Double-Octupole States in ^{208}Pb

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The states of ^{208}Pb are calculated in a 24 orbit model space which includes all excitations up to two particles and two holes beyond the closed shell. All of the known low-lying states are reproduced by this model. The observed distributions of electromagnetic excitation strength from the ground state to low-lying excited states are well described by the model. The double-octupole excitation strength is calculated and found to be concentrated in 0^+, 2^+, and 4^+ states around 5.2 MeV, but it is fragmented over many 6^+ states, in agreement with recent experiments.

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The nucleus ^{208}Pb and the surrounding nuclei present a showcase for the nuclear shell model. All of the single-particle states in the major shells below and above ^{208}Pb are well known from experiment (for a total of 24 orbitals). The nucleus ^{208}Pb itself manifests the whole variety of shell-model phenomena including a low-lying collective octupole state [1], many states up to 7 MeV in excitation which have rather pure particle-hole configurations [2,3], two-particle two-hole pairing vibrations [4], a fragmented magnetic dipole ($M1$) giant resonance [5], and high-lying giant resonances.

In this Letter, I present the first full shell-model calculation which includes all of the orbitals in the four major shells surrounding ^{208}Pb and going up to a full two-particle two-hole ($2p - 2h$) basis. This basis incorporates all of the phenomena mentioned above, except for the high-lying resonances. The calculations involve a basis 368 $1p - 1h$ state and 375 466 $2p - 2h$ states and were carried out with the code OXBASH [6]. The $2p - 2h$ states have been previously treated in terms of a collective response function for specific operators [7]; but in this paper the full wave functions for individual states are obtained, and these can be used to calculate any measured observable. The closest previous work is that of Poppelier and Glaudemans [8] which considered only 14 orbits. Those orbits are not complete with respect to the space needed to describe the spurious (isoscalar dipole) states. The present 24 orbit space is large enough so that the spurious states can be removed by using the Gloeckner-Lawson method [9], in which a center-of-mass Hamiltonian is added to push the spurious states to a high excitation energy.

There are many interesting new results from this calculation, and in this Letter I concentrate on the properties of the double-octupole excitation which has been the focus of several recent experimental searches [10–12]. The low-lying nature of the collective 3^- state in ^{208}Pb as well as the rather extensively studied spectrum of excited states make this a unique nucleus for locating the double-octupole states. But to date the experimental searches have not yielded definitive results. Reasons for this lack of detection will be explored in the present calculation. Previous calculations for the double-octupole states have been carried out with the two-phonon model [13] and a schematic model [14]. This calculation is more ambitious than either of these.

For the Hamiltonian, I start with the M3Y interaction [15] which is based on a one-boson-exchange potential that includes the one-pion-exchange potential fixed to its standard form and strength. The strengths of the short-ranged potential components are determined by reproducing the harmonic-oscillator matrix elements of a G matrix obtained with the Reid NN potential. The two-body matrix elements (TBME) for the M3Y potential were calculated with harmonic-oscillator radial wave functions ($\hbar\omega =$ 6.88 MeV) for all possible combinations of orbitals involving protons in the major shells $A = (1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2})$ and $B = (1h_{9/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{13/2})$, and neutrons in the major shells $C = (1h_{9/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{13/2})$ and $D = (1i_{11/2}, 2g_{9/2}, 2g_{7/2}, 3d_{5/2}, 3d_{3/2}, 4s_{1/2}, 1j_{15/2})$. There are about 35 000 TBME. In addition, the TBME for the Coulomb potential were calculated for the two proton major shells A and B. The particle-particle Hamiltonian involving the B and D major shells has been extensively investigated with the Kuo-Herrling renormalized G matrix [16], and the 4586 TBME labeled KHPE in Ref. [16] were used in place of the M3Y TBME. Also, the hole-hole Hamiltonian involving the A and C major shells has been investigated using the Kuo-Herrling renormalized G matrix, and the 2101 hole-hole TBME were replaced by those discussed in Ref. [17] which is labeled KHHE in OXBASH [6]. These replacements ensure that the energies associated with the $2p$ states in the $A = 210$ nuclei and the $2h$ states in the $A = 206$ nuclei are reproduced, but they do not affect the $1p - 1h$ states.

