High-Accuracy Measurement of the Magnetic Moment Anomaly of the Electron Bound in Hydrogenlike Carbon

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We present a new experimental value for the magnetic moment of the electron bound in hydrogenlike carbon ($^{12}C^{5+}$): $g_{\mathrm{exp}} = 2.001\,041\,596\,(5)$. This is the most precise determination of an atomic g_J factor so far. The experiment was carried out on a single $^{12}C^{5+}$ ion stored in a Penning trap. The high accuracy was made possible by spatially separating the induction of spin flips and the analysis of the spin direction. The current theoretical value amounts to $g_{\mathrm{th}} = 2.001\,041\,591\,(7)$. Together experiment and theory test the bound-state QED contributions to the g_J factor of a bound electron to a precision of 1%.

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Highly charged ions provide a unique testing ground for quantum electrodynamics (QED) in very strong electric and magnetic fields which is not accessible experimentally otherwise [1]. In recent years, very precise measurements and calculations have been performed on the Lamb shift in hydrogenlike and lithiumlike heavy systems and also on the hyperfine structure splitting ([2] and references therein). Another quantity suitable for investigating QED in strong fields is the Zeeman level splitting of a hydrogenlike atom under the influence of an external magnetic field. The energy shift due to this external field is given by $E_{\mathrm{mag}} = \langle -\boldsymbol{\mu} \cdot \boldsymbol{B} \rangle$, where the magnetic moment $\boldsymbol{\mu}$ of a particle with charge q and mass m is related to its angular momentum $\boldsymbol{J}$ by the g_J factor, $\boldsymbol{\mu} = g_J(q/2mc)\boldsymbol{J}$. The g factor of the spin of the free electron was measured to a relative precision of 4×10^{-12} by Van Dyck *et al.* [3]. The value obtained from QED calculations [4] is in very good agreement. Thus investigations on the magnetic moment represent one of the most stringent tests of QED of a free particle. For an electron bound in a hydrogenlike system only the total angular momentum $\boldsymbol{J}$ is an observable [5]. Additional QED corrections due to binding have to be considered [6]. A precise measurement of the g_J factor of the electron in atomic hydrogen [7] probed the leading term of these bound-state QED contributions. An experiment on hydrogenlike helium ($^4He^+$) [8] was sensitive only to the binding modification which was derived by Breit [5] from the Dirac theory.

We have developed an experimental setup which was employed to investigate the g_J factor of $^{12}C^{5+}$ [9,10]. A single ion is stored in the strong magnetic field ($\approx$4 T) of a Penning trap [11], where the confinement in the radial direction is performed by the magnetic field and the axial trapping is achieved by an electric field. The eigenmotion of an ion in a Penning trap is characterized by three oscillation frequencies, ω_+, ω_z, and ω_-, where ω_+ is the trap-modified cyclotron frequency, ω_z is the axial oscillation frequency, and ω_- is the magnetron frequency. The free-space cyclotron frequency $\omega_c = (q/m)B$ is related to these frequencies by

$$\omega_c^2 = \omega_+^2 + \omega_z^2 + \omega_-^2 . \qquad (1)$$

The g factor of an electron can be expressed as the ratio of the Larmor precession frequency $\omega_L = g_J(e/2m_e)B$ and the cyclotron frequency of the electron, ω_c^e,

$$g_J = 2\,\frac{\omega_L}{\omega_c^e} = 2\,\frac{\omega_L}{\omega_c^i}\,\frac{\omega_c^i}{\omega_c^e}\,, \qquad (2)$$

where the experimentally accessible cyclotron frequency of the ion, ω_c^i, was introduced. The ratio $\omega_c^i(^{12}C^{5+})/\omega_c^e$ can be deduced from the measurement of the electron mass by Van Dyck *et al.* [12,13], $\omega_c^i(^{12}C^{5+})/\omega_c^e = 0.000\,228\,627\,210\,33\,(50)$. Therefore the g factor can be determined from measurements of the Larmor frequency ω_L of the electron and of the cyclotron frequency ω_c^i of the ion.

