CPT-Odd Resonances in Neutrino Oscillations

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We consider the consequences for future neutrino factory experiments of small *CPT*-odd interactions in neutrino oscillations. The $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ survival probabilities at a baseline $L = 732$ km can test for *CPT*-odd contributions at orders of magnitude better sensitivity than present neutrino sector limits. Interference between the *CPT*-violating interaction and *CPT*-even mass terms in the Lagrangian can lead to a resonant enhancement of the oscillation amplitude. For oscillations in matter, a simultaneous enhancement of both neutrino and antineutrino oscillation amplitudes is possible.

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I. Introduction.—The discrete symmetries C, P, and T have fundamental importance in elementary particle theory. Violations of C, P, CP, and T by the weak interactions have all been observed [1]. *CPT* invariance is a basic property of local quantum field theory [2] and no evidence of deviations from *CPT* invariance has been found so far. The most stringent limits on *CPT* violation are obtained from the difference between the K^0 and $\bar{K}^0$ masses [3],

$$m_K - m_{\bar{K}} < 0.44 \times 10^{-18}\ \mathrm{GeV}, \tag{1}$$

and from measurements [4] of frequency variations of atomic clocks that undergo orientation changes, which give limits of 10^{-27} and 10^{-31} GeV, respectively, for the proton and neutron. In string theory, the *CPT* invariance may not be manifest due to the extended nature of strings [5–7]. Mechanisms by which string theories could spontaneously break *CPT* have been formulated [5–7]. The search for *CPT* violation is thus of considerable theoretical interest as a means of searching for purely string effects. Neutrino oscillations have been considered as phenomena that could probe *CPT* nonconservation [8]. With growing interest in the construction of neutrino factories to make high-precision measurements of neutrino mass-squared differences and of the *CP*-violating phase in the neutrino sector [9], it is appropriate to undertake a more extensive study of the ability to measure *CPT*-violating effects in neutrino oscillations. We find that a comparison of $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ oscillation probabilities at neutrino factories would give precision tests of *CPT*. A significant result of our study, as reported below, is that *CPT*-violating resonance effects can occur that can magnify small *CPT* violation into a measurable oscillation amplitude.

II. Basic formalism.—Consequences of CP, T, and *CPT* violation for neutrino oscillations have been written previously [10]. We summarize them briefly for the $\nu_\alpha \to \nu_\beta$ flavor oscillation probabilities $P_{\alpha\beta}$ at a distance L from the source. If

$$P_{\alpha\beta}(L) \neq P_{\bar{\alpha}\bar{\beta}}(L), \qquad \beta \neq \alpha, \tag{2}$$

then CP is not conserved. If

$$P_{\alpha\beta}(L) \neq P_{\beta\alpha}(L), \qquad \beta \neq \alpha, \tag{3}$$

then T invariance is violated. If

$$P_{\alpha\beta}(L) \neq P_{\bar{\beta}\bar{\alpha}}(L), \qquad \beta \neq \alpha, \tag{4}$$

or

$$P_{\alpha\alpha}(L) \neq P_{\bar{\alpha}\bar{\alpha}}(L), \tag{5}$$

then *CPT* is violated. When neutrinos propagate in matter, matter effects give rise to apparent *CP and CPT* violation even if the mass matrix is *CP* conserving.

CPT-violating terms that are also Lorentz-invariance violating (LV) have been discussed by Colladay and Kostelecky [6] for Dirac particles and by Coleman and Glashow [8] for Majorana neutrinos. Our analysis applies to either Majorana or Dirac neutrinos. The effective LV *CPT*-violating interaction for neutrinos is of the form

