Quantum Phase Transitions in d-Wave Superconductors

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Motivated by the strong, low temperature damping of nodal quasiparticles observed in some cuprate superconductors, we study quantum phase transitions in $d_{x^2-y^2}$ superconductors with a spin-singlet, zero momentum, fermion bilinear order parameter. We present a complete, group-theoretic classification of such transitions into seven distinct cases (including cases with nematic order) and analyze fluctuations by the renormalization group. We find that only two, the transitions to $d_{x^2-y^2} + is$ and $d_{x^2-y^2} + id_{xy}$ pairing, possess stable fixed points with universal damping of nodal quasiparticles; the latter leaves the gapped quasiparticles along $(1,0)$, $(0,1)$ essentially undamped.

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Recent photoemission [1] and Thz conductivity [2] measurements on $Bi_2Sr_2CaCu_2O_{8+\delta}$, the cuprate superconductor, have indicated anomalously large inelastic scattering of fermionic quasiparticles near the gap nodes in the d-wave superconductor. While many scattering mechanisms and scenarios have been proposed [3–6] for the damping of quasiparticles along the $(1,0)$, $(0,1)$ directions (the "antinodal quasiparticles") above the superconducting critical temperature T_c, the possibilities below T_c at the nodal points are much more restricted and allow us to make sharp distinctions between competing theories. Standard BCS theory predicts a nodal scattering rate $\sim T^3$ from short-range interactions, and this is far too small to account for the observations. In this paper we study a possible explanation [7] due to the proximity to a quantum phase transition to some other superconducting state X (see Fig. 1). We show how global symmetry and field-theoretic considerations permit a classification of all possibilities for X, and we list those that may account for the experiments.

The nodal quasiparticles at the gap nodes have a momentum distribution curve (MDC) with a width proportional to $k_B T$ [1], and there is little change [8] in this behavior when tuning T through T_c. The antinodal quasiparticles are broad and ill-defined above T_c but narrow dramatically below T_c, forming long-lived states with an energy gap of 30–40 meV. A natural possibility, based on other experimental probes [9], is that these effects are due to their proximity to a quantum critical point to magnetic ordering. However, wave vector matching conditions appear to rule this out for the nodal quasiparticles: the magnetic fluctuations are strongest near wave vector $\mathbf{Q} = (\pi, \pi)$, and while they can strongly scatter antinodal quasiparticles above T_c, they do not connect low-energy quasiparticles near the nodes [8].

Rather than exploring the intricate details of the many experiments, this paper performs the following well-posed theoretical task: classify and describe theories in which a d-wave superconductor at [10] $T \ll T_c$ has, with minimal fine-tuning, (i) a nodal quasiparticle MDC with a width $\propto k_B T$ and possibly (ii) negligible scattering of the quasiparticles along $(1,0)$, $(0,1)$. We find that theories that satisfy (i) also have a high frequency tail [11] in the energy distribution curve (EDC) of the nodal quasiparticles, as is experimentally observed [1,5].

Strong scattering of the gapless nodal quasiparticles surely requires their coupling to some low-energy bosonic mode. It is convenient to imagine that we have at our disposal some parameter r (which is possibly the hole concentration δ, but not necessarily so) which we can tune to condense the bosonic mode, leading to a new superconducting state X for $r < r_c$ (Fig. 1). The quantum-critical region of the phase transition at $r = r_c$ and $T = 0$ will satisfy (i) provided the phase transition is below its upper critical dimension, and the nodal fermions are intrinsic (in a sense to be made precise below) degrees of freedom of the critical field theory [12]. Conversely, (ii) requires that the antinodal fermions are merely spectators of the phase transitions and are essentially decoupled from the critical degrees of freedom.

