

## Primordial Galactic Magnetic Fields from Domain Walls at the QCD Phase Transition

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We propose a mechanism to generate large-scale magnetic fields with correlation lengths of 100 kpc. Domain walls with QCD-scale internal structure form and coalesce, obtaining Hubble-scale correlations while aligning nucleon spins. Because of strong  $CP$  violation, the walls are ferromagnetic, which induces electromagnetic fields with Hubble-size correlations. The  $CP$  violation also induces a maximal helicity (Chern-Simons) which supports an “inverse cascade,” allowing the initial correlations to grow to 100 kpc today. We estimate the generated electromagnetic fields in terms of the QCD parameters and discuss the effects of the resulting fields.

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*Introduction.*—The source of cosmic magnetic fields with large-scale correlations has remained somewhat of a mystery [1]. There are two possible origins for these fields: primordial sources and galactic sources. Primordial fields are produced in the earlier universe and then evolve and are thought to provide seeds which gravitational dynamos later amplify. Galactic sources would produce the fields as well as amplify them. Many mechanisms have been proposed [2–5], however, most fail to convincingly generate fields with large enough correlation lengths to match the observed microgauss fields with  $\sim 100$  kpc correlations. We present here a mechanism which, although probably requiring a dynamo to produce microgauss fields, generates fields with hundred kiloparsec correlations. We present this mechanism as an application of our recent understanding of QCD domain walls, which will be described in detail elsewhere [6].

- (1) Sometime near the QCD phase transition,  $T_{\text{QCD}} \approx 1$  GeV, QCD domain walls form.
- (2) These domain walls rapidly coalesce until there remains, on average, one domain wall per Hubble volume with Hubble-scale correlations.
- (3) Baryons interact with the domain walls and align their spins along the domain walls.
- (4) The magnetic and electric dipole moments of the baryons induce helical magnetic fields correlated with the domain wall.
- (5) The domain walls decay, leaving a magnetic field.
- (6) As the universe expands, an “inverse cascade” mechanism transfers energy from small to large scale modes, effectively increasing the resulting correlation length of the observed large scale fields.

We shall start by discussing the inverse cascade mechanism which seems to be the most efficient mechanism for increasing the correlation length of magnetic turbulence. After presenting some estimates to show that this mechanism can indeed generate fields of the observed scales, we shall discuss the domain wall mechanism for generating the initial fields.

*Evolution of magnetic fields.*—As suggested by Cornwall [3], discussed by Son [4], and confirmed by

Field and Carroll [5], energy in magnetic fields can undergo an apparent inverse cascade and be transferred from high frequency modes to low frequency modes, thus increasing the overall correlation length of the field faster than the naïve scaling by the universe’s scale parameter  $R(T)$ . There are two important conditions: turbulence must be supported as indicated by a large Reynolds number  $\text{Re}$ , and magnetic helicity (Abelian Chern-Simons number)  $H = \int \vec{A} \cdot \vec{B} d^3x$  is approximately conserved. The importance of helicity was originally demonstrated by Pouquet and collaborators [7]. The mechanism is thus: the small scale modes dissipate, but the conservation of helicity requires that the helicity be transferred to larger scale modes. Some energy is transferred along with the helicity and hence energy is transported from the small to large scale modes. This is the inverse cascade. The reader is referred to [3–5] for a more complete discussion.

In the early universe,  $\text{Re}$  is very large and supports turbulence. This drops to  $\text{Re} \approx 1$  at the  $e^+e^-$  annihilation epoch,  $T_0 \approx 100$  eV [4]. After this point (and throughout the matter dominated phase) we assume that the fields are “frozen in” and that the correlation length expands as  $R$  while the field strength decays as  $R^{-2}$ . Note that the inverse cascade is supported only during the radiation dominated phase of the universe.

Under the assumption that the field is maximally helical, these conditions imply the following relationships between the initial field  $B_{\text{rms}}(T_i)$  with initial correlation  $l(T_i)$  and present fields today ( $T_{\text{now}} \approx 2 \times 10^{-4}$  eV)  $B_{\text{rms}}(T_{\text{now}})$  with correlation  $l(T_{\text{now}})$  [4,5]:

$$B_{\text{rms}}(T_{\text{now}}) = \left(\frac{T_0}{T_{\text{now}}}\right)^{-2} \left(\frac{T_i}{T_0}\right)^{-7/3} B_{\text{rms}}(T_i), \quad (1)$$

$$l(T_{\text{now}}) = \left(\frac{T_0}{T_{\text{now}}}\right) \left(\frac{T_i}{T_0}\right)^{5/3} l(T_i). \quad (2)$$

As pointed out in [4], the only way to generate turbulence is either by a phase transition  $T_i$  or by gravitational instabilities. We consider the former source. As we shall show, our mechanism generates Hubble-size correlations  $l_i$  at a phase transition  $T_i$ . In the radiation dominated

epoch, the Hubble-size scales as  $T_i^{-2}$ . Combining this with (2), we see that  $l_{\text{now}} \propto T_i^{-1/3}$ ; thus, the earlier the phase transition, the smaller the possible correlations.

