools down as it expands further, a subsequent nonthermal phase transition [2]. The fact that the fluctuations produced during preheating have large occupation numbers implies that they can be considered as interacting classical waves, an important result because it means that the dynamics of fluctuations during and after preheating can be studied using lattice simulations [3]. A concrete example of a nonthermal phase transition occurring after preheating was presented in Ref. [4] (see also Ref. [5]). The phase transition that they found is first order, and depending on the field content of the model being investigated topological defects may form, an intriguing result as it opens up the possibility that inflation can create a defect problem if, for example, they produce gauged monopoles or domain walls [6–8].

Nonthermal phase transitions may even solve the old puzzle of baryon asymmetry in the Universe [9]. Although the baryon number is conserved perturbatively in the standard model, there are nonperturbative interactions that violate this conservation law. The rate of baryon number violation is extremely low at low energies, but it becomes much higher in the high-temperature phase of the electroweak theory. Thus it is possible to generate the observed baryon asymmetry if, for some reason, the fields are out of equilibrium at the electroweak scale and thermalize to a temperature below T_c. This could be the case even in the standard big bang cosmology, if the electroweak phase transition were strongly first order, but at least in the minimal standard model it is not, as lattice simulations have shown [10]. However, in a nonthermal phase transition, the fields are driven out of equilibrium by the oscillations of the inflaton, and baryogenesis may be possible, if the reheating temperature is much lower than T_c [11,12].

Despite the exciting possibility of electroweak baryogenesis, most of the numerical work on nonthermal phase transitions so far has concentrated on scalar fields [4,6–8] or has been restricted to one spatial dimension [11]. In this Letter we present results from simulations of the Abelian Higgs model in two rather different cases. The first case is a direct analog of the simulations in Ref. [4]. The inflaton itself is charged under a gauge group and eventually breaks the gauge symmetry in a first-order phase transition. (For simplicity, we use the terminology of spontaneous symmetry breakdown, although the gauge symmetry is not actually broken in the Higgs phase.) The second case is more relevant for electroweak baryogenesis. We show that even with a Higgs mass that is compatible with experimental bounds, the transition is sharp, and electroweak baryogenesis is therefore possible. Although we restrict ourselves to the Abelian case in our simulations, we expect that our conclusions apply to non-Abelian theories as well.

The Lagrangian of our model is

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^* D_\mu\phi - \lambda(|\phi|^2 - v^2)^2. \quad (1)$$

Here the gauge covariant derivative is $D_\mu\phi = \partial_\mu\phi + ieA_\mu\phi$, and $F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}$. The couplings λ and e are assumed to be small, and we will use $\lambda \sim e^2$ in our estimates.

Ideally, we would like to study the quantum field theory defined by Eq. (1), but solving for the time evolution of even a simple quantum system is a formidable task. Therefore we have to resort to the classical approximation, which is expected to work as long as the dynamics is determined by modes with a macroscopic occupation number [3]. For studying the dynamics, it is convenient to fix the temporal gauge $A_0 = 0$ and use the conformal time η defined by $d\eta \equiv dt/a$ and the rescaled fields $\tilde{\phi} \equiv a\phi$, $\tilde{E}_i \equiv -\partial_\eta A_i$. The equations of motion for $\tilde{\phi}$ and A_i follow from Eq. (1):

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$$\partial_\eta^2 \tilde{\phi} = D_i D_i \tilde{\phi} + (2\lambda v^2 a^2 + \partial_\eta^2 a/a)\tilde{\phi} - 2\lambda|\tilde{\phi}|^2\tilde{\phi}, \tag{2}$$

$$\partial_\eta \tilde{E}_i = \partial_j F_{ij} + 2e\,\mathrm{Im}\tilde{\phi}^* D_i \tilde{\phi}, \tag{3}$$

$$\partial_i \tilde{E}_i = 2e\,\mathrm{Im}\tilde{\phi}^* \partial_\eta \tilde{\phi}. \tag{4}$$

The initial conditions for the fields are those produced by inflation: the gauge field is in vacuum and the covariant derivatives of the Higgs field vanish. This allows us to fix the remaining gauge degree freedom by setting initially $A_i = 0$ and $\phi = \overline{\phi}_0 = \text{const}$.