The most efficient scattering of nodal quasiparticles is provided by a linear, nonderivative coupling between the fermion bilinears and the order parameter; higher order and derivative couplings have been considered recently [7,13] and invariably lead [7] to quasiparticle scattering

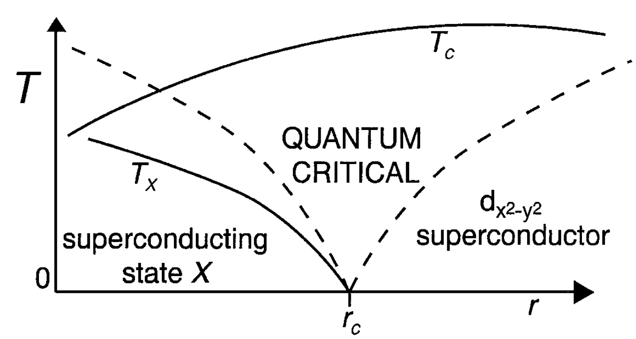

FIG. 1. Phase diagram adapted from Ref. [7]. Superconductivity is present for $T < T_c$. The long-range order associated with the state X vanishes for $T > T_X$, but fluctuations of this order provide anomalous damping of the nodal quasiparticles in the quantum-critical region.

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rates that vanish with superlinear powers of T. Order parameters which carry a net momentum $\mathbf{Q}$, will, by momentum conservation, couple linearly with the nodal fermions only if the spacing between two of the nodal points is exactly $\mathbf{Q}$. Transitions involving the onset of spin [14] or site/bond charge density waves [7] (stripes) do satisfy [7] (i) and (ii) for such values of $\mathbf{Q}$; however, the restriction on $\mathbf{Q}$ could be a fine-tuning condition and is not satisfied by the $\mathbf{Q}$ values observed so far. "Staggered-flux" order [6,13] has a derivative coupling to the nodal fermions, and $\mathbf{Q} = (\pi, \pi)$ which does not connect nodal points: so (i) is not satisfied. Indeed, only the value $\mathbf{Q} = 0$ naturally satisfies the constraints of momentum conservation, and so we limit our attention to order parameters at zero momentum. Furthermore, spin-triplet ordering at $\mathbf{Q} = 0$ implies ferromagnetic correlations which are unlikely to be present, and therefore we further restrict ourselves to spin-singlet fermion bilinears. This means that our order parameter is a component of the complex superconducting pairing function $\Delta_{\mathbf{q}} = \langle c_{\mathbf{q}\uparrow} c_{-\mathbf{q}\downarrow} \rangle$, or the real excitonic (or "particle-hole") pairing function $A_{\mathbf{q}} = \langle c^{\dagger}_{\mathbf{q}a} c_{\mathbf{q}a} \rangle$ ($c_{\mathbf{q}a}$ annihilates an electron with momentum $\mathbf{q}$ and spin $a = \uparrow, \downarrow$). It is useful to decompose the functions $\Delta_{\mathbf{q}}$ and $A_{\mathbf{q}}$ into components which transform under one of the irreducible representations of the symmetry group of the Hamiltonian [15]: this is $C_{4v} \times Z_2$, where C_{4v} is the tetragonal point group (see Table I), and the Z_2 component represents time-reversal symmetry $\mathcal{T}$ (point group symmetry breaking has been considered recently [16,17], as have exciton condensations [13] at nonzero $\mathbf{Q}$). Generically, a second-order transition can occur only by condensation of an irreducible component (multiple components can appear in successive transitions), and this permits a complete classification of inequivalent order parameters. Note that $d_{x^2-y^2}$ pairing is already present for $r > r_c$ (see Fig. 1), and we assume that this ordering remains well formed across the transition; all our subsequent characterizations of possible orderings in state X refer to additional ordering beyond an implicitly assumed background of $d_{x^2-y^2}$ pairing. $A_{\mathbf{q}}$ is necessarily even under $\mathcal{T}$ and so can generate $s, p, \ldots$ exciton ordering; similarly $\Delta_{\mathbf{q}}$ can generate $s, p, \ldots$ pairing or $is, ip, \ldots$ pairing (the latter also break $\mathcal{T}$), leading to a total of 15 possible order parameters for X. Of these, s exciton ordering is equivalent to an innocuous shift in the chemical potential, while p and ip pairing are forbidden by Fermi statistics. Because of the background $d_{x^2-y^2}$ pairing, further $d_{x^2-y^2}$ or $id_{x^2-y^2}$ pairing is not a new ordering, while simple symmetry considerations (e.g., examination of the fermion dispersion relation in state X) show that g excitons, g pairing, and $d_{x^2-y^2}$ excitons are equivalent to d_{xy} pairing, d_{xy} excitons, and s pairing, respectively. Only seven inequivalent order parameters now remain and we discuss their properties shortly.

