Interaction of Localized Structures in an Optical Pattern-Forming System

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We report on the observation and interaction of dissipative localized structures in an optical pattern-forming system. Single localized structures are found to have oscillatory decaying tails originating from diffraction. We observe bound states of two or more constituents. These clusters contain several preferred mutual distances. Numerical simulations show that the corresponding interactions are mediated by the oscillatory tails.

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Spontaneous self-organization in two-dimensional dissipative systems far from equilibrium can produce extended patterns of, e.g., square, hexagonal, or even quasiperiodic symmetry [1]. On the other hand, the emergence of single or multiple spotlike excitations that are localized to a small area has attracted increasing attention in recent years (e.g., [2–5] and references therein). These localized structures (LS) may form clusters, either with or without a well-defined symmetry [4,5]. Understanding the interactions responsible for the selection of the symmetry and the length scale of the clusters is highly desirable and should clarify the relation between LS and extended (delocalized) patterns.

In this Letter we report on the occurrence and the properties of LS in an optical pattern-forming experiment. Nonlinear optical systems appear to be interesting for a detailed investigation of LS [6–10]. On the one hand, this is due to the fact that the structures arise from a balance between nonlinearity and diffraction and thus form a counterpart to localized states occurring in other systems, in which diffraction is not available as a spatial coupling mechanism. On the other hand, interest stems from potential future applications of optical LS as "bits" in all-optical logical components. This task obviously also requires a thorough understanding of their interaction behavior. It has been investigated in theoretical work on different optical [11–15] and nonoptical systems [16] and also experimental results have been reported [17–19]. Here we present experimental evidence of the existence of a number of discrete stable distances of LS that result obviously from a repeated change in the interaction from repulsion to attraction, when the distance is varied, and have been predicted in the theoretical analysis [11–16].

The setup of our experiment (cf. Fig. 1) is an all-optical implementation of the single-mirror feedback scheme analyzed in [20], consisting of a thin nonlinear medium that is irradiated by an intense light field and a plane feedback mirror at a distance d behind the medium. Inside the nonlinear medium the amplitude and phase of the light field is modified locally, if there is a spatial variation of the complex susceptibility. Outside the medium, diffraction takes place during the propagation of the light field to the feedback mirror and back and provides the spatial coupling required for the formation of structures. In this scheme certain spatial modulations of the susceptibility can stabilize themselves, since the susceptibility of the medium depends on the intensity of both the incoming and the reflected beam.

We use sodium vapor in a N_2 buffer gas atmosphere as a nonlinear medium. A circularly polarized light field with a frequency of $\Delta \approx 15$ GHz above the resonance of the Na-D_1 line is injected into the vapor. Under these conditions the nonlinearity has both a dispersive and an absorptive contribution. The laser beam is spatially filtered in order to obtain a smooth profile with a good cylindrical symmetry; the frequency and the intensity are actively stabilized. We observe the transverse intensity distribution of the light field through the slightly transmissive feedback mirror ($R = 0.91$) with the help of an imaging lens and a charge-coupled device (CCD) camera.

The susceptibility of the sodium vapor depends linearly on the population difference between the two Zeeman sublevels of the ground state, which is called "orientation." Optical pumping with circularly polarized light induces a nonzero orientation which saturates already for small intensities. In an oblique magnetic field the orientation can display a nonmonotonous dependence on the intensity: the Zeeman splitting produced by the longitudinal component of the magnetic field can be compensated by the level shift produced by a light field that is detuned from resonance;

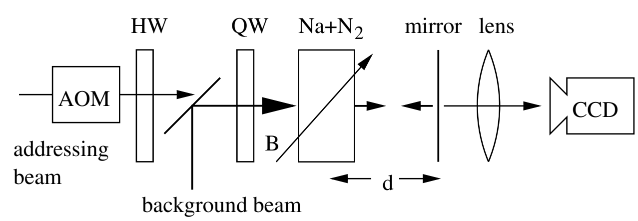

FIG. 1. Schematic diagram of the experimental setup. QW: quarter-wave plate; HW: half-wave plate; AOM: acousto-optic modulator; Na + N_2: sodium cell. The AOM shifts the frequency of the second beam by 140 MHz with respect to the background beam in order to avoid the need for interferometric stability between the two beams.

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in this case of degenerate sublevels the orientation will efficiently be destroyed by the presence of a transverse component of the magnetic field [21]. Thus there can be a pronounced minimum of the orientation for a well-defined finite light intensity. In the presence of optical feedback the corresponding characteristic curve describing the homogeneous solution of the orientation can become very steep and it can even display bistability. This should provide favorable conditions for the occurrence of large amplitude excitations such as LS. In the experiment the power of the background beam is chosen in such a way that the intensity in the area around the beam center is in the minimum of the characteristic.