The last phase transition is the QCD transition,  $T_i = T_{\text{QCD}} \approx 0.2$  GeV with Hubble size  $l(T_{\text{QCD}}) \approx 30$  km. We calculate (9) the initial magnetic field strength to be  $B_{\text{rms}}(T_i) \approx e\Lambda_{\text{QCD}}^2/(\xi\Lambda_{\text{QCD}}) \approx (10^{17} \text{ G})/(\xi\Lambda_{\text{QCD}})$  where  $\xi$  is a correlation length that depends on the dynamics of the system as discussed below and  $\Lambda_{\text{QCD}} \approx 0.2$  GeV. With these estimates, we see that

$$B_{\text{rms}} \sim \frac{10^{-9} \text{ G}}{\xi\Lambda_{\text{QCD}}}, \quad l \sim 100 \text{ kpc} \quad (3)$$

today. One could consider the electroweak transition which might produce 100 pc correlations today, but this presupposes a mechanism for generating fields with Hubble-scale correlations. Such a mechanism does not appear to be possible in the standard model. Instead, the fields produced are correlated at the scale  $T_i^{-1}$  which can produce only  $\sim 1$  km correlations today.

These are crude estimates, and galactic dynamos likely amplify these fields. The important point is that we can generate easily the 100 kpc correlations observed today *provided* that the fields were initially of Hubble-size correlation. Unless another mechanism for amplifying the correlations of magnetic fields is discovered, we suggest that, in order to obtain microgauss fields with 100 kpc correlation lengths, helical fields must be generated with Hubble-scale correlations near or slightly after the QCD phase transition  $T_{\text{QCD}}$ . The same conclusion regarding the relevance of the QCD scale for this problem was also reached in [4,5]. The rest of this work presents a mechanism that can provide the desired Hubble-size fields, justifying the estimate (3). We shall explain the mechanism and give simple estimates here. See [6] for details.

*Magnetic field generation mechanism.*—The key players in our mechanism are domain walls formed at the QCD phase transition that possess an internal structure of QCD scale. We shall present a full exposition of these walls in [6] but to be concrete, we shall discuss an axion wall similar to that described by Huang and Sikivie [8].

We start with a similar effective Lagrangian to that used by Huang and Sikivie except we included the effects of the  $\eta'$  singlet field which they neglected:

$$\mathcal{L}_{\text{eff}} = \frac{f_a^2}{2} |\partial_\mu e^{i\tilde{a}}|^2 + \frac{f_\pi^2}{4} \text{Tr}[\partial_\mu \mathbf{U}]^2 - V(\mathbf{U}, \tilde{a}), \quad (4)$$

where  $\tilde{a} = f_a^{-1}a$  is the dimensionless axion field and the matrix  $\mathbf{U} = \exp(i\tilde{\eta}' + i\tilde{\pi}^f \mathbf{\Lambda}^f)$  contains the pion and  $\eta'$  fields [to simplify the calculations, we consider only the SU(2) flavor group]. Although the  $\eta'$  field is light, it couples to the anomaly and is the dominant player in aligning the magnetic fields. The potential

$$V = \frac{1}{2} \text{Tr}(\mathbf{M}\mathbf{U}e^{i\tilde{a}} + \text{H.c.}) - E \cos\left(\frac{i \ln[\det(\mathbf{U})]}{N_c}\right) \quad (5)$$

was first introduced in [9]. It should be realized that  $i \ln[\det(\mathbf{U})] \equiv i \ln[\det(\mathbf{U})] + 2\pi n$  is a multivalued func-

tion and we must choose the minimum values branch. Details about this potential are discussed in [6,9] but several points will be made here. All dimensionful parameters are expressed in terms of the QCD chiral and gluon vacuum condensates, and are well known numerically:  $\mathbf{M} = -\text{diag}(m_q^i |\langle \bar{q}^i q^i \rangle|)$  and  $E = \langle b\alpha_s/(32\pi)G^2 \rangle$ . This potential correctly reproduces the Veneziano-Witten effective chiral Lagrangian in the large  $N_c$  limit [10]; it reproduces the anomalous conformal and chiral Ward identities of QCD; and it reproduces the known dependence in  $\theta$  for small angles [10]. We should also remark that the qualitative results do not depend on the exact form of the potential: domain walls form naturally because of the discrete nature of the symmetries [8,11,12].