TABLE I. Character table of the irreducible representations of the group C_{4v}. The C_4 rotations are about the z axis, and the I $[I']$ are reflections about the $(1,0)$, $(0,1)$ $[(1,1),(1,-1)]$ directions; the basis functions are chosen to be invariant under translations by reciprocal lattice vectors.

	E	C_4^2	$2C_4$	$2I$	$2I'$	Basis functions
s	1	1	1	1	1	1
p	2	-2	0	0	0	$(\sin q_x, \sin q_y)$
$d_{x^2-y^2}$	1	1	-1	1	-1	$\cos q_x - \cos q_y$
d_{xy}	1	1	-1	-1	1	$\sin q_x \sin q_y$
g	1	1	1	-1	-1	$\sin q_x \sin q_y (\cos q_x - \cos q_y)$

We begin by reviewing the action for low-energy fermionic excitations in a d-wave superconductor. We denote the components of $c_{\mathbf{q}a}$ in the vicinity of the four nodal points $(\pm K, \pm K)$ ($K \approx 0.39\pi$ at optimal doping) by (anticlockwise) $f_{1a}, f_{2a}, f_{3a}, f_{4a}$, and introduce the four-component Nambu spinors $\Psi_1 = (f_{1a}, \varepsilon_{ab} f^{\dagger}_{3b})$ and $\Psi_2 = (f_{2a}, \varepsilon_{ab} f^{\dagger}_{4b})$ where $\varepsilon_{ab} = -\varepsilon_{ba}$ and $\varepsilon_{\uparrow\downarrow} = 1$. Expanding to linear order in gradients from the nodal points, we obtain

$$S_\Psi = \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_1^{\dagger}(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x)\Psi_1 + \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_2^{\dagger}(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x)\Psi_2 . \quad (1)$$

Here ω_n is a Matsubara frequency, τ^α are Pauli matrices which act in the fermionic particle-hole space, $k_{x,y}$ measure the wave vector from the nodal points and have been rotated by 45° from $q_{x,y}$ coordinates in Table I, and v_F, v_Δ are velocities.

We now describe the seven possible order parameters for state X, along with the respective actions for the quantum phase transition.

(A) *is pairing:* This has been considered in Ref. [7]. The state X (with $d_{x^2-y^2} + is$ pairing) has no gapless fermionic excitations, breaks $\mathcal{T}$, but all charge neutral observables (like the charge density or lattice displacements) retain the full C_{4v} symmetry. The order parameter transforms as a real, one-dimensional representation of $C_{4v} \times Z_2$ and so can be represented by a single, real field ϕ; this will also be true for (B)–(F) below, with only (G) requiring a doublet of real fields. On general symmetry grounds, following action for ϕ is obtained after integrating out high-energy fermion modes:

$$S_\phi = \int d^2x \, d\tau \left[\frac{1}{2} (\partial_\tau \phi)^2 + \frac{c^2}{2} (\nabla \phi)^2 + \frac{r}{2} \phi^2 + \frac{u}{24} \phi^4 \right]; \quad (2)$$

here τ is imaginary time, c is a velocity, r tunes the system across the quantum critical point, and u is a quartic self-interaction. By itself, S_ϕ would describe a critical point at $r = r_c$ in the universality class of the classical, three-dimensional Ising model. However, a coupling to the low-energy fermionic modes in (1) can preempt this conclusion [14]: its form can be deduced from the values of the basis function in Table I at the nodal points, and the information that the order parameter is in the particle-particle channel—

$$S_{\Psi\phi} = \int d^2x\, d\tau\, [\lambda\phi(\Psi_1^\dagger M_1 \Psi_1 + \Psi_2^\dagger M_2 \Psi_2)], \quad (3)$$

where λ is the required linear coupling constant between the order parameter and a fermion bilinear, and $M_1 = M_2 = \tau^y$.

