Scaling Law of Resistance Fluctuations in Stationary Random Resistor Networks

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In a random resistor network we consider the simultaneous evolution of two competing random processes consisting in breaking and recovering the elementary resistors with probabilities W_D and W_R. The condition $W_R > W_D/(1 + W_D)$ leads to a stationary state, while in the opposite case, the broken resistor fraction reaches the percolation threshold p_c. We study the resistance noise of this system under stationary conditions by Monte Carlo simulations. The variance of resistance fluctuations $\langle \delta R^2 \rangle$ is found to follow a scaling law $|p - p_c|^{-\kappa_0}$ with $\kappa_0 = 5.5$. The proposed model relates quantitatively the defectiveness of a disordered media with its electrical and excess-noise characteristics.

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Random resistor networks (RRN) have proven to be a useful model of electronic transport through disordered media [1–13]. To this purpose, several percolation approaches have been applied to RRNs to investigate complex interactions where structural or electronic rearrangements modify the properties of the system under investigation. Examples include biological systems [2], electronic transport in composite materials [2,14], amorphous and crystalline semiconductors [2,15], or breakdown of electrical properties [2,14]. Recently, degradation toward failure has been addressed successfully with standard and biased percolation models [16,17].

In this Letter we study a stationary percolation regime of a RRN by focusing on the resistance noise properties. To this purpose, we consider a RRN where two competing processes, defect generation and defect recovery, take place randomly. These two processes result from spontaneous breaking, occurring with probability W_D, and recovery with probability W_R, of elemental network resistors. As such, they are modeled as two standard percolations which evolve in competition. In particular, the two competing mechanisms of breaking and recovery we have introduced can have several physical mechanisms as counterpart. Some illustrative examples are the following: (i) the productions of voids and their healing due to the accumulation of mechanical stress in electromigration phenomena [18–20]; (ii) generation-recombination models of carriers between bands and/or from band to localized states in semiconductors [15,21–24]; (iii) carrier-number fluctuations produced by tunneling or hopping conduction in composite materials [13,14]; (iv) charge trapping and detrapping involved in soft dielectric breakdown of ultrathin dielectrics [18,20,25]. Here we limit our study to the intrinsic property of the RRN. Accordingly, the applied current plays no other role besides that of enabling us to evaluate the network resistance within linear response theory. Monte Carlo (MC) simulations are performed to explore the network evolution as a function of the two model parameters: the probabilities W_D and W_R. In the presence of recovery two possible asymptotic evolutions are expected, namely, (i) failure and (ii) steady state. The case of failure, by leading to divergence of the network resistance, is appropriate to study the electrical breakdown of thin films [1,2,14,16,17,26]. The case of steady state, by leading to fluctuations of the network resistance, is proposed as a new percolative approach able to model resistance fluctuations associated with structural defects [5–26].

The RRN consists of a two-dimensional square-lattice network of resistors of resistance r_α. We take a square geometry $N \times N$ where N determines the linear sizes of the lattice and $N_{\rm tot} = 2N^2$ is the total number of resistors. In practical applications, concerning the study of the electrical noise of thin films, the value of N can be related to the ratio between the size of the sample and that of the characteristic grain. The electrical contacts are realized by perfectly conducting bars at the left- and right-hand sides of the network, through which a constant current I is applied. The network resistance is thus given by [1]

$$R = \frac{1}{I^2} \sum i_\alpha^2 r_\alpha , \tag{1}$$

where i_α is the current flowing in the r_α resistor and the sum is extended to $N_{\rm tot}$. Starting from a perfect network, where all the resistors have the same resistance r_0, we create defects by changing the resistances r_α to the value $r_D = 10^9 r_0$, according to the probability W_D. Defects are then recovered with probability W_R. The sequence, defect creation and defect recovery, is then iterated, until a steady state or the percolation threshold is reached, as specified later. The ensemble average value of defects at the nth iteration, $\bar{p}_n$, is found to be given by

$$\bar{p}_n = W_D(1 - W_R) \frac{1 - [(1 - W_D)(1 - W_R)]^n}{1 - (1 - W_D)(1 - W_R)} . \tag{2}$$