We separate ϕ into the homogeneous zero mode $\overline{\phi}$ and the inhomogeneous fluctuations $\delta\phi = \phi - \overline{\phi}$. We take the quantum nature of the system into account by introducing small fluctuations for the fields A_i and $\delta\phi$ and for their canonical momenta E_i and $\delta\pi \equiv \partial_\eta \delta\phi$. The width of these classical fluctuations is chosen to be equal to the width of quantum fluctuations in the vacuum calculated for free fields. We allow fluctuations in the phase of $\delta\phi$ and fix the associated gauge degree of freedom by choosing $\partial_i A_i = 0$. The longitudinal component of E_i is determined from the Gauss law (4).

In the very beginning of our simulation, when the fields are in vacuum, the conditions required by the classical approximation are not satisfied, but we expect that the final results will be unaffected. What is important is not the precise nature of the initial fluctuations, but that some small fluctuations are present.

As in the scalar theory [2], the time evolution begins with a period of parametric resonance. The resonance parameter q is given by $q \approx e^2/\lambda$. Let us first consider the case $q \gg 1$, in which the resonance is broad, and during the first oscillations, a large amount of energy is transferred from the zero mode $\overline{\phi}$ to the long-wavelength modes $p \sim \lambda^{1/2}\overline{\phi}_0$ of A_i and $\delta\phi$, from which it soon spreads to all modes with $p \lesssim p_* \approx e\overline{\phi}_0$. We can approximate the state of the system after this period by assuming that the modes with $p \lesssim p_*$ thermalize to some effective temperature T_{eff}, but those with $p \gtrsim p_*$ remain in vacuum. Then the energy density in these fluctuations is

$$\rho \approx \int^{p_*} \frac{d^3p}{(2\pi)^3}\, p^2 \frac{T_{\text{eff}}}{p^2} \sim p_*^3 T_{\text{eff}}, \tag{5}$$

and after preheating it is of the same order as the initial energy density in the zero mode $\rho_0 \sim e^2\overline{\phi}_0^4$, which implies $T_{\text{eff}} \sim \overline{\phi}_0/e$. In the reheating picture, the temperature after the equilibration of the fields would be $T_r \sim \sqrt{e}\,\overline{\phi}_0 \ll T_{\text{eff}}$.

Since the occupation number of the long-wavelength modes is $n_p \sim T_{\text{eff}}/p$, which is large when $p \lesssim p_*$ provided that $e \ll 1$, the classical approximation works well after preheating begins.

The zero mode $\overline{\phi}$ continues oscillating around the minimum, but the fluctuations in $\delta\phi$ and A_i induce an effective mass term

$$m_{\text{eff}}^2 \approx -2\lambda v^2 + 4\lambda\langle\delta\phi^2\rangle + e^2\langle A_i^2\rangle. \tag{6}$$

The magnitude of the fluctuation terms is

$$\langle\delta\phi^2\rangle \sim \langle A_i^2\rangle \sim \int^{p_*} \frac{d^3p}{(2\pi)^3}\frac{T_{\text{eff}}}{p^2} \sim p_* T_{\text{eff}} \sim \overline{\phi}_0^2. \tag{7}$$

In the reheating picture, the fluctuation terms would be much smaller, $\langle\delta\phi^2\rangle \sim \langle A_i^2\rangle \sim T_r^2 \sim e\overline{\phi}_0^2$. This shows that m_{eff}^2 can become positive, thereby restoring the symmetry, even if the reheating temperature is below T_c.

When the Universe expands further, the fluctuation terms decrease and the system undergoes a phase transition to the broken phase. The nature of this transition can be studied by calculating the effective potential of $\overline{\phi}$ in the background of the fluctuations $\delta\phi$ and A_i. If $e^2 \gg \lambda$, the contribution from A_i will be more important. Taking the one-loop contribution from the gauge field into account, we have

$$V_{\text{eff}}(\overline{\phi}) \approx -2\lambda v^2\overline{\phi}^2 + \lambda\overline{\phi}^4 + T_{\text{eff}}\int^{p_*}\frac{d^3p}{(2\pi)^3}\log\frac{p^2 + m_A^2}{p^2}, \tag{8}$$

where $m_A \sim e\overline{\phi}$ is the photon mass generated by the Higgs mechanism.