$$\bar{\nu}_L^\alpha b_{\alpha\beta}^\mu \gamma_\mu \nu_L^\beta, \tag{6}$$

where α and β are flavor indices. We assume rotational invariance in the "preferred" frame, in which the cosmic microwave background radiation is isotropic (following Coleman and Glashow [8]). (An experimental limit on *CPT*-violating interactions of the electron has been obtained [11] in studies of torques on a spin polarized torsion pendulum. The bound on b^3 of 10^{-29} GeV translates into a bound on b^0 for electrons of 5×10^{-25} GeV; if $SU(2)_L$ symmetry holds, a similar bound is implied on b_{ee}^0 in Eq. (6), but there are no similar bounds on other elements of b^0 for neutrinos. Following the suggestions in Ref. [12], bounds on space components of b_{ee} at levels of 10^{-29} have also been obtained; see, e.g., [13]. Existing data on muonium ground state hyperfine structure and the muon anomalous magnetic moment could be analyzed to probe $b_{\mu\mu}^0$ at levels of 10^{-22}–10^{-25} GeV [14].) The energies of ultrarelativistic neutrinos with definite momentum p are eigenvalues of the matrix

$$m^2/2p + b^0, \tag{7}$$

where b^0 is a Hermitian matrix, hereafter labeled b; for antineutrinos, $b \to -b$, and for Majorana neutrinos b is symmetric. For string theories, the expected size of the CPT violation [7] is E^2/m_S, where E is a typical energy for the system and m_S is the string scale. If m_S is the Planck mass, $m_{\rm Pl}$, and since the effective neutrino mixing matrix involves mixing of both the charged and neutral leptons, one might expect b as large as $m_\tau^2/m_{\rm Pl} \sim 2.5 \times 10^{-19}$ GeV for terms involving ν_τ or $m_\mu^2/m_P \sim 10^{-22}$ GeV for terms involving ν_μ. If m_S is about 1 TeV and $E = m_\nu$, then, for $m_\nu = 1$ eV, one gets b of order 10^{-21} GeV.

In the two-flavor case the neutrino phases may be chosen such that b is real, in which case the interaction in Eq. (6) is CPT odd. The survival probabilities for flavors α and $\bar\alpha$ produced at $t = 0$ are given by [8]

$$P_{\alpha\alpha}(L) = 1 - \sin^2 2\Theta \sin^2(\Delta L/4)\,, \qquad (8)$$

and

$$P_{\bar\alpha\bar\alpha}(L) = 1 - \sin^2 2\bar\Theta \sin^2(\bar\Delta L/4)\,, \qquad (9)$$

where

$$\Delta \sin 2\Theta = |(\delta m^2/E)\sin 2\theta_m + 2\delta b e^{i\eta} \sin 2\theta_b|\,, \qquad (10)$$

$$\Delta \cos 2\Theta = (\delta m^2/E)\cos 2\theta_m + 2\delta b \cos 2\theta_b\,. \qquad (11)$$

$\bar\Delta$ and $\bar\Theta$ are defined by similar equations with $\delta b \to -\delta b$. Here θ_m and θ_b define the rotation angles that diagonalize m^2 and b, respectively; $\delta m^2 = m_2^2 - m_1^2$ and $\delta b = b_2 - b_1$, where m_i^2 and b_i are the respective eigenvalues. We use the convention that $\cos 2\theta_m$ and $\cos 2\theta_b$ are positive and that δm^2 and δb can have either sign. The phase η in Eq. (10) is the difference of the phases in the unitary matrices that diagonalize δm^2 and δb; only one of these two phases can be absorbed by a redefinition of the neutrino states.

Observable CPT violation in the two-flavor case is a consequence of the interference of the δm^2 terms (which are CPT even) and the LV terms in Eq. (6) (which are CPT odd); if $\delta m^2 = 0$ or $\delta b = 0$, then there is no observable CPT-violating effect in neutrino oscillations. If $\delta m^2/E \gg 2\delta b$ then $\Theta \simeq \theta_m$ and $\Delta \simeq \delta m^2/E$, whereas if $\delta m^2/E \ll 2\delta b$ then $\Theta \simeq \theta_b$ and $\Delta \simeq 2\delta b$. Hence the effective mixing angle and oscillation wavelength can vary dramatically with E for appropriate values of δb.