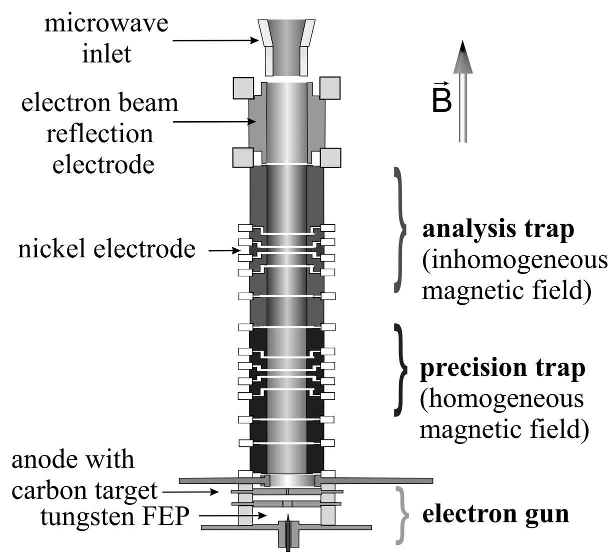

FIG. 1. Sketch of the experimental setup.

In our setup the electric trapping potential is created by a stack of 13 cylindrical electrodes of 0.7 cm inner diameter (Fig. 1). By a proper choice of the electrode voltages, two harmonic electrostatic potential minima can be formed which are spatially separated by a distance of 2 cm. These two potential minima are denoted precision trap and analysis trap, respectively.

The Larmor frequency ω_L is obtained by measuring the spin-flip rate of the electron as a function of the frequency ω_{mw} of a driving microwave field. The spin-flip transitions are observed in the same way as in the electron $g-2$ experiment of Dehmelt *et al.* [3]: An inhomogeneity in the magnetic field produced by a nickel ring electrode in the analysis trap (see Fig. 1) causes a leading quadratic dependence of the magnetic energy E_{mag} on the z coordinate: $E_{\mathrm{mag}} = -\boldsymbol{\mu} \cdot \boldsymbol{B} = -\mu_z(B_0 + B_2 z^2 + \ldots)$. The axial oscillation frequency is given by the sum of the quadratic electric potential and the magnetic potential. Thus it depends on the projection μ_z of the magnetic moment on the z axis, i.e., the spin direction (*continuous Stern-Gerlach effect*) [14]. The axial frequency of $^{12}C^{5+}$ differs by 0.7 Hz for the two spin directions at a total value of 364 kHz (Fig. 2). It is determined by a Fourier transform of the image current induced by the ion's motion at an axial energy of about 5 meV.

Our previous g_J-factor measurement [10] was limited to an accuracy of 10^{-6} by the strong magnetic inhomogeneity ($B_2 = 10$ mT/mm^2) required to obtain a measurable frequency shift between the two spin states. In order to overcome this limitation, we now spatially separate the functions of inducing and detecting the spin flips by a *double-trap* technique: The spin flips are induced in the precision trap and detected in the analysis trap. In the precision trap, the inhomogeneity of the magnetic field is 1000 times smaller than in the analysis trap.

The measurement procedure comprises the following steps: First the direction of the electron spin is determined by stimulating spin flips in the analysis trap (Fig. 3). Then the ion is transferred to the precision trap by adiabatically moving the electric potential minimum. Spin flips are induced by a microwave field at $\omega_{\mathrm{mw}} = 2\pi \times 105$ GHz. Finally, the ion is transported back to the analysis trap, where the direction of the spin is determined again, thus allowing a determination of whether a spin flip has happened in the precision trap or not. It was found that the ion was never lost in the transfer from the analysis trap to the precision trap and vice versa. While the ion is in the precision trap the modified cyclotron frequency ω_+ is determined by a Fourier transform of the image current at a cyclotron energy of about 3 eV. The influence of temporal fluctuations of the magnetic field is strongly reduced by measuring ω_+ while simultaneously applying the microwave field at ω_{mw} to induce spin-flip transitions. In the precision trap the frequencies are $\omega_+ = 2\pi \times 24$ MHz, $\omega_z = 2\pi \times 930$ kHz, and $\omega_- = 2\pi \times 18$ kHz. The magnetron frequency ω_- is measured by coupling the magnetron motion to the axial motion by an rf excitation at $\omega_z - \omega_-$.

