Generalization of the N_pN_n Scheme and the Structure of the Valence Space

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The N_pN_n scheme, which has been extensively applied to even-even nuclei, is found to be a very good benchmark for odd-even, even-odd, and doubly-odd nuclei as well. There are no apparent shifts in the correlations for these four classes of nuclei. The compact correlations highlight the deviant behavior of the $Z = 78$ nuclei and are used to deduce effective valence proton numbers near $Z = 64$ as well as to study the evolution of the $Z = 64$ subshell gap.

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Many physical systems, including atoms, nuclei, and metallic clusters, exhibit shell structure. Indeed, eigenvalues of the three-dimensional Schrödinger equation will tend to cluster in energy groupings (characterized by specific sets of principal and angular momentum quantum numbers) for any reasonable central potential. In the treatment of complex finite many-body systems, a common simplification is to invoke a "mean field" ansatz, replacing the sum of all the two-body interactions by a one-body potential. Generally, such a procedure is only an approximation and various residual interactions need to be incorporated. These will alter the predictions of the independent particle picture and may even lead to a breakdown of the shell structure, shell closures, and shell gaps.

Nuclei provide an ideal venue to study shell structure and residual interactions since they are finite-body systems where the effective number of active bodies (the valence nucleons) is generally quite small (0–30, say) and where one can both count and change this number of bodies (the mass number) in a controlled way. Here, we wish to explore the evolution of collective behavior in nuclei and the associated evolution of shell structure using an empirical correlation scheme of collective observables that stresses the importance of the valence residual p-n interaction.

The importance of the proton-neutron interaction in determining the evolution of nuclear structure was emphasized long ago by de Shalit and Goldhaber [1], and Talmi [2]. Two decades ago, Federman and Pittel [3] emphasized that the driving mechanism in the development of nuclear deformation is the proton-neutron interaction between nucleons in spin-orbit partner orbits. If the proton-neutron interaction is a controlling factor in the determination of nuclear structure, a reasonable estimate of this interaction ought to be a useful systematizing parameter with which the evolution of structure could be correlated.

In 1985 Casten described the N_pN_n scheme for even-even nuclei [4], in which $E_{2_1^+}$, $E_{4_1^+}/E_{2_1^+}$, and $B(E2, 0_1^+ \rightarrow 2_1^+)$ values were plotted against the product of valence proton number and valence neutron number, N_pN_n. The systematics for each observable is very smooth, and similar from region to region. It was found that the quantity N_pN_n provides an excellent scaling factor that allows one to assess the rapidity of different transition regions and to predict the properties of new nuclei [5]. Moreover, the slopes of different observables plotted against N_pN_n are related to the average interaction, per proton-neutron pair, in the highly overlapping orbits whose occupation induces structural change.

However, most papers related to the N_pN_n scheme have concentrated on the even-even case where there is a rich array of compiled nuclear data. It is therefore important to see whether the N_pN_n scheme works, and how well it works, in odd-A and doubly odd nuclei. The N_pN_n concept is more difficult to apply to odd-A and odd-odd cases because there can be a very strong interplay between collective and single particle excitations, and the low-lying excitation structures themselves are more complicated. Moreover, adjacent nuclei differ in ground state and low-lying J^π values so it is sometimes not clear which data to use in a systematic comparison. Finally, observables related to odd-A nuclei and odd-odd nuclei are in general less well, and less systematically, known than those of even-even nuclei.

The most extensive studies for odd-A nuclei to date have been for the $A = 80-100$ region. In [6] the N_pN_n scheme was applied to both even-even and odd-A nuclei in the $A \sim 80$ region; in [7] a few odd-A nuclei with $A \sim 100$ were considered; in [8], it was shown that states based on different single-particle excitations behave differently with N_pN_n. However, there has not yet been any concerted effort towards a unified N_pN_n treatment for even-even, odd-A, and doubly-odd nuclei over large mass regions.

It is therefore the purpose of this Letter to show for the first time that the simple N_pN_n scheme works equally well for large regions of medium-heavy nuclei for even-even, odd-A and the doubly-odd nuclei. This extension to the N_pN_n scheme will significantly expand its usefulness for interpreting the sparse data soon-to-be-obtained on exotic nuclei far from stability. We will also use these results to extract effective valence proton numbers near $Z = 64$

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and $N = 83-91$ in order to study the breakdown of the $Z = 64$ shell gap in even, odd, and odd-odd nuclei.