There are five parameters in Eqs. (8)–(11) (δm^2, θ_m, δb, θ_b, and η) that can be determined by measuring $P_{\alpha\alpha}$ and $P_{\bar\alpha\bar\alpha}$ at different L and E values. A practical way to do this is to measure the energy dependence of $P_{\alpha\alpha}$ and $P_{\bar\alpha\bar\alpha}$ at one L value. Probability conservation in the two-neutrino case implies $P_{\alpha\beta} = 1 - P_{\alpha\alpha}$ and $P_{\bar\alpha\bar\beta} = 1 - P_{\bar\alpha\bar\alpha}$ for $\alpha \neq \beta$, so the parameters could also be determined by measuring off-diagonal neutrino and antineutrino channels.

An approximate direct limit on δb when $\alpha = \mu$ can be obtained by noting that in atmospheric neutrino data the flux of downward-going ν_μ is not depleted [15]. Hence, the oscillation arguments in Eqs. (8) and (9) cannot have fully developed for downward neutrinos. Taking $|\delta b L/2| < \pi/2$ with $L \sim 20$ km for downward events leads to the upper bound $|\delta b_{\mu\tau}| < 3 \times 10^{-20}$ GeV for maximal θ_b mixing. For upward-going atmospheric neutrino events with $L \sim 10^4$ km, the apparent lack of $\nu_\mu \leftrightarrow \nu_e$ oscillations gives a corresponding limit of $|\delta b_{\mu e}| < 5 \times 10^{-23}$ GeV. Reactor neutrino experiments that indicate no $\bar\nu_e \to \bar\nu_e$ oscillations at $L \sim 1$ km give a corresponding bound of $|\delta b_{e\tau}| < 5 \times 10^{-19}$ GeV. Since the CPT-odd oscillation argument depends on L and the ordinary oscillation argument on L/E, more precise direct limits could be obtained by a dedicated study of the energy and zenith angle dependence of the atmospheric neutrino data. The K2K, MINOS, ICANOE, and OPERA experiments with $L = 250$–730 km can improve the limits on $|\delta b_{\mu\tau}|$ by at least 1 order of magnitude, and the Kamland experiment with $L \sim 100$–1000 km can improve the limits on $|\delta b_{e\tau}|$ by about 2 orders of magnitude.

We note that a CPT-odd resonance for neutrinos ($\sin^2 2\Theta = 1$) occurs whenever $\cos 2\Theta = 0$ or

$$(\delta m^2/E)\cos 2\theta_m + 2\delta b \cos 2\theta_b = 0\,, \qquad (12)$$

similar to the resonance due to matter effects [16,17]. The condition for antineutrinos is the same, except δb is replaced by $-\delta b$. The resonance occurs for neutrinos if δm^2 and δb have the opposite sign, and for antineutrinos if they have the same sign. A resonance can occur even when θ_m and θ_b are both small, and for all values of η; if $\theta_m = \theta_b$, a resonance can occur only if $\eta \neq 0$.

If one of ν_α or ν_β is ν_e, then the neutrino propagation is modified in the presence of matter. Then Eq. (11) becomes

$$\Delta \cos 2\Theta = (\delta m^2/E)\cos 2\theta_m + 2\delta b \cos 2\theta_b - 2\sqrt{2}\, G_F N_e \qquad (13)$$

for neutrinos, where N_e is the number density of electrons in matter. For antineutrinos, $\delta b \to -\delta b$ and $N_e \to -N_e$ in Eq. (13).

III. Examples of CPT-violation and CPT-odd resonances.—Hereafter, for simplicity, we assume that m^2 and b are diagonalized by the same angle θ, i.e., $\theta_m = \theta_b \equiv \theta$.