The result is that two different types of axion domain walls form [6]. One is almost identical to the one discussed in [8] with small corrections due to the  $\eta'$ . We shall call this the axion/pion ( $a_\pi$ ) domain wall. The second type, which we shall call the axion/eta' ( $a_{\eta'}$ ) domain wall is a new solution characterized by a transition in both the axion and  $\eta'$  fields. The boundary conditions (vacuum states) for this wall are  $\tilde{a}(-\infty) = \tilde{\eta}'(-\infty) = 0$  and  $\tilde{a}(\infty) = \tilde{\eta}'(\infty) = \pm\pi$  with  $\pi^0 = 0$  at both boundaries. The main difference between the structures of the two walls is that, whereas the  $a_\pi$  domain wall has structure only on the huge scale of  $m_a^{-1}$ , the  $\eta'$  transition in  $a_{\eta'}$  has a scale of  $m_{\eta'}^{-1} \sim \Lambda_{\text{QCD}}^{-1}$ . The reason is that, in the presence of the nonzero axion ( $\theta$ ) field, the pion becomes effectively massless due to its Goldstone nature. The  $\eta'$  is not sensitive to  $\theta$  and so its mass never becomes zero. It is crucial that the walls have a structure of scale  $\Lambda_{\text{QCD}}^{-1}$ : there is no way for the  $a_\pi$  wall to trap nucleons because of the huge difference in scales but the  $a_{\eta'}$  wall has exactly this structure and can therefore efficiently align the nucleons.

The model we propose is this: Immediately after the phase transition, the universe is filled with domain walls of scale  $T_{\text{QCD}}^{-1}$ . As the temperature drops, these domain walls coalesce, resulting in an average of one domain wall per Hubble volume with Hubble-scale correlations [11,13]. It is these  $a_{\eta'}$  domain walls which align the dipole moments of the nucleons producing the seed fields.

The following steps are crucial for this phenomenon: (1) The coalescing of QCD domain wall gives the fields  $\pi^f$ ,  $\eta'$  Hubble-scale correlations. (2) These fields interact with the nucleons producing Hubble-scale correlations of nucleon spins residing in the vicinity of the domain wall. (The spins align perpendicular to the wall surface.) (3) Finally, the nucleons, which carry electric and magnetic moments (due to strong  $CP$  violation), induce Hubble-scale correlated magnetic and electric fields. (4) These magnetic and electric fields eventually induce a nonzero helicity which has the same correlation. This helicity enables the inverse cascade.

*Quantitative estimates.*—As outlined below, we have estimated the strengths of the induced fields in terms of the QCD parameters [6]. We consider two types of interactions. First, the nucleons align with the domain wall.

Here we assume that the fluctuations in the nucleon field  $\Psi$  are rapid and that these effects cancel leaving the classical domain wall background unaltered. Thus, we are able to estimate many mean values correlated on a large scale on the domain walls such as  $\langle \bar{\Psi} \gamma_5 \sigma_{xy} \Psi \rangle$  and  $\langle \bar{\Psi} \gamma_z \gamma_5 \Psi \rangle$  through the interaction  $\bar{\Psi} [i \not{\partial} - m_N e^{i \tilde{\eta}'(z) \gamma_5}] \Psi$ .

To estimate the magnetization of the domain wall, we make the approximation that the wall is flat compared to the length scales of the nucleon interactions. By assuming that momentum is conserved in the wall, we reduce our problem to an effective 1 + 1 dimensional theory (in  $z$  and  $t$ ) which allows us to compute easily various mean value using a bosonization trick [14,15]. The result for the mean value  $\langle \bar{\Psi} \gamma_5 \sigma_{xy} \Psi \rangle$ , for example, is [6]:

$$\langle \bar{\Psi} \sigma_{xy} \gamma_5 \Psi \rangle \simeq \frac{\mu}{\pi} \Lambda_{\text{QCD}}^2, \quad (6)$$

where  $\mu \simeq m_N$  is a dimensional parameter originating from the bosonization procedure of the corresponding 2D system and the parameter  $\Lambda_{\text{QCD}}^2 \sim \int dk_x dk_y$  comes from counting the nucleon degeneracy in the  $x$ - $y$  plane of a Fermi gas at temperature  $T_c \simeq \Lambda_{\text{QCD}}$ . These mean values are only nonzero within a distance  $\Lambda_{\text{QCD}}^{-1}$  of the domain wall and are therefore correlated on the same Hubble scale as the domain wall.

