Quantum Communication between Atomic Ensembles Using Coherent Light

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Protocols for quantum communication between massive particles, such as atoms, are usually based on making use of nonclassical light, and/or superhigh finesse optical cavities are normally needed to enhance interaction between atoms and photons. We demonstrate a remarkable result: by using only coherent light, entanglement can be generated between distant free space atomic ensembles, and an unknown quantum state can thus be teleported from one to another. Neither nonclassical light nor cavities are needed in the scheme, which greatly simplifies its experimental implementation.

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The goal of quantum communication is to transmit an unknown quantum state from one particle to another one at a distant location. This can be obtained either by direct transmission of the state [1], or by disembodied transport, i.e., quantum teleportation [2]. Quantum teleportation of an unknown state from a photon to a photon [3,4], or from a single-mode beam of light to another [5] has been demonstrated experimentally. A desired goal is to perform quantum teleportation of the state of massive particles, since the massive particles are ideal for storage of quantum information, and they play an important role in local quantum information processing, such as quantum computation. At the same time, the information should be transferred from one location to another via optical states, since light is the best long distance carrier of information. There have been several proposals for quantum teleportation of atomic motional or internal states, by transmitting single-photon or nonclassical light [6–8]. Most of these proposals are based on the assumption that atoms are trapped inside high-Q optical cavities, which is difficult to achieve experimentally [6,7]. The recent proposal [8] eliminates this requirement; however, it still requires an external source of entanglement (nonclassical light). Here we propose and analyze an atomic quantum teleportation scheme, in which *quantum entanglement* between *free-space atomic ensembles* is achieved by using only *coherent light*. This result is remarkable, since strong coherent light (light from an ordinary laser) is usually thought to be "purely classical," but via it quantum entanglement can be achieved, and unknown internal states of free-space atomic ensembles can nonetheless be teleported from one location to another.

The system we are considering is a cloud of identical atoms with the relevant level structure shown in Fig. 1. Each atom has two degenerate ground states and two degenerate excited states. The transitions $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |4\rangle$ are coupled with a large detuning Δ to propagating light fields with different circular polarizations. This kind of interaction has been analyzed semiclassically in [9], and recently shown to be applicable for quantum non-demolition measurements [10,11] and teleportation with nonclassical light [8], with an adiabatic Hamiltonian.

We create entanglement between two atomic ensembles through a nonlocal Bell measurement with the schematic setup shown by Fig. 2. We assume a one-dimensional model for the propagating light field. As shown in Ref. [12], this is justified if the atomic ensemble is of a pencil shape with Fresnel number $F = A/\lambda_0 L \approx 1$. Here A and L are the cross section and the length of the ensemble, respectively, and λ_0 is the optical wave length. The input laser pulse is linearly polarized and expressed as $E^{(+)}(z,t) = \sqrt{\hbar \omega_0 / 4\pi \epsilon_0 A} \sum_{i=1,2} a_i(z,t) e^{i(k_0 z - \omega_0 t)}$, where $\omega_0 = k_0 c = 2\pi c/\lambda_0$ is the carrier frequency, and i denotes two orthogonal circular polarizations, with the standard commutation relations $[a_i(z,t), a_j(z',t)] = \delta_{ij}\delta(z - z')$. The light is weakly focused with cross section A to match the atomic ensemble. For a strong coherent input with linear polarization, the initial condition is expressed as $\langle a_i(0,t)\rangle = \alpha_t$, while the total photon number over the pulse duration T satisfies $2N_p = 2c \int_0^T |\alpha_t|^2 dt \gg 1$. The Stokes operators are introduced for the free-space input and output light (light before entering or after leaving the atomic ensemble) by $S_x^p = \frac{c}{2} \int_0^T (a_1^\dagger a_2 + a_2^\dagger a_1) \, d\tau$, $S_y^p = \frac{c}{2i} \int_0^L (a_1^\dagger a_2 - a_2^\dagger a_1) \, d\tau$, $S_z^p = \frac{c}{2} \int_0^L (a_1^\dagger a_1 - a_2^\dagger a_2) \, d\tau$. In free space, $a_i(z,t)$ depends only on $\tau = t - z/c$, and then the Stokes operators satisfy the spin commutation relations $[S_y^p, S_z^p] = iS_x^p$. For our coherent input, we have $\langle S_x^p \rangle = N_p$ and $\langle S_y^p \rangle = \langle S_z^p \rangle = 0$. With a very large N_p, the off-resonant interaction with atoms is only a small perturbation to S_x^p, and we can treat S_x^p
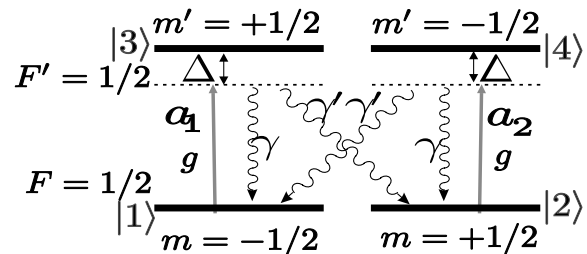