To understand the shape of the potential (8), we expand it both for small and large $\overline{\phi}$,

$$V_{\text{eff}}(\overline{\phi}) \approx \begin{cases} m_{\text{eff}}^2\overline{\phi}^2 - C_1 e^3 T_{\text{eff}}\overline{\phi}^3 + \lambda\overline{\phi}^4, & (\overline{\phi} \ll p_*/e), \\ C_2 T_{\text{eff}} p_*^3 \ln\frac{e\overline{\phi}}{p_*} - 2\lambda v^2\overline{\phi}^2 + \lambda\overline{\phi}^4, & (\overline{\phi} \gg p_*/e), \end{cases} \tag{9}$$

where C_1 and C_2 are numerical factors and m_{eff}^2 is given by Eq. (6).

The origin $\overline{\phi} = 0$ is a local minimum whenever m_{eff}^2 is positive. Assuming first that $p_* > ev$, the cubic term in Eq. (9) induces another minimum for the potential when m_{eff}^2 becomes small enough, and eventually when this new minimum becomes the global one the system enters the Higgs phase in a first-order phase transition. While this phenomenon is present also in equilibrium [13], the transition is stronger in our case, since the cubic term is proportional to $T_{\text{eff}} \gg T_r$. The existence of this minimum requires $e^2 \gtrsim \lambda$, since otherwise the contribution from the scalar loop, which does not contain any cubic term, would dominate in Eq. (9).

Even if $e^2 \lesssim \lambda$, the potential can have two minima, provided that $p_* < ev$ [4]. Then the tree-level minimum is the global one if the logarithmic term in Eq. (9) is smaller than λv^4, i.e., $T_{\text{eff}} p_*^3 \lesssim \lambda v^4$. It is difficult to simultaneously satisfy this inequality, along with the condition $m_{\text{eff}}^2 > 0$, and we were unable to do this in our simulations.

To confirm this expected behavior, we carried out a numerical lattice simulation with our model. We chose $\overline{\phi}_0 = 0.25 M_{\rm Pl}$ and $\lambda = 2 \times 10^{-13}$, $e = 6.4 \times 10^{-6}$, $v = 7.2 \times 10^{-4} M_{\rm Pl}$. The lattice spacing was $\delta x = 9.3 \times 10^5 M_{\rm Pl}^{-1}$, the time step $\delta\eta = 1.2 \times 10^5 M_{\rm Pl}^{-1}$, and the size of the lattice was 320^3. The Universe was assumed to be radiation dominated with $a(\eta) = 1 + \eta H$, where $H = 8.3 \times 10^{-8} M_{\rm Pl}$. Since $\overline{\phi}$ is not a gauge-invariant quantity and can therefore be defined only in the vacuum, we did not measure its value. Instead, we show $|\phi|^2$ as a function of time in Fig. 1. The fact that $|\phi|^2 < v^2$ when $1.5 \times 10^9 M_{\rm Pl}^{-1} \lesssim \eta \lesssim 3 \times 10^9 M_{\rm Pl}^{-1}$ clearly shows that the gauge symmetry is restored and the system is in the Coulomb phase. The amplitude of the oscillations remains quite large, which is probably a finite-size effect. In an infinite system, there would be more infrared modes to which the zero mode of ϕ could decay. Eventually, the system enters the Higgs phase in a first-order transition, as in the scalar theory. The first-order nature of the transition can be seen from the configurations during the transition; for example, by looking at the isosurface of $|\phi|^2$ we would see a growing bubble of the Higgs phase characterized by a larger value of $|\phi|^2$.