The LS can be ignited by injecting a second laser beam (addressing beam in Fig. 1) of smaller width having the same circular polarization as the background beam. The spot remains stable when the addressing beam is switched off. Because of noise and because of the intensity and phase gradient of the Gaussian background beam the spot may wander in the region around the center of the background beam. Repeated injection of the addressing beam at different places leads to the creation of multiple LS; starting from a background beam without LS, up to three spots have been ignited with the addressing beam. By changing the sign of the circular polarization of the addressing beam we can make use of the polarization properties of the light-matter interaction [22] and use the addressing beam to erase LS.

By varying the position of the imaging lens, the evolution of the intensity profile during the free space propagation from the vapor cell to the mirror and back can be examined (Fig. 2). It turns out that the incoming light field experiences a higher transmission at the site of the LS which leads to a small maximum in the transmitted

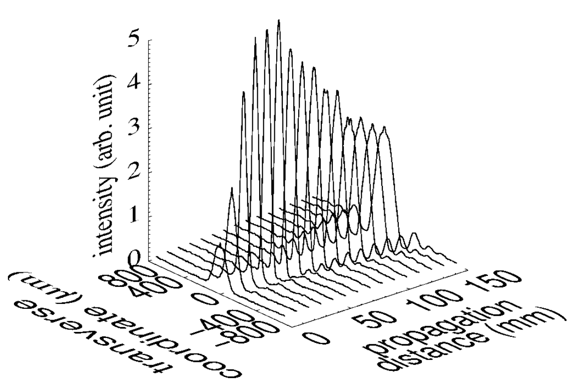

FIG. 2. Variation of the intensity profile produced by a localized structure during the propagation from the sodium vapor to the feedback mirror and back. Parameters: transverse magnetic field $B_{\perp} = 4.17\ \mu$T, longitudinal magnetic field $B_{\parallel} = 8.23\ \mu$T, detuning $\Delta = 13.5$ GHz, mirror distance $d = 70$ mm, length of sodium vapor $L = 15$ mm, N_2 pressure $p = 216$ mbar, and temperature $T = 340\,^{\circ}$C. The profiles are averaged over the azimuthal coordinate to enhance the signal-to-noise ratio.

intensity distribution (see the first profile in Fig. 2). The reflected intensity peak which reenters the sodium vapor has approximately the same diameter as the peak directly behind the vapor. As can be seen from the last profile in Fig. 2, this peak is surrounded by several diffraction fringes; especially the first diffraction minimum is very pronounced. Between these two positions, the light field passes through a kind of focus. From these observations one infers that the incoming light field experiences a wave front deformation indicating that the vapor acts locally as a focusing lens. The corresponding variation of the susceptibility is sustained by the intensity peak in the reflected light. Therefore we conclude that the LS are stabilized by a self-induced lensing effect.

The formation of LS does not require the addressing beam but they form spontaneously when the input power is increased above a critical value P_c. They remain stable when the input power is decreased; the system thus shows a bistable behavior between the unstructured state and the state with LS. In the power range above P_c, the average number of bright spots that pop up spontaneously increases on average with the input power (see Fig. 3). They are not densely packed as in a periodic lattice but may form clusters which do not have an apparent symmetry. The images in Fig. 3 show typical examples of the observed clusters. We observe reorientation between different configurations of spots and the spontaneous appearance and disappearance of spots as well as hysteresis indicating multistability. For higher input powers the LS show the tendency to arrange more densely. For the parameters of Fig. 3 these clusters are stable on a time scale of some tens of milliseconds. The transition time between different clusters is much shorter (less than 100 μs). Since the most important dynamical time scale is given by the inverse of the Larmor frequency of the transverse magnetic field and amounts to some tens of microseconds we conclude that the clusters are essentially stable and stationary structures but that alternation between different clusters occurs due to noise in a highly multistable situation.