The calculation is started with a closed-shell ($0p - 0h$) configuration for ^{208}Pb, where the orbits in the A and C major shells are filled and those in the B and D major shells are empty. The 24 states involving the single-particle and the single-hole state in the surrounding nuclei (^{207}Pb, ^{209}Pb, ^{207}Tl, and ^{209}Bi) were then calculated, and were used to set the single-particle energies of the Hamiltonian to experimental values given in Fig. 1 of Ref. [2]. In the next step, the 368 $1p - 1h$ states in ^{208}Pb were calculated. This gave a low-lying collective 3^- state as well as many rather pure $1p - 1h$ states whose energies agree with those discussed in Ref. [2] to within an rms deviation of about 100 keV, and whose dominant components also agree with the results of the reaction data discussed in Ref. [2]. In addition, the energy of the collective $1p - 1h$ 1^+ state obtained with the M3Y interaction is close to the centroid of the observed M1 strength [5].

The final step was to calculate the $2p - 2h$ states on top of and mixing with the $1p - 1h$ states. This was done by starting with the same Hamiltonian described above, except that the single-particle energies were readjusted to reproduce the energies given in Fig. 1 of Ref. [2] when the neighboring odd-even nuclei are calculated in a model space which includes an additional $1p - 1h$ excitation. For example, the model space for ^{207}Pb was $1h$ plus $1p - 2h$. This "dressing" of the single-particle states gives rise to the dynamical effective mass with respect to the underlying single-particle spectrum which has been considered previously in terms of particle-vibration coupling models [18]. My results correspond to a dynamical effective mass of $m_\omega/m = 1.25$, but in general there will be other contributions from the high-lying resonances not included in the present model space.

The $2p - 2h$ admixture in the ground state lowers its energy by 11 MeV and results in a wave function which is only a 66% closed shell. This describes the breaking of the closed shell in ^{208}Pb which is similar in size to that of nuclei such as ^{16}O [19] and ^{56}Ni [20]. But in order to calculate the correct energies for excited states, one would need to include $3p - 3h$ and $4p - 4h$ configurations, which would have the effect of pushing down the states which are predominantly $2p - 2h$. This is beyond our computational reach. Thus, all excitation energies discussed here are those obtained relative to the energy of the closed-shell $(0p - 0h)$ ground state (model A). Transition rates from the ground state are obtained with the pure closed-shell $0p - 0h$ configuration (model A), and with a correlated $0p - 0h + 2p - 2h$ configuration which will be discussed below (model B).

Model A gives a good account of the energies of the states which are predominantly $1p - 1h$ in ^{208}Pb. These are the "dressed" $1p - 1h$ states and they automatically include a downward shift in the energies due to mixing with $2p - 2h$ states. However, the states which are predominantly $2p - 2h$ appear too high in the energy by about 2.1 MeV. The reason for this is that the higher $3p - 3h$ and $4p - 4h$ states, which would give a downward shift to the energies of the $2p - 2h$ states, are not present in the configuration space. A practical (and approximate) way to account for this truncation is to shift the energy of all $2p - 2h$ states downward by 2.1 MeV in order to bring the energy of the monopole pairing vibration state close to its well-known experimental value of 4.86 MeV [4]. For the excited states, this is the only parameter in the present Hamiltonian, but its physical origin is clear.

The lowest ten levels of each spin and each parity are shown in Fig. 1. The experimental data are shown by crosses; states which are predominantly $1p - 1h$ from Refs. [2] and [3], 3^- and 4^+ from Ref. [21], 2^+ from Refs. [22] and [23], and 6^+ from Ref. [12]. The experimental and theoretical energies agree in detail with an rms deviation of about 100 keV. The largest discrepancy is found for the lowest 2^+, 4^+, and 6^+ states, where the theoretical energies are higher than experiment. The level density is well reproduced (the density of 3^- and 4^+ levels observed in Ref. [21] is a little higher than predicted, but the spin assignments should be confirmed), and there are many theoretical levels left over (especially for 2^-, 3^+, 4^-, and 5^+) which may eventually be matched with experimental levels with uncertain spin assignments [22,24]. The solid circles on the theoretical lines in Fig. 1 indicate the lowest state which has more than 50% $2p - 2h$ configuration. States below the solid circles are predominantly $1p - 1h$ and have wave functions which agree with the configuration assignments found in one-nucleon transfer

