Neutron Radii in Nuclei and the Neutron Equation of State

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The root-mean-square radius for neutrons in nuclei is investigated in the Skyrme Hartree-Fock model. The main source of theoretical variation comes from the exchange part of the density-dependent interaction which can be related to a basic property of the neutron equation of state. A precise measurement of the neutron radius in ^{208}Pb would place an important new constraint on the equation of state for neutron matter. The Friedman-Pandharipande neutron equation of state would lead to a very precise value of 0.16 ± 0.02 fm for the difference between the neutron and the proton root-mean-square radius in ^{208}Pb.

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The proton root-mean-square (rms) radii of nuclei are now very precisely known, typically with an error 0.02 fm or better for many nuclei via the charge radii measured by electromagnetic interactions [1]. In contrast, the neutron rms radii are much less well known because they have been determined by strongly interacting probes whose interaction with the nucleus must be phenomenologically modeled. In general the neutron radii are known only within an error of about 0.2 fm [2]. Thus, it has recently been proposed to perform an experiment at JLAB [3] to measure the neutron radius in ^{208}Pb via the parity violation effect in electron scattering. The parity violation in this experiment comes from the Z^0 exchange between the electron and the nucleus which is dominated by interaction with neutrons. A detailed theoretical investigation of the interpretation of the JLAB experiment in terms of the rms radii of neutrons in the nucleus has been carried out [4].

In this Letter I discuss the interpretation of neutron rms radii within the framework of modern Skyrme Hartree-Fock (SHF) models. I will show that the neutron radius can be related to a new constraint on the neutron equation of states (EOS) which is important for its extrapolation to high density and hence to its use for the properties of neutron stars. Skyrme Hartree-Fock models have been extremely successful in describing the precisely measured proton radii together with binding energy properties of nuclei. The Skyrme interaction is an s and p wave expansion of the effective interaction and the inclusion of a density-dependent interaction [5]. The main part of the Skyrme interaction which is important for this discussion is given by

$$\begin{aligned} V_{\text{Skyrme}} = {} & t_0(1 + x_0 P_\sigma)\delta + \frac{1}{2}\, t_1(1 + x_1 P_\sigma)\,(\mathbf{k}'^2\,\delta + \delta\,\mathbf{k}^2) \\ & + t_2(1 + x_2 P_\sigma)\,\mathbf{k}' \cdot \delta\,\mathbf{k} \\ & + \frac{1}{6}\, t_3(1 + x_3 P_\sigma)\,\rho^\alpha(\mathbf{R})\,\delta\,, \end{aligned} \tag{1}$$

where $\delta = \delta(\mathbf{r}_i - \mathbf{r}_j)$, $\mathbf{k} = (1/2i)\,(\nabla_i - \nabla_j)$ is the relative momentum operator acting on the wave function to the right, and $\mathbf{k}'$ is the adjoint of $\mathbf{k}$. P_σ is the spin-exchange operator and $\mathbf{R} = (\mathbf{r}_i + \mathbf{r}_j)/2$. There is also a generalized spin-orbit interaction and a Coulomb interaction [6]. The binding energy differences of mirror nuclei can be reproduced either by a modification of the Coulomb exchange interaction (SkX [6]) or by the addition of a charge symmetry breaking interaction (SkXcsb [7]). The form of this interaction allows one to express the closed-shell Hartree-Fock potential for finite nuclei and the Fermi-gas properties of nuclear matter and neutron matter analytically in terms of the parameters and the nucleon densities [8].

There is a minimal number of parameters in the Skyrme interaction, and only six of the standard parameters are well determined by measured nuclear properties [6] (t_0, t_1, t_2, t_3, x_0, and the spin-orbit strength). But the accuracy with which SHF can reproduce nuclear properties is remarkable. After taking into account deformation effects, modern SHF can describe the charge (proton) radii to an accuracy of about 0.02 fm for all nuclei. The questions I address in this Letter are (1) how precisely can SHF models predict neutron rms radii, and (2) what part of the SHF interaction is important for neutron radii? I also discuss to what extent a measurement of the neutron radius in a single nucleus such as ^{208}Pb will be able to put universal constraints on other nuclei. An accurate knowledge of the neutron radius is important for the interpretation of the atomic measurements of the weak charge in nuclei such as ^{133}Cs [9].