From now on we treat the expectation value (6) as a background classical field correlated on the Hubble scale. Once these sources are known, one could calculate the generated electromagnetic field by solving Maxwell's equations with the interaction

$$\frac{1}{2} (d_\Psi \bar{\Psi} \sigma_{\mu\nu} \gamma_5 \Psi + \mu_\Psi \bar{\Psi} i \sigma_{\mu\nu} \Psi) F_{\mu\nu} + \bar{\Psi} (iD)^2 \Psi, \quad (7)$$

where  $d_\Psi$  ( $\mu_\Psi$ ) is effective electric (magnetic) dipole moments of the field  $\Psi$ . Because of the  $CP$  violation (nonzero  $\theta$ ) along the axion domain wall, the anomalous nucleon dipole moment in (7)  $d_\Psi \sim \mu_\Psi \sim \frac{e}{m_N}$  is also nonzero [16]. This is an important point: if no anomalous moments were induced, then only charged particles could generate the magnetic field: the walls would be diamagnetic not ferromagnetic as argued in [17], and Landau levels would exactly cancel the field generated by the dipoles.

Solving the complete set of Maxwell's equations, however, is extremely difficult. Instead, we use simple dimensional arguments. For a small planar region of area  $\xi^2$  filled with aligned dipoles with constant density, we know that the net magnetic field is proportional to  $\xi^{-1}$  since the dipole fields tend to cancel, thus for a flat section of our domain wall, the field would be suppressed by a factor of  $(\xi \Lambda_{\text{QCD}})^{-1}$ . For a perfectly flat, infinite domain wall ( $\xi \rightarrow \infty$ ), there would be no net field as pointed out in [17]. However, our domain walls are far from flat. Indeed, they have many wiggles and high frequency modes; thus, the size of the flat regions where the fields are suppressed is governed by a correlation  $\xi$  which describes the curvature of the wall. Thus, the average electric and magnetic fields produced by the domain wall are of the order

$$\langle F_{\mu\nu} \rangle \simeq \frac{[d_\Psi \langle \bar{\Psi} \sigma_{\mu\nu} \gamma_5 \Psi \rangle + \mu_\Psi \langle \bar{\Psi} i \sigma_{\mu\nu} \Psi \rangle]}{\xi \Lambda_{\text{QCD}}}, \quad (8)$$

where  $\xi$  is an effective correlation length related to the size of the dominant high frequency modes.

To estimate what effective scale  $\xi$  has, however, requires an understanding of the dynamics of the domain walls. Initially, the domain walls are correlated with a scale of  $\Lambda_{\text{QCD}}^{-1}$ . As the temperature cools, the walls smooth out and the lower bound  $\xi_-(t)$  for the scale of the walls correlations increases from  $\xi_-(0) \simeq \Lambda_{\text{QCD}}^{-1}$ . This increase is a dynamical feature, however, and is thus slow. In addition, the walls coalesce and become correlated on the Hubble scale generating large-scale correlations. Thus the wall has correlations from  $\xi_-(t)$  up to the upper limit set by the Hubble scale. We expect that  $\xi \ll$  Hubble size at the time that the fields are aligned and so that suppression is not nearly as great as implied in [17]. Note that, even though the effects are confined to the region close to the wall, the domain walls are moving and twisted so that the effects occur throughout the entire Hubble volume.

The picture is thus that fields of strength

$$\langle E_z \rangle \simeq \langle B_z \rangle \sim \frac{1}{\xi \Lambda_{\text{QCD}}} \frac{e}{m_N} \frac{m_N \Lambda_{\text{QCD}}^2}{\pi} \sim \frac{e \Lambda_{\text{QCD}}}{\xi \pi} \quad (9)$$

are generated with short correlations  $\xi$ , but then domains are correlated on a large scale by the Hubble-scale modes of the coalescing domain walls. Thus, strong turbulence is generated with correlations that run from  $\Lambda_{\text{QCD}}$  up to the Hubble scale.