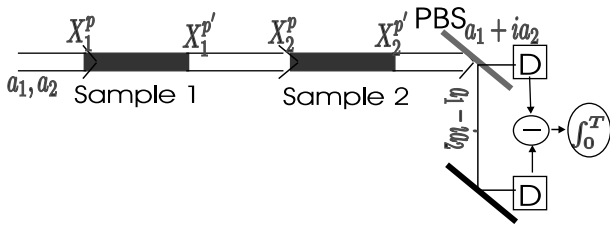FIG. 1.   Level structure of the atoms.

FIG. 2. Schematic setup for Bell measurements. A linearly polarized strong laser pulse (decomposed into two circular polarization modes a_1, a_2) propagates successively through the two atomic samples. The two polarization modes $(a_1 + ia_2)/\sqrt{2}$ and $(a_1 - ia_2)/\sqrt{2}$ are then split by a polarizing beam splitter (PBS), and finally the difference of the two photon currents (integrated over the pulse duration T) is measured.

classically by replacing it with its mean value $\langle S_x^p \rangle$. Then, we define two canonical observables for light by $X^p = S_y^p/\sqrt{\langle S_x^p \rangle}$, $P^p = S_z^p/\sqrt{\langle S_x^p \rangle}$ with a standard commutator $[X^p, P^p] = i$. These operators are the quantum variables we are interested in. Similar operators can be introduced for atoms. For an atomic ensemble with many atoms, it is convenient to define the continuous atomic operators $\sigma_{\mu\nu}(z,t) = \lim_{\delta z \to 0} \frac{1}{\rho A \delta z} \sum_i^{z \leq z_i < z + \delta z} |\mu\rangle_i \langle\nu|$ ($\mu, \nu = 1, 2, 3, 4$) with the commutation relations $[\sigma_{\mu\nu}(z,t), \sigma_{\nu'\mu'}(z',t)] = (1/\rho A)\delta(z - z')(\delta_{\nu\nu'}\sigma_{\mu\mu'} - \delta_{\mu\mu'}\sigma_{\nu'\nu})$. In the definition, z_i is the position of the i atom, and ρ is the number density of the atomic ensemble with the total atom number $2N_a = \rho A L \gg 1$. The collective spin operators are introduced for the ground states of the atomic ensemble by $S_x^a = \frac{\rho A}{2}\int_0^L (\sigma_{12} + \sigma_{12}^\dagger) dz$, $S_y^a = \frac{\rho A}{2i}\int_0^L (\sigma_{12} - \sigma_{12}^\dagger) dz$, $S_z^a = \frac{\rho A}{2}\int_0^L (\sigma_{11} - \sigma_{22}) dz$. All the atoms are initially prepared in the superposition of the two ground states $(|1\rangle + |2\rangle)/\sqrt{2}$ (this can be obtained with negligible noise by applying classical laser pulses with detuning $\Delta \gg \gamma$), which is an eigenstate of S_x^a with a very large eigenvalue N_a. As before, we treat S_x^a classically, and define the canonical operators for atoms by $X^a = S_y^a/\sqrt{\langle S_x^a \rangle}$, $P^a = S_z^a/\sqrt{\langle S_x^a \rangle}$ with $[X^a, P^a] = i$ and an initial vacuum state. As shown below, after the laser pulse passes through the atomic ensemble, the off-resonant interaction changes the canonical operators according to