The recent SkX parametrization [6] makes a prediction of $S = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle} = 0.16$ fm for the rms neutron "skin" ^{208}Pb. The SkX parameters are determined by a least-squares fit to about 100 nuclear data involving binding energies, rms charge radii, and single-particle energies. In addition, SkX was constrained to reproduce the Friedman-Pandharipande [10] (FP) variational calculation for the neutron EOS within an assumed 10% error. If I add a data point for S in the SkX fit, only a very narrow range of S values (from about 0.14 to 0.18 fm) is acceptable in the fit. In contrast, if one inspects the results obtained from a wide range of other Skyrme parametrizations in the

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literature, one finds a much larger range (0.05 to 0.25 fm) of S values. [I will consider the 20 SHF interactions given in Table I of Ref. [11] together with MSkA [12] (21), SkT6 [13] (22), SkP [14] (23), SkSC4 [15] (24), SkX [6] (25), and SkXcsb [7] (26).]

It quickly becomes apparent that the new aspect of SkX which constrains the S value is the FP neutron EOS. The neutron EOS was introduced as a constraint into SkX following the suggestion of Pethick and Ravenhall [16] that the existing Skyrme parameter sets gave an extremely wide range of predictions for the neutron EOS, many of which were far from the "fundamental" FP calculation. Indeed, if the FP neutron EOS is removed from the SkX fit, a much wider range of S values would be acceptable.

In the Skryme interaction fit one linear combination of exchange parameters (dominated by the s wave term x_0) is well determined by nuclear binding energies and radii, whereas another linear combination (dominated by density-dependent term x_3) can only be well determined by additional constraints to the neutron EOS, or as I will show, by the neutron rms radius. (The x_1 and x_2 parameters are much less important for the present problem.) The situation is illustrated in Fig. 1 where the binding energy difference between ^{132}Sn and ^{100}Sn, which is sensitive to the asymmetry $(N - Z)$ energy term in the nuclear binding energy, is plotted against S for ^{132}Sn for the 26 Skyrme parameter sets mentioned above. Whereas most of the SHF parameter sets do fairly well for the binding energy difference, they give a wide range of values for S in ^{132}Sn.

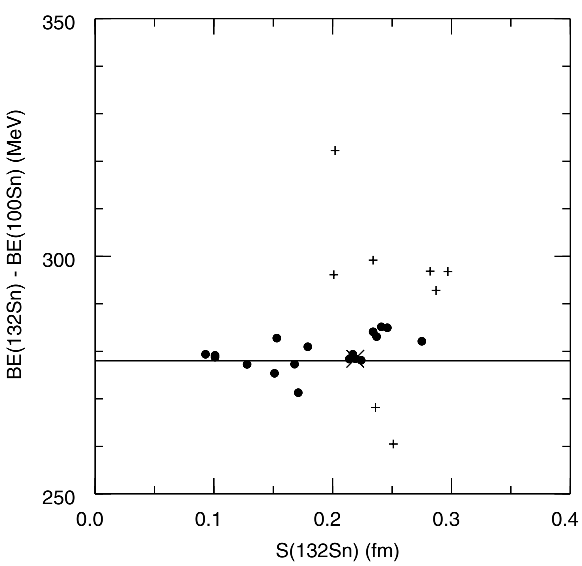

FIG. 1. The binding energy difference between ^{132}Sn and ^{100}Sn plotted vs the S value in ^{132}Sn for 26 Skyrme parameter sets (filled circles and plusses). SkX is indicated by the cross. The horizontal line is the experimental binding energy difference. The filled circles are those 18 sets which are used for the subsequent figures.