We notice that if $\lim_{n\to\infty} \bar{p}_n = p_c$ the connectivity of the network is broken and R diverges, while if $\lim_{n\to\infty} \bar{p}_n < p_c$ a steady state is reached. As here we will essentially focus on the steady state, for the sake of brevity we denote

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$\lim_{n\to\infty}\bar{p}_n$ simply by p, thus

$$\lim_{n\to\infty}\bar{p}_n \equiv p = \frac{W_D(1 - W_R)}{W_R + W_D(1 - W_R)}, \tag{3}$$

where p also gives the average value of p_n under stationary conditions. Since in a two-dimensional square-lattice [1] $p_c = 0.5$, from Eq. (3), stationarity implies

$$W_R > W_D/(1 + W_D). \tag{4}$$

It is convenient to define x as the steady state value of the ratio between the numbers of broken and unbroken resistors, and x thus represents an effective breaking probability which determines p:

$$x \equiv \frac{p}{1 - p} = \frac{W_D(1 - W_R)}{W_R}, \tag{5}$$

which implies $p = x/(1 + x)$.

To investigate the intrinsic resistance noise, we carried out MC simulations using networks with linear size N up to 120. Starting from the perfect lattice with resistance $R_0 = r_0[N/(N + 1)]$ (step zero), defects are generated with probability W_D and recovered with probability W_R. By solving Kirchhoff loop equations the currents i_α and the resistance R are calculated (step 1). This procedure is iterated until the two following possibilities are achieved: (i) defect percolation threshold or (ii) steady state condition (in this case the iteration runs long enough for correlation and fluctuation analyis to be carried out). The network evolution as a function of the number of iterations can be associated with a time evolution, once that time is measured in units of iteration step. If not stated otherwise, for the simulations we used the following parameter values: $N = 75$, $r_0 = 1$ (Ω), $I = 0.1$ (A), $W_D = 3.35 \times 10^{-4}$ while several values of W_R are considered.

Figure 1 reports p_n versus the iteration step (time). For decreasing values of W_R the RRN is found to evolve from a steady state toward failure. Figure 2 reports p as a function of the effective breaking probability x. We can see that at increasing values of x the fraction of defect corresponding to the steady state increases as predicted by Eq. (3). The excellent agreement found between the simulations reported in Figs. 1 and 2 and the theory given in Eqs. (2) and (3) is taken as an internal check of consistency.

For the case of standard percolation, the average resistance of a sufficiently large RRN is related to the fraction of broken resistors by the well known scaling relation: $\langle R\rangle \sim (p_c - p)^{-\mu}$ where the universal exponent $\mu = 1.303$ is known from very accurate calculations [1,3]. Because of the superposition of two opposite processes, each of them obeying standard percolation, this scaling relation should hold also for the steady state of a RRN. In this case, it relates the average network resistance $\langle R\rangle$ with the average fraction of defects. Figure 3 reports $\langle R\rangle$ as a function of $(p_c - p)$ obtained from the simulations. The results are well fitted by the expected scaling law, and the estimate of the critical exponent $\mu = 1.2 \pm 0.1$ we have found is in satisfactory agreement with the value reported in literature [1–3].

We have then calculated the variance of resistance fluctuations under steady state conditions, $\langle \delta R^2\rangle = \langle R^2\rangle - \langle R\rangle^2$, as a function of the average fraction of defects. Figure 4 reports $\langle \delta R^2\rangle/R_0^2$ as a function of $(p_c - p)$ while the inset shows the same quantity as a function of p in the region $p \ll p_c$. The figure shows the existence of two regimes separated by the condition $\langle \delta R^2\rangle/R_0^2 = 1/(2N^2)$: a nearly perfect network regime occurring when $\langle \delta R^2\rangle/R_0^2 < 1/(2N^2)$, and a disordered network regime in the opposite case. In the first regime, the resistance noise is proportional to the fraction of defects. This regime agrees with the prediction of the analogous generation-recombination model for a single trap electrical noise in homogeneous