In order to check that the separation of scales below and above p_* indeed takes place, we measured the effective temperature of different Fourier modes of the electric fields E_i at various times during the simulation. A reason for choosing this quantity rather than the power spectrum of $\tilde{\phi}$ or A_i is that it is gauge invariant. In equilibrium

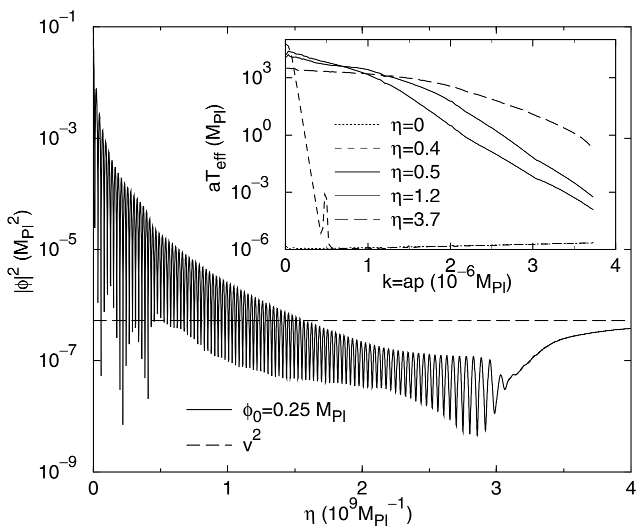

FIG. 1. The time evolution of $|\phi|^2$ in the first simulation on a 320^3 lattice with the initial condition $\overline{\phi}_0 = 0.25 M_{\rm Pl}$. That $|\phi|^2$ is below v^2 indicates that the symmetry is restored. At $\eta \approx 3 \times 10^9 M_{\rm Pl}^{-1}$, the transition to the Higgs phase takes place. The inset shows the effective temperature of different Fourier modes of the electric field $\tilde{E}_i$ measured at various values of η (given in units of $10^9 M_{\rm Pl}^{-1}$). The energy density in the short-wavelength modes is suppressed by a factor of 10^4 relative to the long-wavelength modes even at the end of the simulation, and therefore the discretization errors are expected to be small.

$|\tilde{E}_{i,k}|^2$ is constant and its magnitude is proportional to the temperature. Therefore we can use it to define the effective temperature of a single mode

$$T_{\rm eff}(p = k/a) = \frac{1}{2a} |\tilde{E}_{i,k}^T|^2 \frac{d^3k}{(2\pi)^3}, \qquad (10)$$

where the superscript T indicates that we have included only the transverse component $k_i \tilde{E}_{i,k}^T = 0$, since the longitudinal component is fixed by the Gauss law. The inset of Fig. 1 shows the product $aT_{\rm eff}$, which is the effective temperature of the rescaled fields, as a function of the conformal momentum $k = pa$.

Immediately after preheating, the temperature of the long-wavelength modes is $T_{\rm eff} \sim 10^4 M_{\rm Pl}$ and the occupation number $n_p = T_{\rm eff}(p)/p \sim 10^{10}$ is huge. The cutoff momentum is $p_* = k_*/a \approx 10^{-6} M_{\rm Pl}/a$. With time, the modes with higher and higher k thermalize and the temperature decreases, but since the couplings are small, this process is very slow. The modes with $k \gg k_*$ are strongly suppressed even after the phase transition, and therefore we believe that the lattice approximation remains reliable even at the end of the simulation. Because the modes with the highest momenta do not remain exactly in the vacuum, discretization errors cannot be ruled out completely.

For the electroweak theory, the opposite case $q < 1$ is more relevant, since $q \approx m_W^2/m_H^2$. In this case, the parametric resonance is narrow and the energy transfer is less efficient. However, since the expansion rate of the Universe is much slower in this case, it could still lead to a similar phenomenon. Most of the energy of the inflaton is transferred to a narrow momentum range of the fluctuations, but the long-wavelength modes thermalize and the energy is spread to all long-wavelength modes. After that, the system should behave as in the case with a broad resonance. In equilibrium, the phase transition is not of first order if $e^2 > \lambda$, but as discussed earlier, we expect the transition to be stronger in our case.

The realistic values for the couplings in the electroweak theory would be $\lambda \sim e^2 \sim 1$, but in that case our simulations are not reliable. With these couplings, the interactions are important even in the vacuum state, and the classical approximation cannot be trusted. Therefore, we have used slightly smaller couplings, $\lambda = 0.04$ and $e = 0.14$, which allow us to use the classical approximation. The initial value of the Higgs field was $\overline{\phi}_0 = 1$ TeV. We also chose $v = 246$ GeV and $a(\eta) = 1 + \eta H$ with $H = 0.7$ GeV. The lattice spacing was $\delta x = 1.4\ {\rm TeV}^{-1}$, time step $\delta\eta = 0.14\ {\rm TeV}^{-1}$, and the lattice size 240^3.