A *g-factor resonance* is obtained by plotting the spin-flip probability versus the ratio of the microwave and the cyclotron frequency $2\omega_{\mathrm{mw}}/\omega_c^e$ (Fig. 4). The full width of the resonance curve is 14×10^{-9} and the center can be determined within 1% of the linewidth using a Gaussian least squares fit. Several systematic uncertainties have to be considered (Table I). The largest contribution arises from the limited knowledge of the atomic mass of the electron. For finite ion oscillation energies, the resonances are broadened and shifted due to the residual inhomogeneity of the magnetic field in the precision trap. To investigate the shifts, we varied the cyclotron energy as well as the axial energy and extrapolated the measured g factor to zero energy. The influence of the magnetron energy is negligible. A variation of the cyclotron energy E_+ yields $\Delta g/\Delta E_+ = (-1.09 \pm 0.05) \times 10^{-9}$ eV^{-1} (Fig. 5). The result coincides with the slope derived from the frequency ratio ω_z/ω_+, $(\Delta g/\Delta E_+)' = (-1.14 \pm 0.10) \times 10^{-9}$ eV^{-1}.

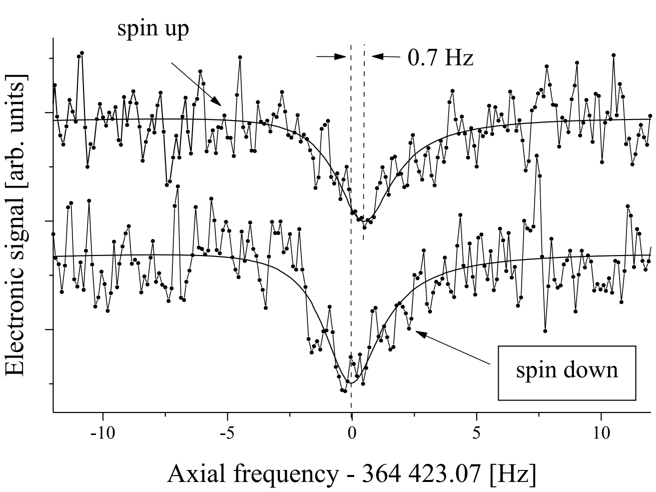

FIG. 2. Determination of the spin orientation by measuring the axial frequency ω_z in the analysis trap. The averaging time for each spectrum was 60 s.

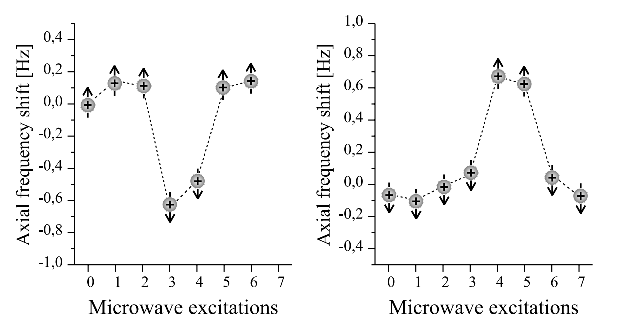

FIG. 3. Determination of the spin orientation in the analysis trap by subsequent microwave irradiations. Left: up → down → up transition; right: down → up → down transition. The initial orientation can be concluded from the sign of the frequency shift.