Finally, we note that this turbulence should be highly helical. This helicity arises from the fact that both electric and magnetic fields are correlated together along the entire domain wall,  $\langle \vec{E} \rangle \sim \langle \vec{A} \rangle / \tau$  where  $\langle \vec{A} \rangle$  is the vector potential and  $\tau$  is a relevant time scale for the fields to align (we expect  $\tau \sim \Lambda_{\text{QCD}}^{-1}$  as we discuss below). The magnetic helicity density thus grows:

$$h = \vec{A} \cdot \vec{B} \sim \tau \langle E_z \rangle \langle B_z \rangle \sim \tau \frac{e^2}{\pi^2} \frac{\Lambda_{\text{QCD}}^2}{\xi^2} \quad (10)$$

and will saturate on a similar time scale to the other interactions. Note carefully what happens here: The total helicity was zero in the quark-gluon-plasma phase and remains zero in the whole universe, but the helicity is separated so that in one Hubble volume, the helicity has the same sign. The reason for this is that, as the domain walls coalesce, initial perturbations cause either a soliton or an antisoliton to dominate and fill one Hubble volume. In the neighboring space, there will be other solitons and antisolitons so that there is an equal number of both, but they are spatially separated which prevents them from annihilating. This is similar to how a particle and antiparticle may be created and then separated so they do not annihilate. In any case, the helicity is a pseudoscalar and thus has the same sign along the domain wall: The entire Hubble volume has helicity of the same sign. This is the origin of the Hubble-scale correlations

in the helicity and in  $B^2$ . The correlation parameter  $\xi$  which affects the magnitude of the fields plays no role in disturbing this correlation.

Eventually, the electric field will be screened. The time scale for this is set by the plasma frequency for the electrons (protons will screen much more slowly)  $\omega_p \sim \Lambda_{\text{QCD}}$ . The nucleons, however, also align on a similar time scale  $\Lambda_{\text{QCD}}^{-1}$ , and the helicity will saturate at on this scale too, so the screening will not qualitatively affect the mechanism. Finally, we note that the turbulence requires a seed which remains in a local region for a time scale set by the conductivity  $\sigma \sim cT/e^2 \sim \Lambda_{\text{QCD}}$  where for  $T = 100$ ,  $c \approx 0.07$  [18] and is smaller for higher  $T$ . Thus, even if the domain walls move at the speed of light (due to vibrations), there is still time to generate turbulence.

For this mechanism to work and not violate current observations, it seems that the domain walls must eventually decay. Several mechanisms have been discussed for the decay of axion domain walls [11,19] and the time scales for these decays are much larger than  $\Lambda_{\text{QCD}}^{-1}$ , i.e., long enough to generate these fields but short enough to avoid cosmological problems. QCD domain walls [6] are quasistable and may nicely solve this problem. We shall present these in [6]. We assume that some mechanism exists to resolve the domain wall problem in an appropriate time scale. Thus, all the relevant time scales are of the order  $\Lambda_{\text{QCD}}^{-1}$  except for the lifetime of the walls and thus, although the discussed interactions will affect the quantitative results, they will not affect the mechanism or substantially change the order of the effects.

*Conclusion.*—We have shown that this mechanism can generate the magnetic fields (3) with large correlations, though galactic dynamos should still play an important amplification role. It seems that the crucial conditions for the dynamo to take place are fields  $B > 10^{-20}$  G with large (100 kpc) correlations. From (3) we see that we have a huge interval  $1 \leq \xi \Lambda_{\text{QCD}} \ll 10^{10}$  of  $\xi$  to seed these dynamos. Also, if  $\xi$  is small, then this mechanism may generate measurable extra-galactic fields.

We mention two new points that distinguish this mechanism from previous proposals [20]. First, the key nucleon is the neutron which generates the fields due to an anomalous dipole moment induced by the  $CP$  violating domain walls. The nucleons thus make the wall ferromagnetic, not diamagnetic as discussed in [17]. Second, the interaction between the domain walls and nucleons is substantial because of the QCD scale of the  $\eta'$  transition in the  $a_{\eta'}$  domain wall. There is no way that axion domain walls with scales  $\sim m_a^{-1}$  can efficiently align nucleons at a temperature  $T_{\text{QCD}}$ .

We should also note that the magnitudes of the fields generated by this mechanism are small enough to satisfy the constraints placed by nucleosynthesis and CMB distortions. Thus, domain walls at the QCD phase transition, in particular those described in [6], provide a nice

method of generating magnetic fields on 100 kpc correlations today (3).

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