(A) $\eta = 0$: For $\eta = 0$ we have

$$\Theta = \theta\,, \qquad (14)$$

$$\Delta = (\delta m^2/E) + 2\delta b\,. \qquad (15)$$

For $\theta_m = \theta_b$, $\eta = 0$, a resonance is not possible. The oscillation probabilities become

$$P_{\alpha\alpha}(L) = 1 - \sin^2 2\theta \sin^2\left\{\left(\frac{\delta m^2}{4E} + \frac{\delta b}{2}\right)L\right\}, \qquad (16)$$

$$P_{\bar\alpha\bar\alpha}(L) = 1 - \sin^2 2\theta \sin^2\left\{\left(\frac{\delta m^2}{4E} - \frac{\delta b}{2}\right)L\right\}. \qquad (17)$$

For fixed E, the δb terms act as a phase shift in the oscillation argument; for fixed L, the δb terms act as a modification of the oscillation wavelength.

The difference between $P_{\alpha\alpha}$ and $P_{\bar{\alpha}\bar{\alpha}}$,

$$P_{\alpha\alpha}(L) - P_{\bar{\alpha}\bar{\alpha}}(L) = -2\sin^2 2\theta \sin\left(\frac{\delta m^2 L}{2E}\right)\sin(\delta b L), \tag{18}$$

can be used to test for CPT violation. In a neutrino factory, the ratio of $\bar{\nu}_\mu \to \bar{\nu}_\mu$ to $\nu_\mu \to \nu_\mu$ events will differ from the standard model (or any local quantum field theory model) value if CPT is violated. Figure 1 shows the event ratios $N(\bar{\nu}_\mu \to \bar{\nu}_\mu)/N(\nu_\mu \to \nu_\mu)$ versus δb for a neutrino factory with 10^{19} stored muons and a 10 kton detector at several values of stored muon energy, assuming $\delta m^2 = 3.5 \times 10^{-3}$ eV^2 and $\sin^2 2\theta = 1.0$, as indicated by the atmospheric neutrino data [15]. The error bars in Fig. 1 are representative statistical uncertainties. The node near $\delta b = 8 \times 10^{-22}$ GeV is a consequence of the fact that $P_{\alpha\alpha} = P_{\bar{\alpha}\bar{\alpha}}$, independent of E, whenever $\delta b L = n\pi$, where n is any integer; the node in Fig. 1 is for $n = 1$. A 3σ CPT-violation effect is possible in such an experiment for δb as low as 3×10^{-23} GeV for stored muon energies of 20 GeV. Although matter effects also induce an apparent CPT-violating effect, the dominant oscillation here is $\nu_\mu \to \nu_\tau$, which has no matter corrections in the two-neutrino limit and very small matter corrections for the three-neutrino case. Fundamental CPT-violating effects via δb can be disentangled from matter effects at short baselines, where matter effects are small and calculable.

We have also checked the observability of CPT violation at other distances, assuming the same neutrino factory parameters used above. For $L = 250$ km, the $\delta b L$ oscillation argument in Eq. (18) has not fully developed and the ratio of $\bar{\nu}$ to ν events is still relatively close to the standard model value. For $L = 2900$ km, a δb as low as 10^{-23} GeV may be observable at the 3σ level. However, at longer distances, matter effects may simulate CPT violation.

(B) $\eta = \pi/2$: For $\eta = \pi/2$ we have

$$P_{\alpha\alpha} = 1 - \sin^2 2\Theta \sin^2\{\Delta L/4\}, \tag{19}$$

$$P_{\bar{\alpha}\bar{\alpha}} = 1 - \sin^2 2\bar{\Theta} \sin^2\{\bar{\Delta} L/4\}, \tag{20}$$

where

$$\tan 2\Theta = \frac{\sqrt{(\delta m^2/E)^2 + (2\delta b)^2}}{(\delta m^2/E) + 2\delta b}\tan 2\theta, \tag{21}$$

$$\Delta^2 = [(\delta m^2/E) + 2\delta b]^2 - 4(\delta m^2/E)\delta b \sin^2 2\theta, \tag{22}$$

and $\bar{\Theta}$ and $\bar{\Delta}$ are defined similarly with $\delta b \to -\delta b$. Here the resonance condition for neutrinos is $\delta m^2/E + 2\delta b = 0$. Figure 2 shows the effective oscillation amplitude $\sin^2 2\Theta$ and oscillation argument Δ versus δb with $\delta m^2 = 3.5 \times 10^{-3}$ eV^2 and $\sin^2 2\theta = 0.1$ (which may be appropriate for $\nu_e \to \nu_e$ oscillations) for several values of neutrino energy. Although the above example assumed $\eta = \pi/2$, such a resonance can occur in this $\theta_b = \theta_m$ example for any value of η in the open interval $(0, 2\pi)$.

