

Solving the Coincidence Problem: Tracking Oscillating Energy

Scott Dodelson,^{1,2} Manoj Kaplinghat,² and Ewan Stewart^{1,3}

¹NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500

²Department of Astronomy & Astrophysics, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637-1433

³Department of Physics, KAIST, Taejeon 305-701, South Korea

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Recent cosmological observations strongly suggest that the Universe is dominated by an unknown form of energy with negative pressure. Why is this dark energy density of order the critical density today? We propose that the dark energy has periodically dominated in the past so that its preponderance today is natural. We illustrate this paradigm with a model potential and show that its predictions are consistent with all observations.

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Introduction.—A variety of evidence accumulated over the past several years points to the existence of an unknown, unclumped form of energy in the Universe. First was an apparent concordance [1] of different measurements: the age of the Universe; the Hubble constant; the baryon fraction in clusters, and the shape of the Galactic power spectrum. Second came the stunning observations [2] of tens of distant type-Ia supernovae, which found a distance-redshift relation in accordance with a cosmological constant, but in strong disagreement with a matter dominated Universe. Finally, this past year has seen analyses [3] of the experiments measuring anisotropies in the cosmic microwave background (CMB). Taken together, the CMB experiments plot out a rough shape for the power spectrum, one that is in accordance with a flat Universe, but in disagreement with an open Universe. If we believe the estimates of matter density coming from observations of clusters [4], the only way to get a flat Universe, and hence account for the CMB measurements, is to have an unclumped form of energy density pervading the Universe.

Perhaps the simplest explanation for these data is that the unclumped form of energy density corresponds to a positive cosmological constant [5]. A nonzero but tiny constant vacuum energy density (cosmological constant) could conceivably be explained by some unknown string theory symmetry (that sets the vacuum energy density to zero) being broken by a small amount. However, to explain in this way a constant vacuum energy density of 2×10^{-59} TeV⁴, which is not only small but is also just the right value that it is just beginning to dominate the energy density of the Universe *now*, would require an unbelievable coincidence. A different possibility is to give up the dream of finding a mechanism which would set the vacuum energy density to exactly zero and resort to believing that anthropic considerations select among $\geq 10^{100}$ string vacua to find one with a vacuum energy density sufficiently fine-tuned for life.

An alternative is to assume that the true vacuum energy density is zero, and to work with the idea that the unknown, unclumped energy (quintessence) is due to a scalar field ϕ which has not yet reached its ground state. This idea has received much attention [6] over the past several years. How-

ever, two problems still remain. First, the field's mass has to be extremely small, less than or of order of the Hubble constant today $\sim 10^{-33}$ eV, to ensure that it is still rolling to its vacuum configuration. This is in general difficult because scalar fields tend to acquire masses greater than or of order of the scale of supersymmetry breaking suppressed by at most the Planck scale: $m \geq F/m_{\text{Pl}} \geq \text{TeV}^2/m_{\text{Pl}} \sim 10^{-3}$ eV. Although difficult, this could be achieved using pseudo-Nambu-Goldstone bosons [7] or perhaps with an additional symmetry. The second, and perhaps even more serious, problem is that almost all of these models require that we live in a special epoch today, when the quintessence is just starting to dominate the energy density of the Universe, and furthermore this specialness cannot even be justified by the use of anthropic arguments.

In recent years a lot of progress has been made in understanding the behavior of quintessence fields. A broad class of solutions, called tracker solutions [8], has been discovered in which the final value of the quintessence energy density is insensitive to the initial conditions. For example, potentials such as $V = V_0 \phi^{-n}$ or $V = V_0 \exp(1/\phi)$ can, for suitable choices of V_0 , catch up with the critical density late in the evolution of the Universe for a wide range of initial conditions and thus provide a natural setting for explaining the current acceleration of the Universe. However, the suitable choice of V_0 must be of the order of the critical energy density today, i.e., we are back to the problem of living at a special epoch today and not even being able to use anthropic arguments to justify this specialness.