$$X^{p\prime} = \sqrt{1 - \varepsilon_p}(X^p - \kappa P^a) + \sqrt{\varepsilon_p} X_s^p,$$
$$X^{a\prime} = \sqrt{1 - \varepsilon_a}(X^a - \kappa P^p) + \sqrt{\varepsilon_a} X_s^a, \qquad (1)$$
$$P^{\beta\prime} = \sqrt{1 - \varepsilon_\beta} P^\beta + \sqrt{\varepsilon_\beta} P_s^\beta, \qquad (\beta = a, p),$$

where the symbols with (without) a prime denote the operators after (before) the interaction, and X_s^a, P_s^a and X_s^p, P_s^p are the standard vacuum noise operators with variance $1/2$. The interaction and damping coefficients κ, ε_p, ε_a are given, respectively, by $\kappa = -2\sqrt{N_p N_a}|g|^2/\Delta c$, $\varepsilon_p = N_a|g|^2\gamma/\Delta^2 c$, $\varepsilon_a = N_p|g|^2\gamma'/\Delta^2 c$, where g is the coupling constant and γ, γ' are spontaneous emission rates

(see Fig. 1). Equation (1) is obtained under the conditions $\varepsilon_{p,a} \ll 1$ and $\kappa \ll \sqrt{N_{p,a}}$. For our applications, we would like to have $\kappa \gtrsim 1$. This is possible if we choose $N_p \sim N_a \gg 1$ and $\Delta \gg \gamma$. The number matching condition $N_p \sim N_a$ is an important requirement obtained here to minimize the noise effect, since we have $\kappa = 2\sqrt{\varepsilon_p \varepsilon_a} \Delta/\sqrt{\gamma\gamma'}$ and the best choice is $\varepsilon_p \sim \varepsilon_a$ to increase the signal-to-noise ratio.

Before we proceed to demonstrating Eq. (1), we show that this transformation allows us to generate entanglement, and to achieve quantum communication between atomic ensembles using only coherent light. Entanglement is generated through a nonlocal Bell measurement of the EPR operators $X_1^a - X_2^a$ and $P_1^a + P_2^a$ with the setup depicted by Fig. 2. This setup measures the Stokes operator $X_2^{p\prime}$ of the output light. Using Eq. (1) and neglecting the small loss terms, we have $X_2^{p\prime} = X_1^p + \kappa(P_1^a + P_2^a)$, so we get a collective measurement of $P_1^a + P_2^a$ with some inherent vacuum noise X_1^p. The efficiency $1 - \eta$ of this measurement is determined by the parameter κ with $\eta = 1/(1 + 2\kappa^2)$. After this round of measurements, we rotate the collective atomic spins around the x axis to get the transformations $X_1^a \to -P_1^a$, $P_1^a \to X_1^a$ and $X_2^a \to P_2^a$, $P_2^a \to -X_2^a$. The rotation of the atomic spin can easily be obtained with negligible noise by applying classical laser pulses with detuning $\Delta \gg \gamma$. After the rotation, the measured observable of the first round of measurement is changed to $X_1^a - X_2^a$ in the new variables. We then make another round of collective measurement of the new variable $P_1^a + P_2^a$. In this way, both the EPR operators $X_1^a - X_2^a$ and $P_1^a + P_2^a$ are measured, and the final state of the two atomic ensembles is collapsed into a two-mode squeezed state with variance $\delta(X_1^a - X_2^a)^2 = \delta(P_1^a + P_2^a)^2 = e^{-2r}$, where the squeezing parameter r is given by