(B) id_{xy} *pairing:* This is very similar to A, with the main change arising from the new basis function in Table I, which now implies $M_1 = -M_2 = \tau^y$.

(C) *ig pairing:* Also related to (A), but now the basis function in Table I vanishes at the nodal points. Consequently, the coupling between Ψ and ϕ requires at least one spatial derivative and is irrelevant [7]. The action S_ϕ in (2) is the entire critical theory of the transition, and the scattering of the nodal fermions is weak, arising only from irrelevant couplings, and violates (i).

(D) *s pairing:* $\mathcal{T}$ remains unbroken, but the symmetry of charge neutral observables is broken to C_{2v}, so that X (with $d_{x^2-y^2} + s$ pairing) is a superconducting nematic [16,17]. The nematic order is polarized along the $(1,0)$ or $(0,1)$ directions. For weak ordering, the state X retains gapless nodal fermionic excitations, but the nodal points are at $(\pm K', \pm K)$ with $K' \neq K$; for a sufficiently large s component, the nodal points disappear upon colliding in pairs as $\min(K', K) \to 0$, in a separate quantum critical point which is not of interest here. As in (A) and (B), coupling of the order parameter is described by (3), but with $M_1 = M_2 = \tau^x$.

(E) d_{xy} *excitons:* This is as in (D), but symmetry of charge neutral observables in X is broken to a different C_{2v} subgroup of C_{4v}, with the nematic now polarized along the diagonal $(1, \pm 1)$ directions. The nodal points in X are at $\pm(K, K)$ and $\pm(K', -K')$ with $K \neq K'$. In the action (3), we now have $M_1 = -M_2 = \tau^z$.

(F) d_{xy} *pairing:* Such an ordering in X moves the nodal points clockwise (or anticlockwise) from $(\pm K, \pm K)$, reducing the C_{4v} symmetry to C_4, while preserving $\mathcal{T}$. Again the action (3) describes the order parameter/fermion coupling, but with $M_1 = -M_2 = \tau^x$.

(G) *p excitons:* The order parameter transforms under a two-dimensional representation of C_{4v}, requiring a doublet of real fields, (ϕ_x, ϕ_y), to describe the low-energy bosonic modes. The state X retains $\mathcal{T}$ and the gapless nodal points, but has C_{4v} broken to Z_2. The action (2) is replaced by

$$\tilde{S}_\phi = \int d^2x\, d\tau \Big[\frac{1}{2}\{(\partial_\tau\phi_x)^2 + (\partial_\tau\phi_y)^2 + c_1^2(\partial_x\phi_x)^2 + c_2^2(\partial_y\phi_x)^2 + c_2^2(\partial_x\phi_y)^2 + c_1^2(\partial_y\phi_y)^2 + e(\partial_x\phi_x)(\partial_y\phi_y) + r(\phi_x^2 + \phi_y^2)\} + \frac{1}{24}\{u(\phi_x^4 + \phi_y^4) + 2v\phi_x^2\phi_y^2\}\Big], \quad (4)$$

while the coupling between $\phi_{x,y}$ and $\Psi_{1,2}$ is

$$\tilde{S}_{\Psi\phi} = \int d^2x\, d\tau\, [\lambda(\phi_x \Psi_1^\dagger \Psi_1 + \phi_y \Psi_2^\dagger \Psi_2)]. \quad (5)$$

We now make a few general remarks on the field theories above. Upon integrating out the fermion fields, we find a finite one-loop renormalization of the tuning parameter, r. This should be contrasted with the behavior in a system with a Fermi surface, where we would find the BCS infrared logarithmic divergence in the analogous term: this is, of course, the reason that a $T = 0$ Fermi liquid is unstable to superconductivity for any attractive interaction. In the present situation, the background $d_{x^2-y^2}$ superconductivity has reduced the Fermi surface to four Fermi points, and so further pairing or excitonic instabilities occur at finite values of r and λ. Indeed, this feature allows a nontrivial quantum critical point, with a universal quantum-critical region (Fig. 1); the fluctuations in this region will satisfy (i) provided the quantum-critical point at $r = r_c$, $T = 0$ is described by a fixed point of the renormalization group (RG) transformation at which λ approaches a nonzero and finite fixed point value—then the scattering rate of the nodal fermions will be determined by T alone [18].