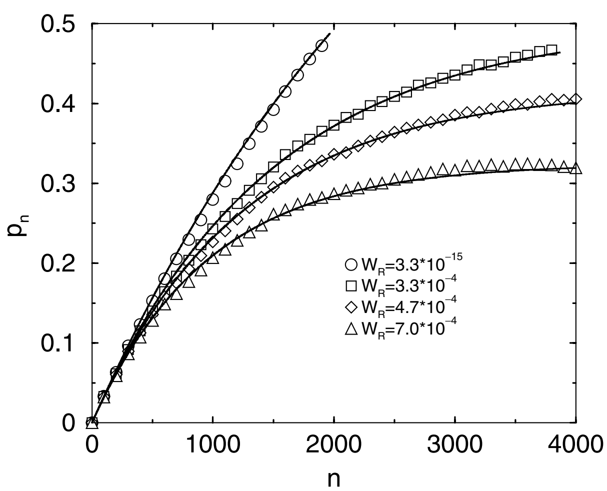

FIG. 1. Evolution of the fraction of broken resistors p_n as a function of the iteration step n. Symbols refer to MC simulations for a single realization and continuous lines to the ensemble average values obtained from Eq. (2).

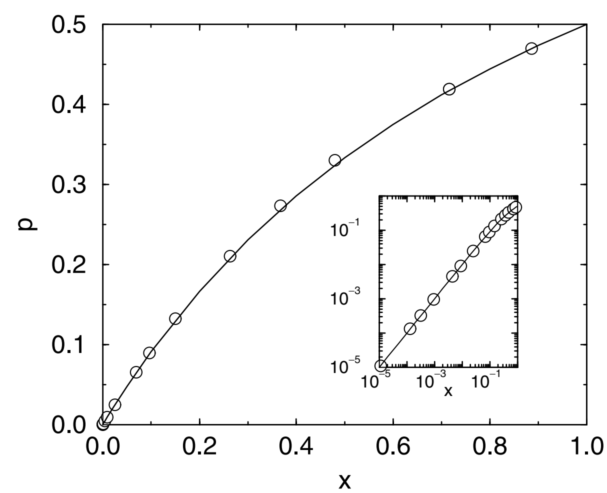

FIG. 2. Average fraction of broken resistors p as a function of x. Symbols refer to MC simulations and continuous lines to theory, respectively. The inset shows the same quantity comparing the theory to simulations at small values of x.

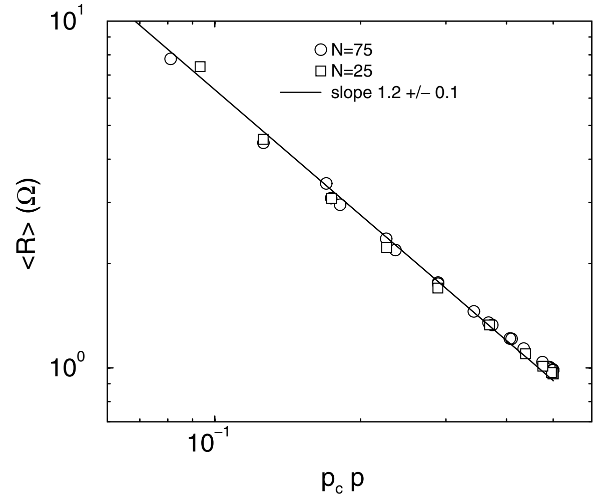

FIG. 3. Average resistance $\langle R \rangle$ as a function of $p_c - p$.

semiconductors [27]. In the disordered network regime, the breaking (and the recovering) of the backbone resistors results in an enhancement of resistance fluctuations. We note that the first regime disappears in the limit of an infinitely large network (i.e., when $N \rightarrow \infty$), thus playing the role of a finite size effect. In the second regime the data follow closely the scaling relation

$$\frac{\langle \delta R^2 \rangle}{R_0^2} \sim (p_c - p)^{-\kappa_0}, \tag{6}$$

where for the exponent κ_0 we have found $\kappa_0 = 5.5$ (see Fig. 4). For the sake of completeness, Fig. 5 reports the steady state resistance noise normalized to the square of the average RRN resistance, $\langle \delta R^2 \rangle / \langle R \rangle^2$, as a function of $(p_c - p)$. The behavior is similar to that shown in Fig. 4. Again, in the disordered network regime,

$$\frac{\langle \delta R^2 \rangle}{\langle R \rangle^2} \sim (p_c - p)^{-\kappa}, \tag{7}$$