$$r = \frac{1}{2}\ln(1 + 2\kappa^2). \qquad (2)$$

Thus, using only coherent light, we generate continuous variable entanglement [13] between two nonlocal atomic ensembles. With the interaction parameter $\kappa \approx 5$, a high squeezing (and thus a large entanglement) $r \approx 2.0$ is obtainable. Note that entanglement generation is the key step for many quantum protocols, and is the basis of quantum communication, quantum cryptography, and tests of Bell inequality. In the following, we show as an example how to achieve indirect quantum communication, i.e., quantum teleportation, between distant atomic ensembles using only coherent light.

We consider unconditional quantum teleportation of continuous variables [5,14,15] from one atomic ensemble to the other since we have continuous variable entanglement. To achieve quantum teleportation, first two distant atomic samples 1 and 2 are prepared in a continuously entangled state using the nonlocal Bell measurement described above. Then, a Bell measurement with the same

setup as shown by Fig. 2 on the two local samples 1 and 3, together with a straightforward displacement of X_2^a, P_2^a on the sample 2, will teleport an unknown collective spin state from the atomic sample 3 to 2. The teleported state on the sample 2 has the same form as that in the original proposal of continuous variable teleportation using squeezing light [15], with the squeezing parameter r replaced by Eq. (2) and with an inherent Bell detection inefficiency $\eta = 1/(1 + 2\kappa^2)$. The teleportation quality is best described by the fidelity, which, for a pure input state, is defined as the overlap of the teleported state and the input state. For any coherent input state of the sample 3, the teleportation fidelity is given by

$$F = 1 \left/ \left(1 + \frac{1}{1 + 2\kappa^2} + \frac{1}{2\kappa^2}\right)\right. . \qquad (3)$$

Equation (3) shows, if there is no extra noise, a high fidelity $F \approx 96\%$ would be possible for the teleportation of the collective atomic spin state with the interaction parameter $\kappa \approx 5$.

Next we will include noise and derive expressions for the squeezing and the fidelity under realistic experimental conditions. Before we analyze the effects of noise, let us first demonstrate Eq. (1) with a full quantum approach. The demonstration of Eq. (1) including the spontaneous emission noise is necessary in the following context: First, it is not clear that the spontaneous emission is indeed negligible

through a simple estimation of the noise, since during the interaction approximately $\frac{N_p N_a |g|^2 \gamma}{\Delta^2 c}$ atoms in the atomic ensemble (normally much larger than 1) will be subjected to quantum jumps caused by the spontaneous emission [14]. We need to show that quantum jumps of individual atoms have negligible influence on the collective spin operators which are the quantities of interest. Second, the maximally allowable interaction parameter κ is mainly limited by the noise. We need a balance between the desired interaction and the noise to maximize the squeezing and the teleportation fidelity. Third, some subtle experimental requirements, such as the number matching condition $N_p \sim N_a$, is obtainable only by considering the noise.

With introduction of the continuous atomic operators, the interaction between atoms and the propagating light $E^{(+)}(z, t)$ is described by the following Hamiltonian (in the rotating frame):

$$H = \hbar \sum_{i=1,2} \int_0^L \{\Delta \sigma_{i+2,i+2}(z, t)$$
$$+ [g e^{ik_0 z} a_i(z, t)\sigma_{i+2,i}(z, t)$$
$$+ \text{H.c.}]\}\rho A \, dz, \qquad (4)$$

where the coupling constant $g = \sqrt{\omega_0/4\pi\hbar\epsilon_0 A}\, d$ and d is the dipole moment of the $|i\rangle \rightarrow |i + 2\rangle$ transition. Corresponding to this Hamiltonian, the Maxwell-Bloch equations are written as [16]