In this case, ϕ cannot be the inflaton, because its couplings are much too strong. However, the homogeneous initial condition for ϕ may arise from a previous preheating phase, in which ϕ couples to the inflaton with a coupling constant that is much smaller than e. Then the parametric resonance will transfer a large amount of energy to modes of ϕ with very long wavelengths. The alternative

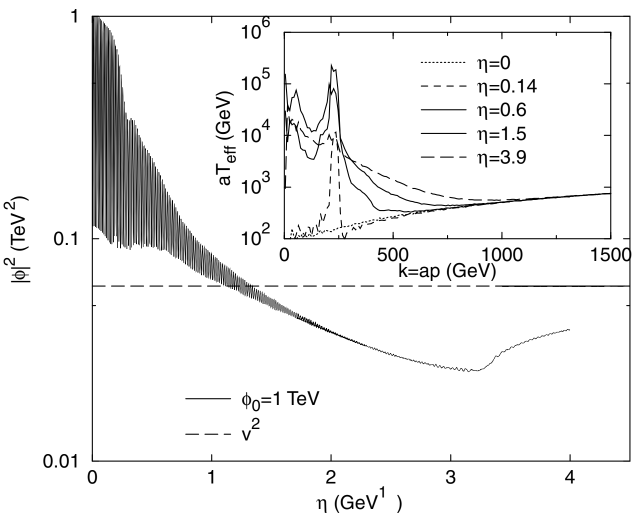

FIG. 2. The time evolution of $|\phi|^2$ in the second simulation on a 240^3 lattice with the initial condition $\overline{\phi}_0 = 1$ TeV. Again, the symmetry is restored, and at $\eta \approx 3\ \mathrm{GeV}^{-1}$, the transition to the Higgs phase takes place. The effective temperature measured at various η (given in units of GeV^{-1}) in the inset shows that in this case the resonance is narrower, but the temperature still grows to high values.

possibility is that quantum fluctuations of ϕ give it a large spatial average during inflation.

As in the earlier case for the inflaton, we show $|\phi|^2$ and the effective temperature of different modes of E_i in Fig. 2. This time, the energy is transferred into a narrow band of gauge field modes. Nevertheless, the long-wavelength modes thermalize, and we reach a similar situation to that in the first case, in which the long-wavelength modes $k \lesssim 300$ GeV have an effective temperature $T_{\mathrm{eff}} \approx 10^4$ GeV, and the symmetry is restored. At $\eta \approx 3\ \mathrm{GeV}^{-1}$, the system undergoes a phase transition to the Higgs phase. The transition is not of first order, but it is still rather sharp.

In the electroweak theory, the conservation of baryon number would be violated by sphaleron configurations with a rate $\Gamma_{\mathrm{sph}} \sim \alpha_W^5 T_{\mathrm{eff}}^5 / p_*$ [14] when the symmetry is temporarily restored, and as discussed in Ref. [15], the oscillations of the Higgs field could create a large baryon asymmetry. If the transition to the Higgs phase is sharp enough, the baryon number violation ceases instantaneously, and the produced baryon asymmetry remains.

Our simulations show that a gauge-Higgs system exhibits the same behavior as the scalar model considered in Ref. [4]. The first case we considered shows that a nonthermal phase transition is possible if the inflaton is charged under a gauge group. Although we restricted ourselves to an Abelian model, we believe that the qualitative features of our results would be the same in non-Abelian theories. In many models, this phase transition would lead to the formation of cosmic strings or other topological defects.

The second case we considered has the qualitative features of the electroweak theory, and we find that the symmetry gets restored although the parametric resonance is narrow, provided that the expansion of the Universe is slow enough. Unlike in the standard thermal phase transition scenario, the transition to the Higgs phase is sharp, which makes it possible to preserve the produced baryon asymmetry. This supports the picture of electroweak baryogenesis at preheating.

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