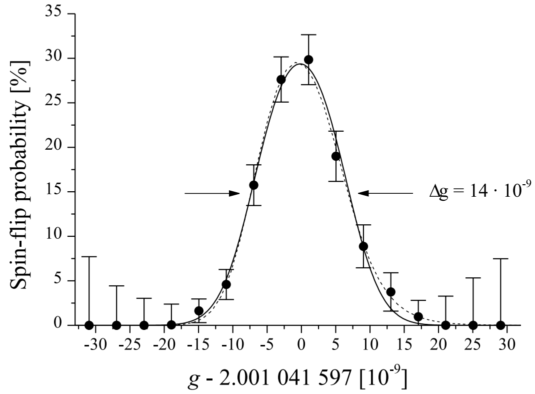

FIG. 4. g-factor resonance measured in the precision trap. Plotted is the ratio $g = 2\omega_{\mathrm{mw}}/\omega_c^e$, corrected for the cyclotron energy E_+ according to Fig. 5 by $1.09 \times 10^{-9} E_+ \ \mathrm{eV}^{-1}$. The solid line is a fit of a Gaussian. The dashed line is a fit of a convolution of a Gaussian and a Boltzmannian distribution. Both models take saturation effects into account. The error margins are deduced by assuming a binomial distribution of the spin-flip probability.

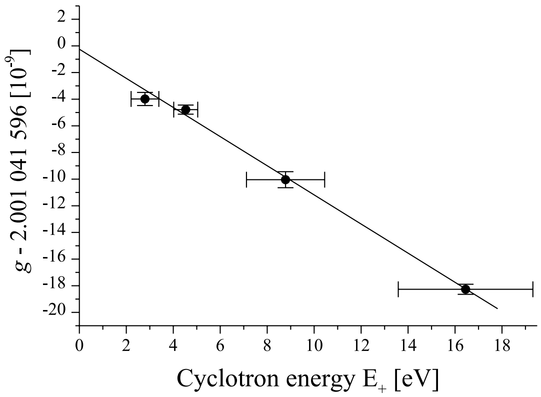

FIG. 5. Extrapolation of the g factor to zero cyclotron energy.

For a detailed discussion we refer to a forthcoming publication.

The variation of the axial energy between 60 and 350 K had no influence on the measured g factor, because the expectation values of both the cyclotron and Larmor frequency are determined by the average magnetic field. This motional averaging was investigated by Brown [15] with regard to the $g - 2$ experiments on the free electron. According to his model the line shapes of the Larmor and the cyclotron resonance differ substantially for our experimental parameters because the Larmor frequency is 3 orders of magnitude higher than the cyclotron frequency. We adapted Brown's model to our g-factor resonance and in addition took into account fluctuations of the measured cyclotron frequency in the residual inhomogeneity of the magnetic field. The asymmetry of the resulting line shape is governed by the ratio of the inhomogeneity and the fluctuations of the magnetic field. A fit of the line shape is shown in Fig. 4 (dashed line). Employing a maximum asymmetry of the resonance consistent with the data gives a maximum relative deviation of 2×10^{-10} compared to the g factor extracted with a Gaussian fit (solid line). We quote this as the resulting uncertainty due to the inhomogeneity of the magnetic field in the precision trap.

The free-space cyclotron frequency, Eq. (1), is shifted due to image charges even for single ions [16]. We calculated the relative shift of the cyclotron frequency by solving the boundary problem for our cylindrical trap. The relative shift amounts to 3×10^{-10} for a single ion. A summary of all applied corrections to the g factor is compiled in Table II.

Our experimental result for the g_J factor in $^{12}C^{5+}$ is

$$g_J(^{12}\mathrm{C}^{5+}) = 2.001\,041\,596\,4\,(8)\,(6)\,(44)\,. \qquad (3)$$

The first number in parentheses indicates the statistical uncertainty (extracted from Fig. 5), the second number is an estimate of possible systematic shifts, and the last number is the contribution of the electron's atomic mass, which dominates our error budget. Hence, the final result is $g_J(^{12}\mathrm{C}^{5+}) = 2.001\,041\,596\,(5)$.