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)a_i(z, t) = -ig^* e^{-ik_0 z}\rho A \sigma_{i, i+2}(z, t),$$

$$\frac{\partial}{\partial t}\sigma_{\mu\nu} = -\frac{i}{\hbar}[\sigma_{\mu\nu}, H] - \frac{\gamma_{\mu\nu}}{2}\sigma_{\mu\nu} + \sqrt{\gamma_{\mu\nu}}(\sigma_{\nu\nu} - \sigma_{\mu\mu})F_{\mu\nu}(\mu < \nu), \qquad (5)$$

where the spontaneous emission rates (see Fig. 1) are, respectively, $\gamma_{13} = \gamma_{24} \equiv \gamma = \frac{\omega_0^3 |d|^2}{3\pi\epsilon_0 \hbar c^3}$, $\gamma_{14} = \gamma_{23} \equiv \gamma'$, and $\gamma_{12} = 0$ (the ground state coherence time is much longer than the interaction time scale considered here under realistic experimental conditions [9,10]). The Doppler broadening caused by the atomic motion is negligible, since it is eliminated for off-resonant interactions with the collinear input and output light. Assuming that the spontaneous emission is independent for different atoms (because the distance between atoms is larger than optical wave length), the vacuum noise operators $F_{\mu\nu}$ satisfy the δ-commutation relations $[F_{\mu\nu}(z, t), F_{\mu'\nu'}^\dagger(z', t')] = (1/\rho A)\delta_{\mu\mu'}\delta_{\nu\nu'}\delta(z - z')\delta(t - t')$. To simplify Eq. (5), first we change the variables by $\tau = t - z/c$, and then adiabatically eliminate the excited states $|3\rangle$ and $|4\rangle$ of atoms in the case of a large detuning, i.e., $\Delta \gg g\langle a_i(z, t)\rangle \sim g\sqrt{N_p/(cT)}$. The resultant equations read

$$\frac{\partial}{\partial z}a_i(z, \tau) = \frac{i|g|^2\rho A\sigma_{ii}}{\Delta c}a_i(z, \tau) - \frac{|g|^2\rho A\gamma\sigma_{ii}}{2\Delta^2 c}a_i(z, \tau) + \frac{g^* e^{-ik_0 z}\rho A\sqrt{\gamma}\,\sigma_{ii}}{\Delta c}F_{i, i+2}(z, \tau),$$

$$\frac{\partial}{\partial \tau}\sigma_{12} = \frac{i|g|^2(a_2^\dagger a_2 - a_1^\dagger a_1)}{\Delta}\sigma_{12} - \frac{|g|^2\gamma'(a_2^\dagger a_2 + a_1^\dagger a_1)}{2\Delta^2}\sigma_{12} + \frac{\sqrt{\gamma'}}{\Delta}(g^* e^{-ik_0 z}a_2^\dagger\sigma_{11}F_{14} + g e^{ik_0 z}a_1\sigma_{22}F_{23}^\dagger). \qquad (6)$$

The physical meaning of the above equation is quite clear: The first term on the right-hand side is the phase shift caused by the off-resonant interaction between light and atoms, and the second and the third terms represent the damping and the corresponding vacuum noise caused by the spontaneous emission, respectively. In Eq. (3), the σ_{ii} and $a_i^\dagger a_i$ are approximately constant

operators, only with a small damping caused by the spontaneous emission. To consider the spontaneous emission noise to the first order, it is reasonable to assume constant σ_{ii} and $a_i^\dagger a_i$ for Eq. (6). Then, this equation can easily be solved by integrating over z, τ on both sides. In this way we obtain Eq. (1) with the introduced