The results of our RG analysis of (A)–(G) are simple and remarkable. Only for (A), (B), and (C) do we find a fixed point, accessed by tuning the parameter r; such a fixed point describes a second-order quantum phase transition at the critical point $r = r_c$. For all other cases, we find runaway flows of the couplings, with no nontrivial fixed points, which suggests first-order transitions. As we have already noted, the fixed point for (C) is the Ising model—the nodal fermions are decoupled from the critical degrees of freedom in the scaling limit, so that (i) is not satisfied. Only (A) and (B) satisfy (i), with the couplings λ and u approaching nonzero fixed point values: the nodal fermions and ϕ are strongly coupled in the critical theory, and the anomalous dimension of the fermion field leads to a large ω tail in its EDC [7,18]. The (A) and (B) fixed points are also Lorentz invariant—the dynamic exponent $z = 1$, and the velocities renormalize to $v_F = v_\Delta = c$ in the scaling limit. Indeed, these fixed points were discussed earlier [7], but only for almost equal velocities; here we have established that the equal-velocity fixed point is the only one for arbitrary initial velocities. However, the crossover exponent which determines how rapidly the velocities approach each other is negligible [7] (≈ 0.05), so that a transient regime with unequal velocities will be realized over essentially all of the experimentally accessible regime.

The methodology of our RG is standard and details appear elsewhere [19]. The familiar momentum-shell method, in which degrees of freedom with momenta between Λ and $\Lambda - d\Lambda$ are successively integrated out, fails here: the combination of momentum dependent renormalizations at one loop, the direction-dependent velocities (v_F, v_Δ, $c\ldots$), and the hard momentum cutoff generate unphysical nonanalytic terms in the effective action. So we obtained the RG equations by using a soft cutoff at scale Λ and by taking a Λ derivative of the renormalized vertices and self-energies. We obtained equations for all the velocities, the dynamic exponent z, and the field anomalous dimensions to one-loop order in the nonlinearities λ, u, v. For (D), (E), and (F) a simple and robust effect preempts a fixed point: the structure of $M_{1,2}$ produces opposite sign renormalizations for $v_{F,\Delta}$, in a manner that both flow equations cannot simultaneously be at a fixed point; (G) required a more detailed analysis.

Our main result is that, among the seven transitions considered here, only for those involving onset of $d_{x^2-y^2} + is$ or $d_{x^2-y^2} + id_{xy}$ pairing in a $d_{x^2-y^2}$ superconductor did we find a universal critical theory of coupled fermionic and bosonic order parameter modes below its upper critical dimension. Such transitions naturally satisfy (i). Upon further imposing condition (ii), case (B), with $d_{x^2-y^2} + id_{xy}$ pairing, is uniquely selected: from the basis functions in Fig. 1 we see that ϕ couples to fermions in all directions for (A), while the fermionic coupling vanishes along the antinodal directions for (B)—so the gapped antinodal fermions will [will not] lose the sharp quasiparticle peak below T_c by emission of multiple ϕ quanta, for (A) [(B)].

Pairing in the $d_{x^2-y^2} + id_{xy}$ channel has been considered in numerous works recently [20], with the order in the ground state either global (induced spontaneously or by an external magnetic field) or local (in the vicinity of defects [21], surfaces [22], or vortices [23]). Here we require only strong fluctuations of such order, induced by a proximity to a hypothetical point in the phase diagram where global order arises. While experimental discovery of such a point is of course preferable, tests of our proposal would also be provided by signals of ϕ fluctuations. This is a spin-singlet mode with d_{xy} symmetry, odd under time reversal, and at $T = 0$ it has spectral weight with mean frequency of order $\sim(r - r_c)^{z\nu}$ (where ν is the usual correlation length exponent)—we estimate this scale is $\sim$5–10 K; in the quantum-critical region the characteristic energy scale is $k_BT/\hbar$. Fluctuations of ϕ should lead to anomalies in Raman scattering [24] and Hall transport [25]: these issues will be discussed in future work.

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