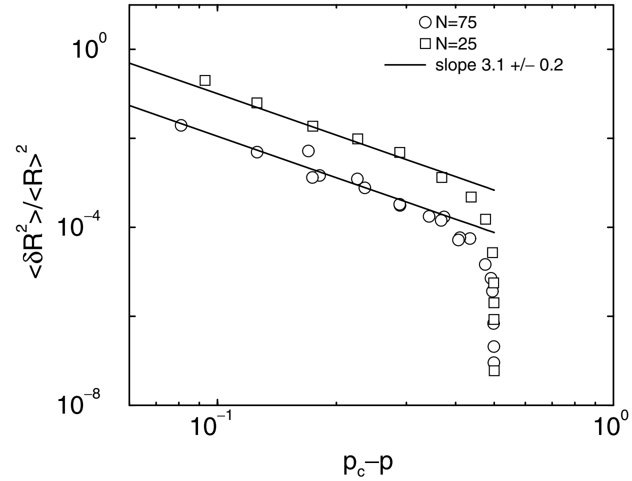

FIG. 5. Resistance noise normalized to the average network resistance $\langle \delta R^2 \rangle / \langle R \rangle^2$ as a function of $p_c - p$.

where $\kappa = 3.1$, consistently with $\kappa = \kappa_0 - 2\mu$. The value of the κ exponent is thus significantly higher than the value reported in literature for the exponent $\kappa_f = 1.12$ associated with flicker $(1/f)$ resistance noise in RRNs [5].

By combining Eq. (7) with $R \sim (p_c - p)^{-\mu}$, we obtain, in the disordered network regime,

$$\frac{\langle \delta R^2 \rangle}{\langle R \rangle^2} \sim \langle R \rangle^{-s} \tag{8}$$

with $s = 2.6$ as reported in Fig. 6.

We have then investigated the dependence on p of the correlation time τ (in units of iteration steps) of resistance fluctuations. For this purpose we have calculated the correlation function of resistance fluctuations and found an exponential decay with time (Lorentzian shape in frequency) as expected by analogy with the two level generation-recombination model in semiconductors [27]. By analogy

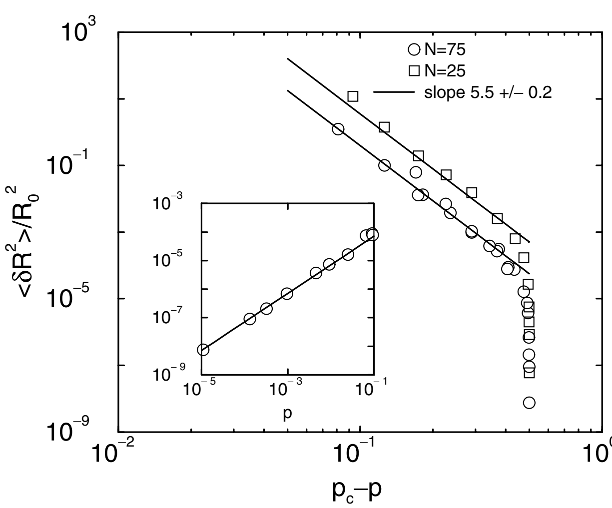

FIG. 4. Resistance noise normalized to the perfect network resistance $\langle \delta R^2 \rangle / R_0^2$ as a function of $p_c - p$. The inset shows the same quantity as a function of p when $p \ll p_c$.