(C) CPT-odd term with matter: In the presence of matter, the effective ν_e oscillation amplitude and argument are defined by Eqs. (10) and (13). Again, assuming

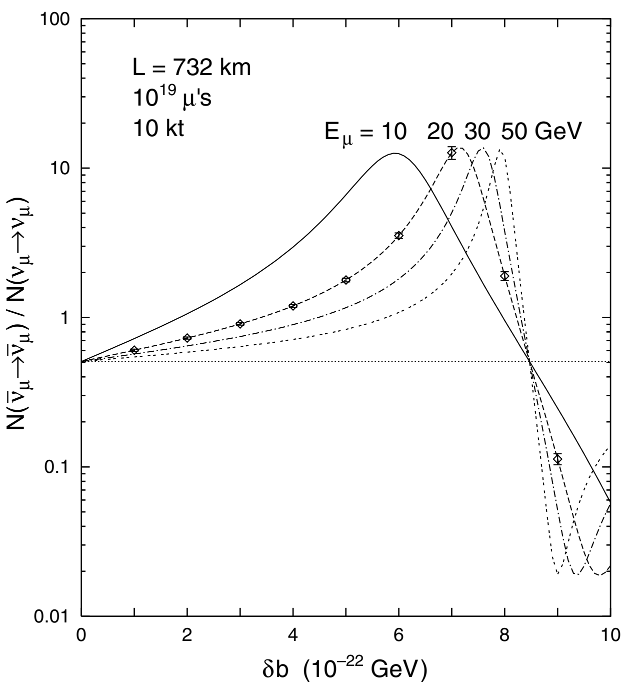

FIG. 1. The ratio of $\bar{\nu}_\mu \to \bar{\nu}_\mu$ to $\nu_\mu \to \nu_\mu$ event rates in a 10 kton detector for a neutrino factory with 10^{19} stored muon with energies $E_\mu = 10$, 20, 30, and 50 GeV for baseline $L = 732$ km versus the CPT-odd parameter δb with $\theta_m = \theta_b \equiv \theta$ and phase $\eta = 0$. The neutrino mass and mixing parameters are $\delta m^2 = 3.5 \times 10^{-3}$ eV^2 and $\sin^2 2\theta = 1.0$. The dotted line indicates the result for $\delta b = 0$, which is given by the ratio of the $\bar{\nu}$ and ν charge-current cross sections. The error bars are representative statistical uncertainties. The ratio does not approach unity in the limit $\delta b \to 0$ because of the different neutrino and antineutrino cross sections.

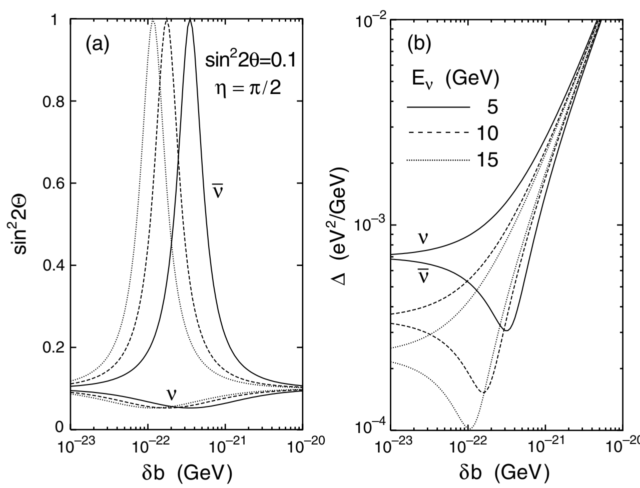

FIG. 2. Resonance effects in $\nu \to \nu$ and $\bar{\nu} \to \bar{\nu}$ oscillations shown versus CPT-odd parameter δb for various values of neutrino energy E with $\delta m^2 = 3.5 \times 10^{-3}$ eV^2, $\sin^2 2\theta = 0.1$, and phase $\eta = \pi/2$: (a) oscillation amplitude $\sin^2 2\Theta$ in Eq. (21) and (b) oscillation argument Δ in Eq. (22).