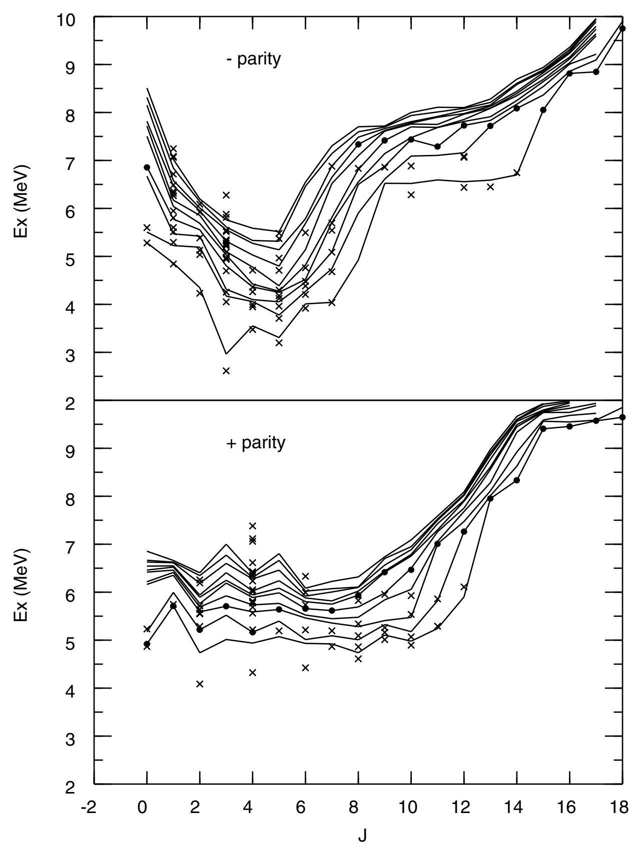

FIG. 1. Calculated energy levels for the lowest ten states of each spin with the nth one of each connected by lines. The experimental energies are shown by crosses. The filled circles indicate the first level which is predominantly $2p - 2h$ in structure.

reactions [2]. The jump after 12^+ and 14^- reflects the fact that these are the highest spin states which can be made in the $1p - 1h$ model space.

The lowest state which is mostly $2p - 2h$ is dominated by the 0^+ neutron monopole pairing vibration observed in a ^{210}Pb(p,t) experiment [4]. The second excited 0^+ state has some mixing with the neutron pairing vibration, in agreement with its weaker observation in the (p,t) experiment. But its main structure is the double octupole. The third 0^+ state about 1 MeV higher is dominated by the proton monopole pairing vibration. These results are similar to those obtained in the schematic model of Ref. [14]. Below 6 MeV the 2^+ states are characterized by two which are mainly $1p - 1h$ (at 4.74 and 5.94 MeV), one which is mainly double octupole at 5.22 MeV, and then, starting at 5.59 MeV, many which have a predominant neutron $2p - 2h$ component. The 2^+ states which are assigned $2p - 2h$ in character in experiment start at 5.55 MeV [4].