In [9,13,23] evidence was given that LS drift along phase and intensity gradients of the background beam. In our case the incoming Gaussian laser beam induces an inhomogeneous distribution of the orientation. This gradient forms the background for the LS and influences the phase of the transmitted beam in such a way that the LS drift away from the beam center. This effect is counterbalanced by the intensity gradient of the light field which induces a motion towards the region of higher intensity. As a result a single LS is positioned at some place on a ring around the center of the background beam (Fig. 3a). The same holds for clusters of two LS (Fig. 3b), whereas in clusters with more constituents more general configurations are realized (Figs. 3c–3e), presumingly resulting from a compromise between the restriction imposed by the background and the interaction behavior of the LS (see below). A detailed investigation of the LS shows that their

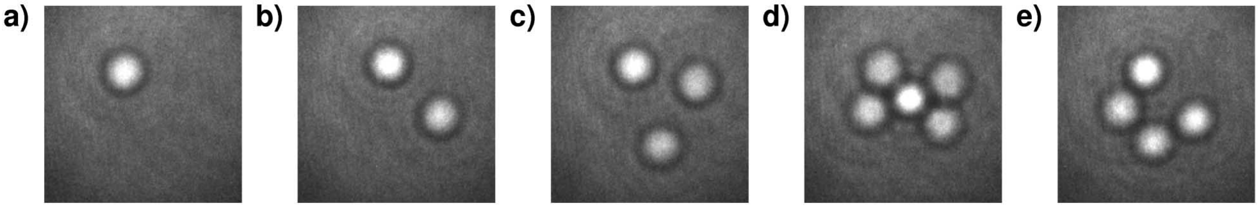

FIG. 3. Stable clusters of localized structures. The image plane shows the reentrant light distribution. The images are overexposed in order to emphasize the diffraction fringes surrounding each localized structure. Parameters: $B_\perp = 0.78\ \mu$T, $B_\parallel = 14.40\ \mu$T, $\Delta = 18.6$ GHz, $d = 70$ mm, $p = 310$ mbar, and $T = 315\,°$C. Intensities: (a) 131 mW, (b) 133 mW, (c) 133 mW, (d) 135 mW, and (e) 138 mW. The transverse size of the images is 2580 μm.

parameters vary in dependence on the distance from the beam center: The maximum intensity decreases with increasing distance, whereas the diameter of the central spot and the diameters of the diffraction fringes increase slightly.

An analysis of the clusters suggests that certain distances between single LS are preferred by the system. For a detailed analysis we recorded a large number of images for fixed experimental parameters at a constant time interval of 200 ms. Because of the spontaneous transitions of the LS into different configurations, the observed configurations varied in the number of constituents (from 1 to 3) and in the position of the LS on the preferred ring around the beam center. For the histogram of distances displayed in Fig. 4 the subset of images containing only two LS has been considered.

A preference for three discrete values of the distance is clearly visible from Fig. 4. By weighted averaging over the three humps we find the values 445, 573, and 717 μm. Figures 5a–5c show typical examples of these configurations. During the experiment the system switched randomly between the three configurations. The peaks for larger distances between two LS are broader than the first peak; the reason for this result is probably that the configurations with a larger distance are more sensitive to noise or parameter fluctuations. We remark that one important noise source is ac stray magnetic fields. A variation of the mirror distance d in the experiment shows that the preferred distances between two LS as well as the diameter of the diffraction fringes are increasing with increasing d.

An analysis of the clusters with more than two constituents in Fig. 3 reveals that most of the distances observed between the constituents appear already in the bound states of two single LS for identical parameters; the deviations are characteristic for the type of cluster.

The LS and the existence of a discrete set of stable mutual distances can be reproduced in numerical simulations of the microscopic model described in [21] in a parameter range similar to the experimental parameters. The LS appear in a region where the homogeneous solution is bistable or nearly bistable due to the light-shift induced level crossing mentioned above. In this region, the system shows a subcritical modulational instability towards an extended pattern. Slightly above threshold the band of unstable wave vectors becomes very broad and includes zero wave number for the case of a bistability of the homogeneous solution. This combination of a subcritical modulational instability and a plane wave optical bistability is

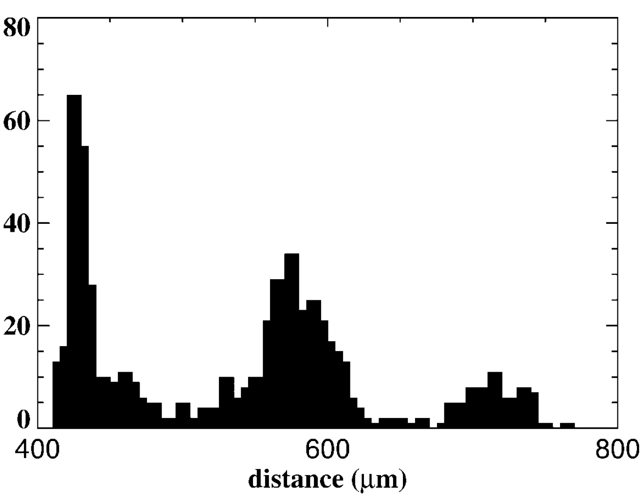

FIG. 4. Histogram of distances between two localized structures; 669 images are evaluated. Parameters as in Fig. 5.