We proceed by studying the deformation parameter e_2 against N_pN_n. The e_2 values are taken from the macroscopic-microscopic calculations of [9] for nuclei with known ground and excited states. These deformations act as surrogates for directly measured observables, and therefore allow us to compare even and odd Z and N nuclei on the same footing. These calcualtions are highly refined, and widely used. For nuclei in or near the valley of stability, such as those considered here, they should provide an excellent guide to realistic deformations, although it would be useful to check them by experiment. Of course, far from stability, the importance of various residual interactions changes, as does the mean field itself, and hence care should be taken in extending these results to new regions. In any case, for known nuclei, we believe that the approximations used in [9] are reasonably good individually, and fully adequate for a systematic study in large regions. Moreover, by using the deformation rather than excitation energies to gauge the structure, one avoids problems with comparing levels with different spins.

In Fig. 1, we present the quadrupole deformation parameter in the Nilsson perturbed-spheroid parametrization, e_2, vs N_pN_n for the nuclei in four different regions ranging from $Z = 50$ to 104, namely the $50 < Z \leq 66$, $82 < N \leq 104$ region, the $66 < Z < 82$, $82 < N \leq 104$ region, the $66 < Z < 82$, $104 < N < 126$ region, and the $82 < Z \leq 104$, $126 < N < 155$ region. The correlation between e_2 and N_pN_n is extraordinarily compact not only for the even-even nuclei but also for the even-odd, odd-even and odd-odd cases as well [see solid symbols in Fig. 1(a) and the full set of points in Figs. 1(b)–1(d)]. Moreover, the correlations are independent of the even-even, even-odd, odd-even, or odd-odd nature of the nuclei considered. No discernible bias for these classes of nuclei is visible except for a slight difference between the points for even-proton number and odd-proton number for N_pN_n values less than 50 in Fig. 1(c).

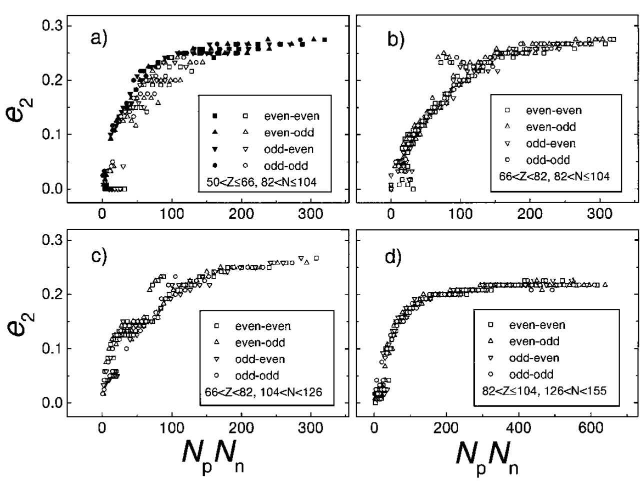

FIG. 1. The deformation parameter e_2 vs N_pN_n. (a) For nuclei with $50 < Z \leq 66$ and $82 < N \leq 104$. Open symbols for $Z = 59-66$ and $N \leq 91$. Solid symbols for all other nuclei (i.e., $50 < Z \leq 58$ for all neutron numbers and $59 \leq Z \leq 66$ for $N \geq 92$.); (b) $66 < Z < 82$ and $82 < N \leq 104$; (c) $66 < Z < 82$ and $104 < N < 126$; (d) $82 < Z \leq 104$ and $126 < N < 155$. Note the scale change in part (d) to accomodate the larger N_pN_n values in this mass region.

Among the correlations shown, Fig. 1(a) shows a greater broadening near $N_pN_n \sim 50-100$ than the other regions. This is a region where there is a subshell at $Z = 64$ which we did not take into account. That is, we used the proton magic numbers 50 and 82 for all nuclei. Below, we will examine the validity of these choices. To facilitate that discussion, Fig. 1(a) uses open symbols for nuclei with $N \leq 91$ and $59 \leq Z \leq 66$. Another interesting point in Fig. 1(a) is that there are a number of data points with $e_2 = 0$, which correspond to the $N = 84$ isotones. These isotones are very soft, which means that the shallow part of the potential energy against the deformation parameter is wide. Hence there can be a large difference between the equilibrium deformation and the expectation value of the deformation.