$\theta_b = \theta_m \equiv \theta$ and $\eta = 0$, we have

$$\tan 2\Theta = \frac{[(\delta m^2/E) + 2\delta b]\sin 2\theta}{[(\delta m^2/E) + 2\delta b]\cos 2\theta - 2\sqrt{2}\, G_F N_e}, \tag{23}$$

$$\Delta^2 = \{[(\delta m^2/E) + 2\delta b]\cos 2\theta - 2\sqrt{2}\, G_F N_e\}^2 + [(\delta m^2/E) + 2\delta b]^2 \sin^2 2\theta \tag{24}$$

for neutrinos, with $\delta b \to -\delta b$ and $N_e \to -N_e$ for antineutrinos. Thus a resonance ($\sin^2 2\Theta = 1$) occurs for neutrinos when $[(\delta m^2/E) + 2\delta b]\cos 2\theta = 2\sqrt{2}\, G_F N_e$, and for antineutrinos when $[(\delta m^2/E) - 2\delta b]\cos 2\theta = -2\sqrt{2}\, G_F N_e$. A resonance can occur simultaneously for neutrinos *and* antineutrinos only in the limit when $\delta m^2/E \ll 2\delta b$ and the CPT-odd effects dominate. However, it is possible to have an effective oscillation amplitude that is significantly enhanced for both neutrinos and antineutrinos even when $\delta m^2/E$ is not small compared to $2\delta b$. For $N_e = 1.67 N_A/\mathrm{cm}^3$ (the electron density appropriate for the upper mantle of the Earth) and vacuum amplitude $\sin^2 2\theta = 0.1$, the effective oscillation amplitudes $\sin^2 2\Theta$ for $\nu_e \to \nu_e$ and $\sin^2 2\bar{\Theta}$ for $\bar{\nu}_e \to \bar{\nu}_e$ can both be greater than 0.5 when δb and δm^2 satisfy both $0.0002\ \mathrm{eV}^2/\mathrm{GeV} < 2\delta b + (\delta m^2/E) < 0.0004\ \mathrm{eV}^2/\mathrm{GeV}$ and $0.0002\ \mathrm{eV}^2/\mathrm{GeV} < 2\delta b - (\delta m^2/E) < 0.0004\ \mathrm{eV}^2/\mathrm{GeV}$. These conditions are satisfied when $\delta b \simeq 1\text{–}2 \times 10^{-22}$ GeV and with $|\delta m^2/E|$ as large as $10^{-4}\ \mathrm{eV}^2/\mathrm{GeV}$. By assuming $\delta m^2 \simeq 3.5 \times 10^{-3}\ \mathrm{eV}^2$, such enhancements in $\nu_e \to \nu_e$ and $\bar{\nu}_e \to \bar{\nu}_e$ are possible for $E > 35$ GeV, provided that $\delta b > 0$. Although here we have only considered $\eta = 0$, similar enhancements are possible for any value of η since they rely on the denominator of Eq. (23) being small, which is independent of η.

IV. Summary.—We have shown that small CPT-odd interactions of neutrinos can have measurable consequences in neutrino oscillations. Resonant enhancements of the oscillation amplitude for either neutrinos or antineutrinos (but not both) are possible if the unitary matrices which diagonalize the neutrino mass term and the CPT-odd term are not the same. A resonance can occur for any relative phase between the CPT-even mass term and the CPT-odd interaction, but, if the rotation angles in the two sectors are the same, a resonance is possible only if the relative phase is not zero. In matter, significant enhancements are possible for both neutrinos and antineutrinos. Measurement of $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ oscillation probabilities in neutrino factories can place stringent limits on the CPT-odd interaction.