Our current experimental value has to be compared with the theoretical prediction of $g_J = 2.001\,041\,590\,7\,(71)$ [17]. The contributions to $g_J(^{12}\mathrm{C}^{5+})$ are those to the free electron's g factor, i.e., the Dirac value of 2 plus the quantum electrodynamical corrections for the free electron (for an overview, cf. [4]) and the modifications due to binding. For an electron bound in the $1S_{1/2}$ state of a hydrogenlike ion, the Dirac value of $g_S = 2$ is modified, $g_J = (2/3)[1 + 2\sqrt{1 - (Z\alpha)^2}]$. Bound-state QED corrections of order (α/π) were evaluated by Grotch

TABLE I. Systematic errors of the g_J determination which are considered. All uncertainties are given in relative units.

Asymmetry of resonance	2×10^{-10}
Measurement of cyclotron energy	2×10^{-11}
Electric field imperfections	1×10^{-10}
Magnetron energy	1×10^{-11}
Relativistic corrections	1×10^{-12}
Shift by standing microwave field	$<10^{-14}$
Stability of quartz oscillators	1×10^{-10}
Grounding of apparatus	4×10^{-11}
Interaction with image charges	3×10^{-11}
Saturation of spin-flip transition	5×10^{-12}
Spectral purity of microwaves	5×10^{-13}
Cavity QED shifts	$\approx 10^{-13}$
Damping of ion motion	$\approx 10^{-20}$
Total (quadrature sum)	3×10^{-10}

TABLE II. Corrections which are included in the final evaluation of the experimental g_J factor.

Experimental value	2.001 041 596 95
Interaction with image charges	−0.000 000 000 58
Shift due to grounding	−0.000 000 000 15
Cyclotron energy measurement	+0.000 000 000 15
Final experimental value	2.001 041 596 37

TABLE III. Theoretical contributions to $g_J(^{12}C^{5+})$, taken from [17].

Dirac theory (incl. binding)	1.998 721 354 2
Finite-size correction	+0.000 000 000 4
Recoil	+0.000 000 087 5 (9)
QED, free, up to order $(\alpha/\pi)^4$	+0.002 319 304 4
QED, bound, order (α/π)	+0.000 000 844 2 (12)
QED, bound, order $(\alpha/\pi)^2$, estimate	±0.000 000 002 0 (50)
Total theoretical value:	2.001 041 590 7 (71)

[6] to the lowest orders in $(Z\alpha)$ to be $(\alpha/\pi)(Z\alpha)^2/6$. Persson *et al.* [18] have presented values for all orders in $(Z\alpha)$ which were obtained nonperturbatively employing strong-field QED calculation methods. A more detailed description of this approach was recently presented by Beier *et al.* [17] where the total QED contribution of order (α/π) to g_J was calculated. By subtracting the corresponding value for the g factor of the free electron, (α/π), the effect of binding on the QED contributions of order (α/π) can be isolated. It amounts to 8.442×10^{-7} which has to be compared with Grotch's result from the $(Z\alpha)$ expansion, 7.422×10^{-7}. Our experiment is clearly sensitive to this difference. Details on nuclear corrections are given in Ref. [17]. All theoretical contributions are listed in Table III, where the errors were linearly added in order not to underestimate any systematic numerical effect.

In conclusion, we have performed a high-accuracy test of QED in hydrogenlike carbon. It was achieved by a new double-trap technique, where the induction and detection of spin flips are spatially separated. Our accurate experimental result for $g_J(^{12}C^{5+})$ is in very good agreement with the theoretical calculations which take into account all orders in $(Z\alpha)$ for the bound-state QED contributions. The first term of a $(Z\alpha)$ expansion is not sufficient to describe our value even for nuclear charge numbers as low as $Z = 6$. We consider our present result as a first step towards higher Z where electronic g_J factors are up to now only accessible via lifetime measurements in hyperfine transitions with precisions of about 10^{-3} [19]. The current experiment clearly proves its feasibility to test QED in strong fields with high accuracy.

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