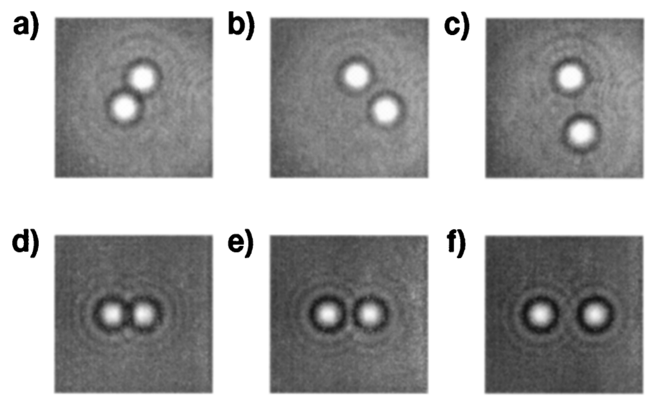

FIG. 5. Stable configurations of two localized structures. (a)–(c): Experiment, parameters: $B_\perp = 2.36\ \mu$T, $B_\parallel = 10.98\ \mu$T, $\Delta = 13.9$ GHz, $d = 63$ mm, $p = 200$ mbar, and $T = 316\,°$C. (d)–(f): Numerical simulation, parameters $B_\perp = 3.94\ \mu$T, $B_\parallel = 15.72\ \mu$T, $\Delta = 9.5$ GHz, $d = 63$ mm, and sodium particle density $N = 0.3 \times 10^{14}$ mm^{-3}. The transverse size of the images is 2020 μm. The experimental images are overexposed to visualize the diffraction fringes.

a typical situation for LS in optical systems [13,17]. The evolution of the light field during the propagation in free space shows qualitatively the same behavior as the experiment reported in Fig. 2. This gives a hint that the assumptions of the model are justified [21]. Among others the model requires a thickness of the nonlinear medium small enough that the light transverses it undergoing negligible diffraction [20]. In this approximation the nonlinear effects and the diffraction, whose interplay is at the origin of the LS formation, occur in separate spatial regions.

For studying the interaction behavior calculations are carried out under the assumption of a plane wave background beam. In this way the drift of the LS in a gradient of the background is eliminated. It turns out that—depending on the initial distance between two LS—they either attract or repel each other until a stable distance is achieved. Figures 5d–5f show the configurations for the three smallest stable distances between two LS. The corresponding distances are 393, 533, and 691 μm. We find even larger distances than these three, but they are rather unstable with respect to noise. The simulations show that the characteristic distances between two LS increase with increasing diameter of the diffraction fringes. This dependence has been checked with two different methods: first the diameter of the diffraction fringes can be varied by changing the position of the feedback mirror. The second approach consists in cutting off higher spatial Fourier components of the light field; in this way the diameter of the diffraction fringes can be increased by $\approx$20% for a given set of parameters.

As mentioned before, the existence of several stable distances between two interacting LS has been predicted in various systems [11,12,14–16]. In the present one the corresponding spatial oscillations in the interaction arise from the multiple diffraction fringes created by the LS. Thus, in this interpretation, the existence of prevailing distances has a similar origin as the motion of a single LS in the profile of a Gaussian beam: the LS experience forces due to the intensity and phase gradients of the light field. The modulations in the diffraction fringes surrounding the LS give substantial contributions to these gradients. In the presence of more than one LS, the corresponding forces induce a drift of the LS until a self-consistent equilibrium position is reached. Because the LS are surrounded by several diffraction fringes, a large number of equilibrium positions can be expected between two LS. Only the smallest distances, however, are observed in the limited beam. Furthermore we observe numerically that the stability of a bound state of LS against perturbations decreases with increasing distance, which is consistent with the fact that the amplitude of the fringes decreases.

Since the emergence of extended patterns as well as the fringes stems from diffraction, it should not be surprising that the smallest distance observed corresponds roughly to the period of the most unstable mode of the linear analysis of the stability against periodic perturbations. The larger distances, however, have no counterpart in the linear stability analysis of the homogeneous system, but are the result of the interaction between the strongly nonlinear localized modes.

We remark that the importance of oscillatory tails for the stability of bound states of solitary pulses was established before in quite general systems like the perturbed nonlinear Schrödinger equation and the quintic Ginzburg-Landau equation [14,15]. This finding illustrates the generality of the observed phenomena.

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