In Figs. 1(b) and 1(c), several data points clearly stand out to the upper left of the correlations. Nearly all have $Z = 78$ (Pt) and lie in a complex region with large γ softness, oblate shapes, prolate shapes, and transition regions between them. Nevertheless, other regions also show sharp shape changes but are not anomalous in the N_pN_n plots. Therefore, it is worth further effort to understand the behavior of the $Z = 78$ Pt region and whether these anomalous points reflect a different role for the p-n interaction in these nuclei or a shortcoming in the calculated deformations in [9].

While the concept of the valence space is important in understanding the structure of nuclei, in many cases the conventional counting of valence protons and neutrons is inadequate. For example, near $A = 100$ and 150, the $Z = 40$ and 64 proton numbers take on magic character for certain neutron numbers but not for others [10]. Likewise, the neutron number $N = 20$ is no longer magic for the neutron rich nucleus ^{32}Mg [11]. Indeed, it is expected that magicity may well be a fragile construct far from stability. This fragility is a result both of changes to the mean field and to the valence p-n residual interaction [12,13]. Its effects might be expected to show up in the N_pN_n scheme. Indeed, in even-even nuclei, effective N_p values have been discussed for both the $A = 100$ and 150 regions [4,14–17].

The present results give us the opportunity to probe this issue more deeply, by extracting effective N_p values in the $A = 150$ region from even, odd, and odd-odd nuclei simultaneously and in a unified way.

In Fig. 1(a), the solid symbols are for the $59 \leq Z \leq 66$ and $N \geq 92$ nuclei, and all nuclei with $50 < Z \leq 58$. They form an extraordinarily compact trajectory, while the $59 \leq Z \leq 66$ and $N \leq 91$ nuclei deviate strongly to the right. This arises because, for these latter nuclei, $Z = 64$

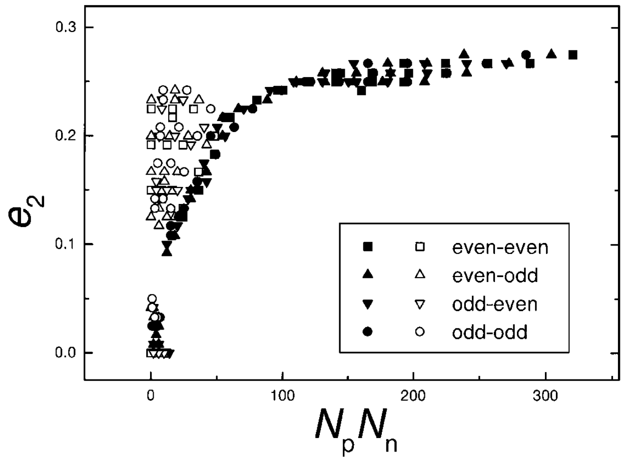

FIG. 2. Similar to Fig. 1(a) except $Z = 64$ is used as a magic number instead of 82 for $N \leq 91$.

acts as a magic or partially magic number whereas Fig. 1(a) was constructed using $Z = 50$ as magic. Hence these nuclei were plotted at inappropriately large N_pN_n values. The opposite assumption, that $Z = 64$ is magic for $N \leq 91$ is also too extreme. As shown in Fig. 2, this leads to an overshoot of these points to the left.

Clearly, by assuming the validity of the compact correlation for nuclei not affected by a $Z = 64$ gap, that is those marked by solid symbols in Fig. 1(a), and shifting the "deviant" nuclei leftward to this correlation, we can extract the effective N_p values for these nuclei and thereby assess the breakdown and dissolution of the $Z = 64$ gap. Equivalently, one can shift the anomalous data points in Fig. 2 to the right. The process is similar to that used in [4] for even-even nuclei but now is extended uniformly to all species.

Figure 3 illustrates how this approach works by looking at a subset of the points in Fig. 2—those for even-odd nuclei. Here, the solid symbols are the nuclei unaffected by a $Z = 64$ gap. The open symbols lie at various distances from the main correlation: consistently, the $Z = 64$, 66 isotopes lie farthest, and the $Z = 62$ and 60 isotopes occur successively closer. The amount of shifting required for each point is determined by fitting an exponential function to the normal (solid symbol) data in Fig. 3, and such a fitting curve is used as a guide to deduce the appropriate N_p value for that e_2. The resulting effective N_p values for all the data of Fig. 1(a) are summarized in Table I and shown in Fig. 4. They are given in the Table to the nearest odd(even) integers for odd(even)-Z nuclei. Note that in Table I we do not present effective valence proton numbers for the $N = 84$ isotones since, as discussed above, these nuclei are soft and the equilibrium and mean deformations may differ considerably, and also the calculated deformations can be very sensitive to small perturbations. The results in Table I demonstrate a gradual breakdown of the $Z = 64$ shell gap, which accelerates near $N = 90$, and consistency regardless of whether the nuclei are even-even, odd-even, even-odd, or odd-odd.