In a subset of these tracking models, which we call the exact tracker solutions [9,10], the scalar field energy density is always related to the ambient energy density in the Universe: if the dominant component in the Universe is radiation, then the tracking field's energy density also falls off as a^{-4} , where a is the scale factor of the Universe. If the dominant component is matter, then the field's energy density scales as a^{-3} . This behavior arises from an exponential potential for ϕ (regardless of the value of V_0). Since the energy density in this field is always comparable to the background density, we are not living at a special epoch: any observer in the distant past or future would

also see the tracking field's energy density. However, these tracking solutions run into two problems. First, if their energy density today truly is dominant, then it should also have been dominant at the time of big bang nucleosynthesis (BBN). Constraints from observations of light element abundances preclude such an additional form of energy density at early times. Second, tracking models have the wrong equation of state at present since the tracking field behaves like matter, with zero pressure, instead of having the necessary negative pressure to accelerate the Universe.

In this Letter we ask the question, what if the Universe has been accelerating periodically in the past? Then the fact that the Universe is accelerating today would not be surprising. If the Universe does accelerate periodically, then it is, in fact, reasonable to expect it to accelerate today.

The potential and the field evolution.—Consider a scalar field ϕ with potential $V(\phi) = V_0 \exp(-\lambda\phi\sqrt{8\pi G})$. It is well known [9] that such a potential leads to an attractor solution with $\Omega_\phi \equiv \rho_\phi/(\rho_\phi + \rho_0) = n/\lambda^2$, where ρ_0 is the energy density in the other component of the Universe, which is assumed to scale as a^{-n} . Thus, no matter what the initial conditions are for ϕ , it always evolves so that it tracks the rest of the density in the Universe.

Now consider the potential

$$V(\phi) = V_0 \exp(-\lambda\phi\sqrt{8\pi G}) [1 + A \sin(\nu\phi\sqrt{8\pi G})]. \quad (1)$$

This potential serves to modulate the tracking behavior. Figure 1 shows the resultant evolution of ϕ and its energy density for a particular set of the parameters A, ν . (The normalization V_0 can be set to G^{-2} by shifting the initial value of ϕ .) Also shown is the tracking solution for this particular value of λ without the modulation. As

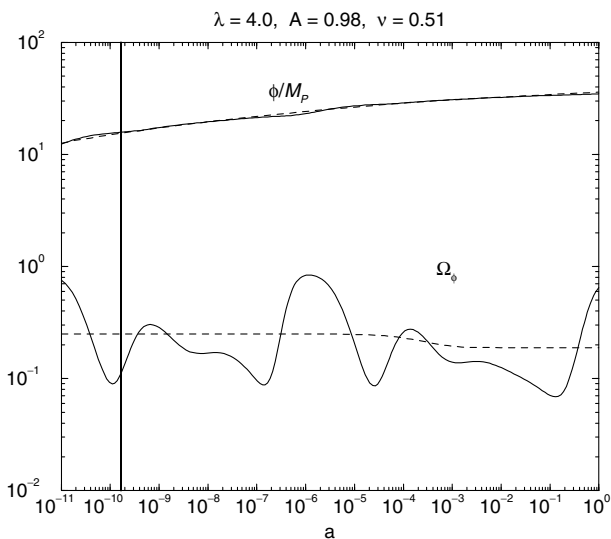


FIG. 1. The fraction of the critical density in ϕ for the potential in Eq. (1). The dashed line shows the corresponding tracking solution ($A = 0$). The upper set of curves shows the evolution in ϕ for the TOE and the tracking models.

expected, the sinusoidal term in the potential leads to oscillations about this tracking behavior. We call this type of energy *tracking, oscillating energy*, or TOE. One can obtain analytic solutions for the dynamics of the potential in Eq. (1) during radiation ($n = 4$) or matter ($n = 3$) domination in the limit that A is small by perturbing about the corresponding exact tracker model which has $\phi\sqrt{8\pi G} = \frac{n}{\lambda} \ln a$. The sine in Eq. (1) provides a periodic forcing term with period $\ln a = \frac{2\pi\lambda}{n\nu}$, while the natural period [9] of the damped oscillations about the exact tracker solution is $\ln a = 8\pi\lambda/\sqrt{(6-n)[3(3n-2)\lambda^2 - 8n^2]}$ with decay e -life $\ln a = 4/(6-n)$. Although the above results are strictly valid only for small A , they account remarkably well for the behavior shown in Fig. 1. The forced period corresponds to the longer period of 5.4 units ($n = 4$) and somewhere between 5.4 units and 7.1 units ($n = 3$), while the natural period corresponds to the shorter period of 1.6 units ($n = 4, 3$) of the damped oscillations which are presumably excited by the nonlinear effects that appear when A is not small.