canonical operators. The vacuum noise operators in Eq. (1) are defined from the integration of $F_{\mu\nu}(z,\tau)$, $X_s^p = \sqrt{c/4N_pN_a|g|^2} \int_0^T \int_0^L \rho A[ig^*e^{-ik_0z}(a_2^\dagger\sigma_{11}F_{13} - a_1^\dagger\sigma_{22}F_{24}) + \text{H.c.}]\,dz\,d\tau$, for instance. It should be noted that the damping term cannot be directly neglected in Eq. (6) compared with the phase shift term, even when $\Delta \gg \gamma$, since $\langle a_2^\dagger a_2 + a_1^\dagger a_1\rangle \gg \langle a_2^\dagger a_2 - a_1^\dagger a_1\rangle$. What is remarkable is that due to the collective effect, the phase shift term obtains another large prefactor $\sqrt{N_{p,a}}$ when we perform the integration in Eq. (6), which makes this contribution well exceed the noise term. It is helpful to give an estimation of the relevant parameters for typical experiments. The interaction parameter κ can be rewritten as $\kappa = (3\rho\lambda_0^2 L\gamma)/(8\pi^2\Delta)$ with $N_p = N_a$. For an atomic sample of density $\rho \sim 5 \times 10^{12}$ cm^{-3} and of length $L \sim 2$ cm, $\kappa \sim 5$ is obtainable with the choice $\Delta \sim 300\gamma$, and at the same time the loss $\varepsilon_p \sim \varepsilon_a < 1\%$.

As our last point, let us return to the analysis of the influence of some important noise terms on the teleportation fidelity. The noise includes the spontaneous emission noise described by Eq. (1), the detector inefficiency, and the transmission loss of the light from the first sample to the second sample. The spontaneous emission noise can be included partly in the transmission loss and partly in the detector efficiency, so we do not analyze it separately. The effect of the detector inefficiency η_d is to replace κ^2 in Eqs. (2) and (3) with $\kappa^2(1 - \eta_d)$, and the teleportation fidelity is decreased by a term η_d/κ^2, which is very small and can be safely ignored. The most important noise comes from the transmission loss. The transmission loss is described by $X_2^p = \sqrt{1 - \eta_t}\,X_1^{p'} + \sqrt{\eta_t}\,X_s^t$ (see Fig. 2), where η_t is the loss rate and X_s^t is the standard vacuum noise. The transmission loss changes the measured observables to be $\sqrt{1 - \eta_t}\,X_1^a - X_2^a$ and $\sqrt{1 - \eta_t}\,P_1^a + P_2^a$. These two observables do not commute, and the two rounds of measurements influence each other. To minimize the influence on the teleportation fidelity, we choose the following configuration (for simplicity, we assume we have the same loss rate η_t from the sample 1 to 2 and from 1 to 3): In the nonlocal Bell measurements on the samples 1 and 2 (the entanglement generation process), we choose a suitable interaction coefficient κ_2 (where its optimal value will be determined below) for the second round measurement, whereas κ_1 for the first round of measurement is large with $\kappa_1^2 \gg \kappa_2^2$ (the interaction coefficient can be easily adjusted, for instance, by changing the detuning). In the local Bell measurement, we choose the same κ_2 for the first round of measurement and the large κ_1 for the second round of measurement. For a coherent input state of the sample 3, the teleported state on the sample 2 is still Gaussian, and the teleportation fidelity F' is found to be

$$F' \approx 2 \left/ \left(2 + \frac{1}{\kappa_2^2} + \kappa_2^2\eta_t\right)\right. \leq 1/(1 + \sqrt{\eta_t}), \quad (7)$$

which is still independent of the coherent input state with suitable gain for the displacements [5,15]. The optimal value for κ_2 is thus given by $\kappa_2 = 1/\sqrt[4]{\eta_t}$. Even with a notable transmission loss rate $\eta_t \sim 0.2$, quantum teleportation with a remarkable high fidelity $F \sim 0.7$ is still achievable. It is known that for coherent inputs a fidelity exceeding $1/2$ has ensured quantum teleportation [17].

In summary, we have shown that quantum communication between free-space atomic ensembles can be achieved using only coherent laser beams. Quantum teleportation of the atomic spin state is observable even in the presence of significant noise. This result, together with the much simplified experimental setup proposed here, suggests that efficient quantum communication between atomic samples is within reach of present experimental conditions.

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