Figure 1 also serves to eliminate from further consideration eight of the 26 Skyrme parameter sets which do not reproduce the binding energy difference.

The neutron EOS for the remaining 18 parameter sets are shown in Fig. 2. One observes the extreme variation discussed by Pethick and Ravenhall. The FP EOS is shown by the filled circles. Although some of the parameter sets come quite close to the FP points (the SkX and SkXcsb by design), most of them deviate significantly. It is observed that there is a family of curves which can be distinguished by their derivative at some value of the density near that found in nuclei which we will take at $\rho_n = 0.10$ neutron/fm^3. A plot of this derivative vs the S value for ^{208}Pb is shown in Fig. 3. There is a very tight correlation. Thus, within the wide range of Skyrme parametrizations which have been explored, an experimental S value for ^{208}Pb will provide a new and important constraint on the neutron EOS. It will be necessary to further explore within the Skyrme model and within other mean-field models whether or not the correlation between the neutron skin and the neutron-matter derivative is as unique as it appears to be in Fig. 3.

The FP EOS for nuclear matter and neutron matter are based upon variational calculations using the v_{14} nucleon-nucleon (NN) potential which reproduces NN scattering data and a phenomenological three-nucleon (NNN) interaction (which was modeled as a density-dependent NN potential) and adjusted to reproduce the properties of nuclear matter. The NNN interaction for neutron matter is not known from experiment, and FP made several assumptions about the isospin dependence of the NNN interaction in order to obtain their neutron EOS. The results of more recent neutron matter calculations are

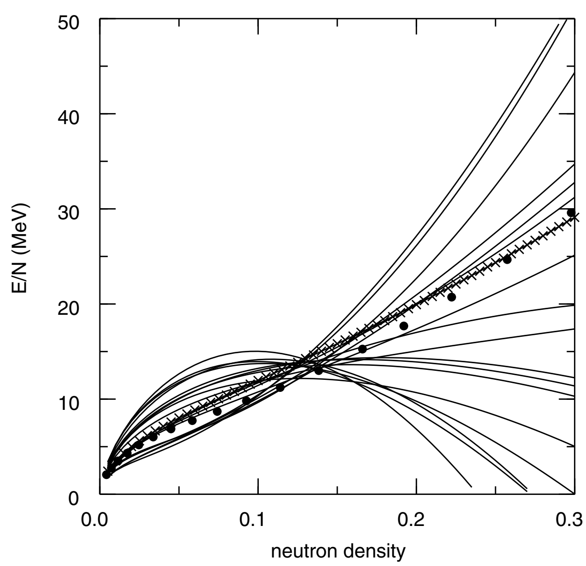

FIG. 2. The neutron EOS for 18 Skyrme parameter sets. The filled circles are the Friedman-Pandharipande (FP) variational calculations and the crosses are SkX. The neutron density is in units of neutron/fm^3.

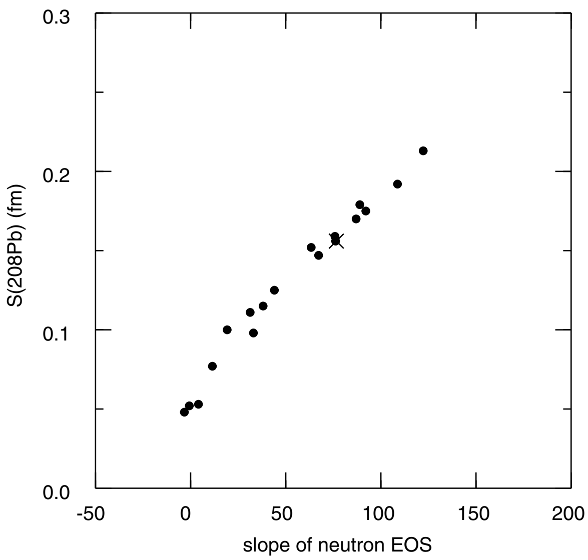

FIG. 3. The derivative of the neutron EOS at $\rho_n = 0.10$ neutron/fm^3 (in units of MeV fm^3/neutron) vs the S value in ^{208}Pb for 18 Skyrme parameter sets. The cross is SkX.