In order to observe the double-octupole features, I calculated the $B(E3)$ from the 3^- to the states with $J = 0^+$ to 6^+. The $B(E3)$ were calculated with the Hartree-Fock radial wave functions obtained with the SKX Skyrme interaction [25] and with effective charges of $e_p = 1.6$ and $e_n = 0.6$ which are determined to match the experimental $B(E3, 3^- \to 0^+, gs)$ value. The origin of the effective charge is in the perturbative mixing with the "$3\hbar\omega$" component of the octupole strength which is missing from the model space. The ratio $B(E3, J^+ \to 3^-)/B(E3, 3^- \to 0^+, gs)$ summed over the lowest ten states of each J^+ are 2.03, 3.8×10^{-4}, 2.18, 14×10^{-4}, 2.24, 23×10^{-4}, and 2.13 for $J = 0^+$ to 6^+, respectively. These are very close to the ratio of 2 for even states and 0 for odd states expected in the boson model [26]. The $B(E3, 3^- \to J^+)$ distributions (for the even J) are shown in Fig. 2. The single-octupole strength is concentrated in the lowest 3^- state, but there is some spreading into a "low-energy octupole" region as observed in α and proton scattering [21]. The double-octupole strength is rather well concentrated in a single state for each J near 5.2 MeV, except for 6^+ where it is fragmented over states which have a predominantly $1p - 1h$ structure. About one-third of the total 6^+ double-octupole strength is in the lowest 6^+ state, and this may be consistent with the recent double Coulomb excitation experiment to $J = 6^+$ states [12]. The interpretation of the experimental data in Ref. [12] is complicated because of the assumptions which are made concerning the decay of the 6^+ states populated. In the future these wave functions can be used to construct a detailed decay scheme which might help in the experimental interpretation. The present results suggest that there are missed 6^+ states (beyond the five discussed by Ref. [12]).

The present results are similar to those of the quasiparticle-phonon model [13], but they differ in detail. The present basis also includes all possible configurations involving $2p$ states (in the $A = 210$ nuclei) coupled to the $2h$ states (in the $A = 206$ nuclei), the lowest of which are

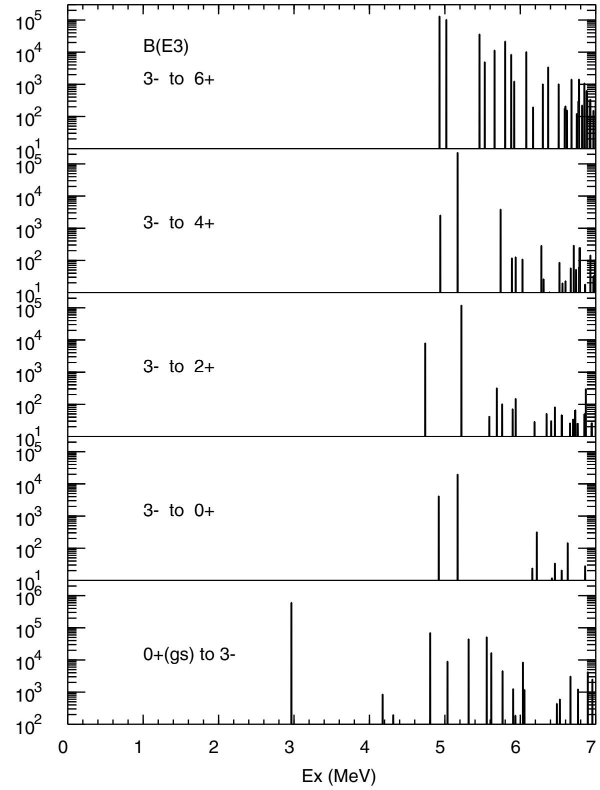

FIG. 2. The $B(E3,\uparrow)$ from the ^{208}Pb ground state to the 3^- states and from the lowest 3^- to the 2^+, 4^+, and 6^+ states. The units e^2 fm^6.

the well-known monopole and quadrupole pairing vibrations [4]. It has been suggested that the double-octupole nature of the 2^+ and 4^+ states might show up as a relatively strong $E1$ transition to the 3^- state [10]. I have calculated the $B(E1)$ between the single- and double-octupole states and find that they are not enhanced but have a hindrance which is typical of those found for transitions between noncollective states (between 10^{-3} and 10^{-5} e^2 fm^2).