The energy density due to ϕ is relatively small at the time of BBN and relatively large today for the parameter set in Fig. 1. It is, of course, clear that, in order to get the right behavior at BBN and today, one has to pick the “correct” parameter sets. This involves a bit of fine-tuning which, as we argue below, is quite reasonable and natural. If one thinks of the parameter set as being randomly selected, then there is a finite probability that the Universe will be accelerating today and that the energy density of ϕ will be subdominant at BBN. What is this probability? If one selects A, ν , and λ randomly, the chance of getting a Universe like ours is of the order of 1 in 100. The exact number (for this potential) depends on how stringently we define “a Universe like ours.” For example, the tight constraints $0.4 < \Omega_\phi < 0.8$, $w_\phi < -0.5$, and $(\rho_\phi/\rho_0)_{\text{BBN}} < 0.1$ give a probability of 1 in 450, while the relaxed constraints $0.1 < \Omega_\phi < 0.9$ and $w_\phi < -0.25$, and $(\rho_\phi/\rho_0)_{\text{BBN}} < 0.2$ give a probability of 1 in 26. It is also very important to note that whatever the extent of fine-tuning, all of it is in *dimensionless* numbers. There are no energy scales in this scenario which are to be set by the present expansion rate of the Universe.

Power spectra.—To compare with CMB and large scale structure observations, we compute the power spectra of the perturbations in a TOE model. Perturbations evolve differently in the presence of the scalar field energy density. For example, perturbations typically grow only when the Universe is matter dominated. Therefore, we expect a nonzero Ω_ϕ to lead directly to power suppression on the scales inside the horizon, with increased suppression for larger Ω_ϕ .

The prediction for the CMB angular power spectrum is plotted in Fig. 2. The primeval power spectrum is scale invariant with adiabatic initial conditions. Also plotted for comparison is a model (Λ CDM) with cosmological constant $\Omega_\Lambda = \Omega_\phi$ today and the rest of the cosmological

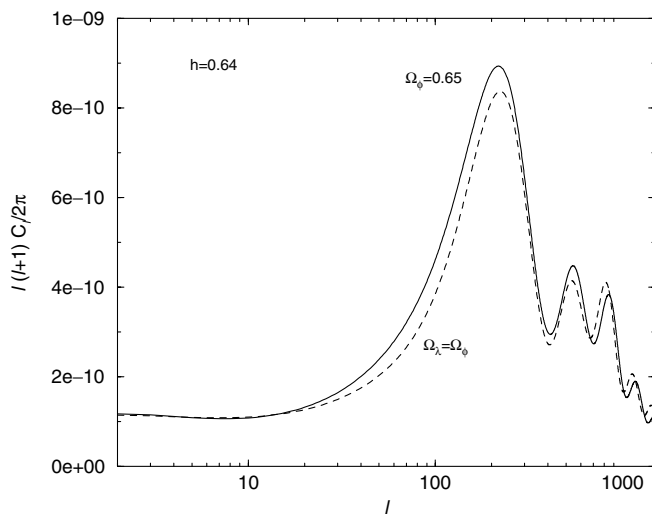


FIG. 2. The angular photon power spectrum from the TOE model of Fig. 1. Also shown is a cosmological constant model with all other parameters equal.

parameters also being the same. In further discussions we will contrast the results from the TOE model against this Λ CDM model. A noteworthy feature in Fig. 2 is the increase in the heights of the first two peaks compared to that of the Λ CDM model. This stems from the fact that the gravitational potential decays more in the presence of the additional quintessence energy density. The decay of the potential at and after recombination (the so-called integrated Sachs-Wolfe, or ISW, effect) leads [11] to enhanced power on scales $l \lesssim 600$, after which the potential becomes irrelevant. Note that the increase in the amplitude of both the first and second peaks cannot be mimicked by adding more baryons, which raise the odd peaks but lower the even ones.