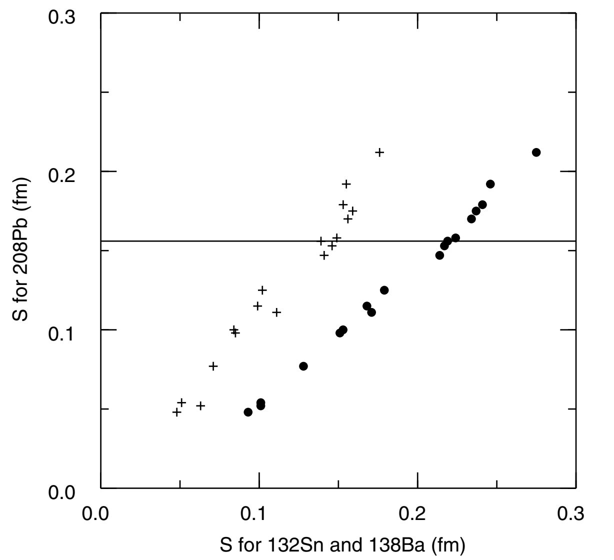

FIG. 4. The S value for ^{208}Pb vs the S values for ^{132}Sn (filled circles) and ^{138}Ba (plusses) for 18 Skyrme parameter sets. The horizontal line is the SkX value for ^{208}Pb.

provided by Wiringa, Fiks, and Fabrocini [17] and Akmal and Pandharipande [18]. Generally the agreement with FP is good up to about $\rho_n = 0.10$ neutron/fm^3. At higher density the differences in the various NN potentials [17] and the very uncertain NNN potential become important. Thus, although the FP neutron EOS serves as a reasonable starting point, we do not have a truly fundamental theory for neutron EOS. Any constraints coming from the properties of nuclei such as the neutron radii are extremely important.

Given the difficulty of the JLAB measurement, it is important to know to what extent a measurement of S in one nucleus such as ^{208}Pb will be applicable to other nuclei. There are two points to investigate: the dependence of S on mass and the dependence of S on the asymmetry in the Fermi energy for protons and neutrons. For the first case, I compare in Fig. 4 the S values for two nuclei near the valley of stability (where the Fermi energies for protons and neutrons are about equal to each other), those for ^{208}Pb and ^{138}Ba. One observes a nearly linear relationship which starts at $S = 0$. For the second case, I compare in the same figure the S value in ^{208}Pb to the S value for ^{132}Sn where the neutrons at the Fermi surface are bound about 8 MeV less than the protons (see Figs. 4 and 5 in Ref. [6]). Again there is a tight correlation, but the asymmetry in the Fermi energy produces a systematic increase in the neutron skin for all of the 18 SHF parameter sets. Thus there are two clear mechanisms for producing a neutron skin. One which is related to the asymmetry in the Fermi energy is well determined within SHF, and another which depends on the neutron EOS is undetermined unless one adds a constraint to the neutron EOS. It is the Fermi-energy asymmetry effect which dominates the increase in the matter radii of neutron-rich light nuclei such as in the Na isotopes [11]. Thus it is most important to accurately determine the neutron rms radius in a stable nucleus such as ^{208}Pb. The neutron rms radius of ^{208}Pb will provide an important new constraint on the neutron EOS models which are used to calculate the properties of neutron stars [17]. The results discussed here are based upon a wide variety of parametrizations for the Skyrme Hartree-Fock model for finite nuclei and nucleon matter. It will be important to explore the generality of these conclusions within the Skyrme model as well as in other mean-field models.

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