Finally, I examine the $E2$ strength from the ^{208}Pb ground state shown in Fig. 3. Model A corresponds to the results with a closed shell for ^{208}Pb and model B includes the mixing of $2p - 2h$ into the ^{208}Pb ground state. The $B(E2)$ were calculated with the SKX HF radial wave functions and with an isoscalar effective charge of $e_p = 2.2$ and $e_n = 1.2$ which is needed to reproduce the $B(E2)$ value to the lowest 2^+ state measured in Ref. [23]. This large effective charge is consistent with the average values of $e_p = 2$ and $e_n = 1$ needed for single-particle and single-hole states around ^{208}Pb [27]. These effective charges have been been reproduced in calculations [27] which take into account core excitations of which the most important is the $E2$ giant resonance near 10 MeV, which lies outside the present model space (see Fig. 4.24 in Ref. [18]). The experimental values in Fig. 3 are from Refs. [22,23]. In addition to the strong transition to the lowest 2^+ state, there is some strength around 6.2 MeV with a total of 30% of that of the lowest state in both experiment and theory.

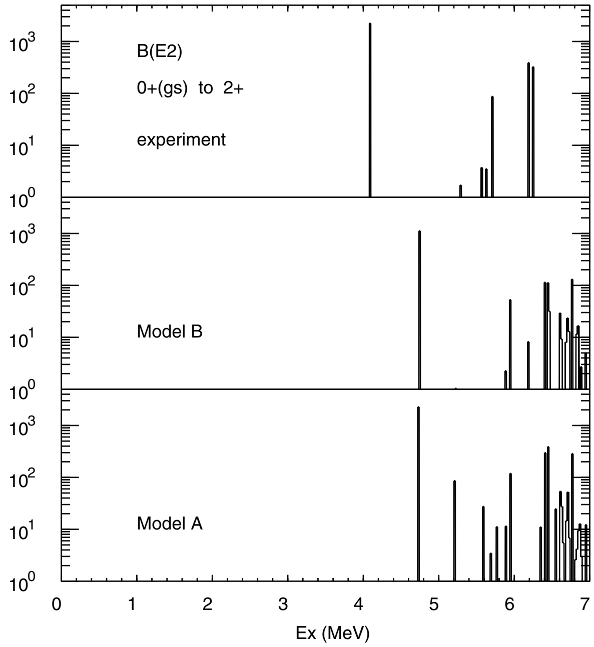

FIG. 3. $B(E2,\uparrow)$ from the ^{208}Pb ground state. The units are $e^2\,\mathrm{fm}^4$. The experimental data is from Refs. [22,23].

To obtain a correlated ground state for model B, the $2p - 2h$ energies are moved down until the first excited 0^+ state comes at about 4.86 MeV [4]. The resulting ^{208}Pb ground state is lowered in energy by 20 MeV and has a 32% closed-shell component. (The large ground-state energy shift means that the $2p - 2h$ energies need to be lowered by about 20 MeV in order to account for the missing push of the $4p - 4h$ components.) The strength to the double-octupole state near 5.2 MeV is very sensitive to the $2p - 2h$ ground-state correlations. The result with correlations (B) gives a nearly vanishing $B(E2)$ value of 1.0 $e^2\,\mathrm{fm}^4$ (below scale in Fig. 3) which is in qualitative agreement with the small value found in experiment [22,23]. The spreading of the $M1$ and $E1$ strength distributions in this model are also interesting and will be discussed in a future paper. A detailed description of the $M1$ distribution is important for the recent suggestion of using ^{208}Pb as a supernova neutrino detector [28].

In summary, I have presented the first complete calculations for ^{208}Pb which include up to $2p - 2h$ excitations involving four major shells (24 orbitals). The calculation appears to account for all known levels up to 8 MeV to within an rms of about 100 keV, except for the lowest 2^+, 4^+, and 6^+ states. This latter problem might be improved with the use of an expanded set of single-particle states, or with the use of a renormalized G matrix. The results for the double-octupole states are consistent with the present experimental situation. The 6^+ component of the double octupole is fragmented. There are 0^+, 2^+, and 4^+ states with a concentrated double-octupole strength, but their actual decay is by weak $E1$ and $E2$ transitions—which in themselves are not strong evidence for their special double-octupole nature. Experimental candidates for these states have been suggested [10]. The detailed decay of these states which can be part of a future calculation based on these wave functions may provide further clues.

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