On smaller scales ($l \gtrsim 600$), the TOE model has smaller anisotropies. Here there are two competing effects. First, the difference between the TOE and the Λ CDM models (around recombination when Λ is insignificant) is the presence of the extra quintessence energy density, which leads to the expansion rate in the two models being related as

$$H_{\text{TOE}}(a) = H_{\Lambda\text{CDM}}(a) \times [1 - \Omega_\phi(a)]^{-1/2}. \quad (2)$$

Equation (2) implies that all of the relevant scales at recombination (which occurs at $a_r \approx 10^{-3}$) are smaller in the TOE model by a factor of about $\sqrt{1 - \Omega_\phi(a_r)}$. In particular, the damping scale is smaller, which increases in power on small scales for the TOE model relative to the Λ CDM model. The second effect is the large scale normalization of the two models [12], and this second effect more than compensates for the first. Cosmic Background Explorer (COBE) normalization is sensitive to scales about $\ell = 10$ for which the differences in the two models with regard to the late-ISW effect is important. In particular, since Λ domination occurs very late, the ISW contribution at about $\ell = 10$ is much larger in the TOE model. This in

turn implies that the normalization of the primeval power spectrum is smaller, a fact noticeable in the smaller amplitude of the photon power spectrum for the TOE model at small scales (and also the matter power spectrum, as we will soon see). One last effect that is worth pointing out concerns the difference in the peak positions in the two models (though, unlike the peak amplitudes, it is probably not easily discerned). In particular, the TOE model has the acoustic features in its angular power spectrum shifted to smaller scales. This directly traces to the decrease in the angular diameter distance to the last scattering surface, for the TOE model. Of course, there is also the competing effect of the decrease in the size of the sound horizon at last scattering for the TOE model, which minimizes the effect.

The prediction for the matter power spectrum is plotted in Fig. 3. The difference in power at the largest scales is due to COBE normalization and the difference in the superhorizon growth factor (which is sensitive to the equation of state of the cosmic fluid) for the perturbation. As one moves to smaller scales, which entered the horizon well before the present, the differences in the evolution of the matter perturbation become more pronounced. The presence of the extra quintessence energy stunts the growth of perturbation once a mode enters the horizon. So, the earlier the mode enters the horizon, the larger the growth suppression relative to the Λ CDM model. In other words, smaller modes are monotonically more suppressed (something that may not be noticeable in the log plot) compared to the same modes in the Λ CDM model. It might also be surprising that the ϕ domination at about $a = 10^{-6}$ does not cause a more appreciable feature (i.e., suppression) in the power spectrum. The reason is that the smallest scales in Fig. 3 have just entered the horizon at the time of ϕ domination ($a \sim 10^{-6}$).

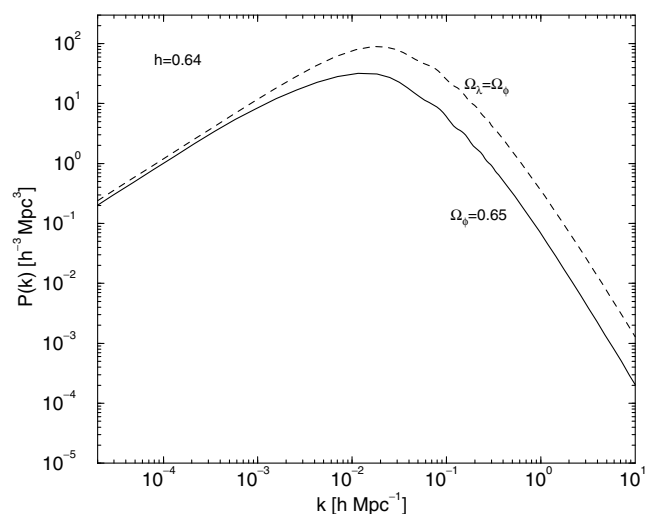


FIG. 3. The matter power spectrum from the TOE model of Fig. 1. Also shown is a cosmological constant model with all other parameters equal. Power is significantly smaller in the TOE model.

The normalization on the small scales is generally quoted in terms of σ_8 , the rms mass fluctuation within a $8h^{-1}$ Mpc sphere. For the parameters in Fig. 1, the TOE model has $\sigma_8 = 0.4$. This is several sigma smaller than the preferred value (see e.g. [13]) of ~ 0.8 , but could be rectified by a small blueshift in the primordial spectrum [14].

Conclusions.—We have constructed a model wherein the energy density tracks the dominant component in the Universe; satisfies the BBN constraints; and has the proper equation of state today. Further, this model makes definite predictions for large scale structure and for the CMB.

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