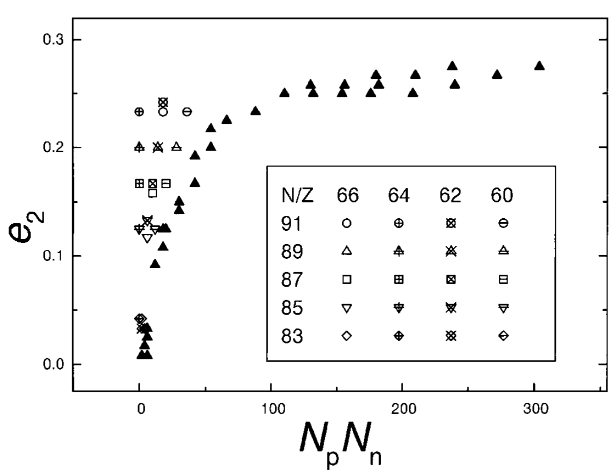

FIG. 3. Extract from Fig. 2 for even-odd nuclei, where different symbols are used to denote nuclei with $60 \leq Z \leq 66$ and $N \leq 91$.

To summarize, the N_pN_n scheme, which has been extensively studied for even-even nuclei, is found to be equally applicable to all species of medium-heavy nuclei: even-even, odd-even, even-odd, and odd-odd. The N_pN_n correlations are not sensitive to the odd-even difference. This supports the idea that the proton-neutron interaction plays a similar role regardless of the even-odd character of the nuclei, and suggests that the average strength of the valence proton-neutron interaction is almost constant between even-even and their odd-A/odd-odd neighbors. The extremely compact N_pN_n trajectories highlight a few deviant nuclei. Finally, effective valence proton numbers were extracted from these correlations and found to be also insensitive to the category of nucleus. This gives a deeper view of the breakdown of the $Z = 64$ magicity near neutron number 90.

The present work extends the realm of application of the N_pN_n scheme to all types of nuclei. Given that compact correlation schemes, such as N_pN_n, magnify anomalous behavior (e.g., the $Z = 78$ nuclei discussed above), and probe the valence space (i.e., the effective valence nucleon numbers), the present results and approach can provide a

TABLE I. Effective proton numbers for nuclei near the $Z = 64$ subshell.

Z/N	83	85	86	87	88	89	90	91	92
59	5	7	7	7	7	7	9	9	9
60	4	6	6	6	8	8	10	10	10
61	5	7	7	7	7	7	9	11	11
62	4	6	6	6	8	8	10	10	12
63	7	7	7	7	7	7	9	11	13
64	4	6	6	6	8	8	10	12	14
65	5	7	7	7	7	7	9	11	15
66	4	6	6	6	8	8	10	12	16

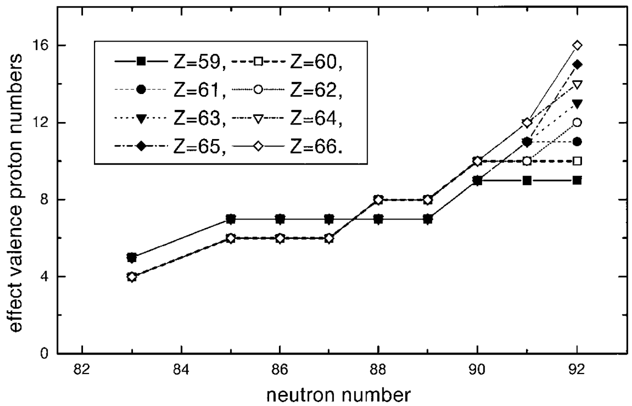

FIG. 4. Summary of the effective valence proton numbers obtained in this work.

more general tool to disclose new and different types of shell structure or structural evolution (e.g., changes in shell structure and magicity) in exotic nuclei.

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