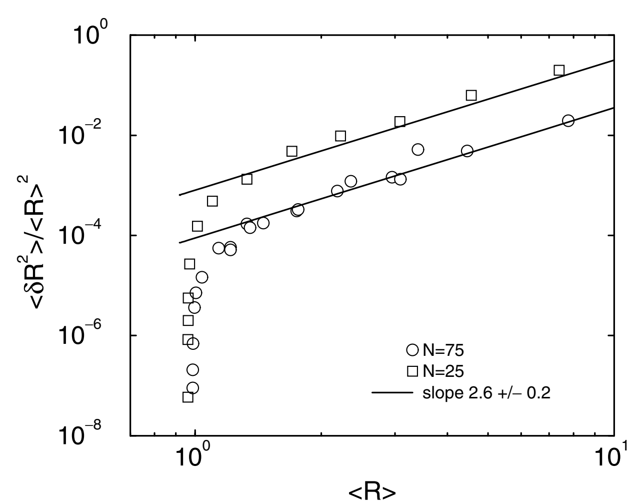

FIG. 6. Resistance noise normalized to the square of the average RRN resistance $\langle \delta R^2 \rangle / \langle R \rangle^2$ as a function of the average resistance $\langle R \rangle$.

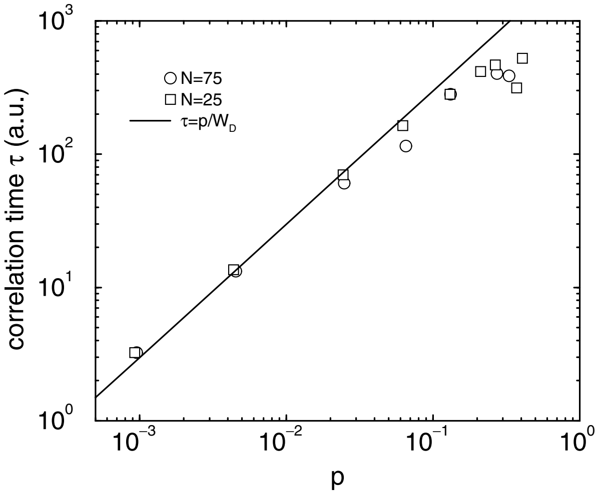

FIG. 7. Correlation time of resistance fluctuations τ as a function of the average fraction of broken resistors p. Symbols refer to MC simulations, continuous line to the values obtained from Eq. (9).

with the same two level model, τ takes the expression

$$\frac{1}{\tau} = W_D + \frac{W_R}{(1 - W_R)} = \frac{W_D}{p}. \qquad (9)$$

Figure 7 reports τ as a function of p. Symbols refer to MC simulations, while the continuous line refers to the expression in Eq. (9). The agreement between the predictions of Eq. (9) and numerical results is excellent in a wide range of decades, while theory tends to overestimate the simulations near $p = p_c$, where finite size effects can be the source of the discrepancy. The overall agreement is considered to be satisfactory, thus validating the theoretical interpretation of the simulations. In concluding, we have studied a resistive network in which two competing random processes are present: breaking and recovering of the elemental resistors. Depending on the probabilities of the two processes, W_D and W_R, failure (associated with the percolation threshold of broken resistors) or steady state of the RRN is reached. In the case of failure, i.e., $W_R < W_D/(1 + W_D)$, the average time to failure (ATTF) can be easily obtained by using Eq. (2), thus giving

$$\text{ATTF} = \frac{\ln[\frac{1}{2}(1 - \frac{W_R}{W_D(1-W_R)})]}{\ln[(1 - W_D)(1 - W_R)]}. \qquad (10)$$

In the case of steady state, i.e., $W_R > W_D/(1 + W_D)$, we have studied the resistance noise as a function of the average fraction of defects. We have found the existence of two regimes: a nearly perfect network regime, corresponding to an homogeneous conducting system, and a disordered network regime, corresponding to a system near the percolation threshold. Remarkably, the first regime disappears in the limit of networks with an infinite large size. The disordered regime is found to exhibit a scaling relation for the relative variance of resistance fluctuations $\langle \delta R^2 \rangle / \langle R \rangle^2 \sim (p_c - p)^{-\kappa}$ with $\kappa = 3.1$. These results confirm that noise is much more sensitive than resistance in probing the dynamical defectiveness of a sample [5–13,15,21–26]. The stationary RRN represents an attractive physical model to investigate excess noise associated with resistance fluctuations.

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