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[1] See, e.g., I.I. Bigi and A.I. Sanda, *CP Violation* (Cambridge University Press, Cambridge, UK, 2000); G.C. Branco, L. Lavoura, and J.P. Silva, *CP Violation* (Clarendon Press, Oxford, 1999), and references therein.

[2] See, e.g., S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, UK, 1996), p. 244, and references therein.

[3] Particle Data Group, C. Caso *et al.,* Eur. J. Phys. C **3**, 1 (1998).

[4] R.L. Walsworth *et al.,* Report No. physics/0007063.

[5] V.A. Kostelecky and R. Potting, Nucl. Phys. **B359**, 545 (1991); Phys. Rev. D **51**, 3923 (1995).

[6] D. Colladay and V.A. Kostelecky, Phys. Rev. D **55**, 6760 (1997); **58**, 116002 (1998).

[7] J. Ellis, J.L. Lopez, N.E. Mavromatos, and D.V. Nanopoulos, Phys. Rev. D **53**, 3846 (1996).

[8] S. Coleman and S.L. Glashow, Phys. Rev. D **59**, 116008 (1999).

[9] V. Barger, S. Geer, R. Raja, and K. Whisnant, hep-ph/0003184, 2000; A. De Rujula, M.B. Gavela, and P. Hernandez, Nucl. Phys. **B547**, 21 (1999); M. Campanelli, A. Bueno, and A. Rubbia, hep-ph/9905240; A. Cervera, A. Donini, M.B. Gavela, J.J. Gomez Cadenas, P. Hernandez, O. Mena, and S. Rigolin, hep-ph/0002108; I. Mocioiu and R. Shrock, hep-ph/9910554; hep-ph/0002149; M. Freund, M. Lindner, S.T. Petcov, and A. Romanino, hep-ph/9912457.

[10] N. Cabibbo, Phys. Lett. **72B**, 333 (1978); V. Barger, K. Whisnant, and R.J.N. Phillips, Phys. Rev. Lett. **45**, 2084 (1980); S. Pakvasa, in *High Energy Physics,* edited by L. Durand and L.G. Pondrom, AIP Conf. Proc. No. 68 (AIP, New York, 1981), Vol. 2, p. 1164.

[11] E.G. Adelberger *et al.* (unpublished), as quoted by N. Russell, R. Blum, and V.A. Kostelecky, Bull. Am. Phys. Soc. **45**, 140 (2000).

[12] R. Bluhm and V.A. Kostelecky, Phys. Rev. Lett. **84**, 1381 (2000).

[13] V.A. Kostelecky, in Proceedings of the International Conference on Orbis Scientiae 1999, Fort Lauderdale, FL 1999 (hep-ph/0005280).

[14] R. Bluhm, V.A. Kostelecky, and C. Lane, Phys. Rev. Lett. **84**, 1098 (2000).

[15] Kamiokande Collaboration, K.S. Hirata *et al.,* Phys. Lett. B **280**, 146 (1992); Y. Fukuda *et al.,* Phys. Lett. B **335**, 237 (1994); IMB Collaboration, R. Becker-Szendy *et al.,* Nucl. Phys. Proc. Suppl. B **38**, 331 (1995); Soudan-2 Collaboration, W.W.M. Allison *et al.,* Phys. Lett. B **391**, 491 (1997); MACRO Collaboration, M. Ambrosio *et al.,* Phys. Lett. B **434**, 451 (1998); Super-Kamiokande Collaboration, Y. Fukuda *et al.,* Phys. Rev. Lett. **81**, 1562 (1998); Phys. Rev. Lett. **82**, 2644 (1999).

[16] V. Barger, K. Whisnant, S. Pakvasa, and R.J.N. Phillips, Phys. Rev. D **22**, 2718 (1980).

[17] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S.P. Mikheyev and A. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1986)].