## Transverse Momentum Distributions and Their Forward-Backward Correlations in the Percolating Color String Approach

M. A. Braun<sup>1</sup> and C. Pajares<sup>2</sup>

<sup>1</sup>Department of High Energy Physics, University of St. Petersburg, 198904 St. Petersburg, Russia

<sup>2</sup>Departamento de Física de Partículas, Universidade de Santiago de Compostela, 15706-Santiago de Compostela, Spain

(Received 19 July 2000)

The forward-backward correlations in the  $p_T$  distributions at midrapidity, which present a clear signature of nonlinear effects in particle production, are studied in the model of percolating color strings. Quantitative predictions are given for these correlations at SPS, RHIC, and LHC energies. Interaction of strings also naturally explains the flattening of  $p_T$  distributions and increase of  $\langle p_T \rangle$  with energy and atomic number for nuclear collisions.

PACS numbers: 13.85.Hd, 12.40.-y

The study of  $p_T$  distributions in hadronic and nuclear reactions at high energies offers a unique opportunity to observe nonlinear effects in high-density nuclear matter. Indeed in a simple picture in which particle production goes via formation of several independent emitters, color strings stretched between the projectile and target, the  $p_T$ distribution in the central region is independent of the number of strings and coincides with the  $p_T$  distribution from a single color string. It is well known that both energy-momentum conservation and transverse movement of string ends introduce certain correlations between the number of strings and  $p_T$  distribution. However, this effect is restricted to the fragmentation regions and even there quickly saturates with energy E and/or atomic number Aof the colliding particles [1]. As a result, in the picture with independent strings the observed  $p_T$  distribution in the central region does not depend on E nor on A (see, e.g., [2], where even with extraordinarily high values for the partonic  $\langle p_T \rangle = 1.4 \text{ GeV}/c$  the average  $p_T$  of secondaries does not practically change with energy from 63 to 540 GeV). Experimentally this prediction is fulfilled only in a very crude manner. In fact the average  $\langle p_T \rangle$  grows with energy and also with the atomic number of the participants (the "Cronin effect"). This behavior clearly shows that particles are produced by different strings, not independently. In other words, strings interact with each other and these nonlinear effects result in the experimentally observed behavior of the  $p_T$  distributions.

Some time ago the authors introduced a simple model which introduces the interaction between strings via their fusion and percolation [3,4]. In this note we study the  $p_T$  distributions in the central region in this model. We show that the model describes both qualitatively and quantitatively the behavior of the  $p_T$  distributions of the produced particles with the growth of both the energy and atomic number of the participants. We also study the forward-backward correlations (FBC) in the  $p_T$  distributions (also in the central region). We find that their mere existence is a signature of nonlinear effects in particle production. Our model predicts a very concrete form of these correlations, which can be tested in present-day and future experiments.

We start from the particle spectra produced by a single color string. The standard idea exploited in their description is that the observed particles are formed via quarkantiquark pair emissions by the color field of the string. According to [5], the probability rate for this production then has a Gaussian form as a function of  $p_T$ . As soon as a  $q\bar{q}$  pair is created, the color charge Q of the string diminishes, so that at the next step the production rate is different (although also of the Gaussian form in  $p_T$ ). Detailed calculations show that the resulting  $\langle p_T^2 \rangle$  of particles produced after the complete breakup of the string is proportional to Q [6]

$$\langle p_T^2 \rangle_Q = Q \langle p_T^2 \rangle_1, \qquad (1)$$

where  $\langle p_T^2 \rangle_1$  is the average  $p_T^2$  of particles produced by a "single" string, created by color  $Q_0$  corresponding to a single  $q\bar{q}$  pair. In the future we measure all colors in units  $Q_0$ , which is equivalent to putting  $Q_0 = 1$ . To simplify the notation we also omit the subindex T in the momentum distribution, since we shall consider only the transverse momenta.

The Gaussian distribution in p can be, however, theoretically supported only for infinitely high energies, which correspond to a string infinitely long in rapidity. For realistic strings with a finite length in rapidity one expects corrections due to energy conservation. Also fluctuations of the string tension may change the form of the p distribution, as advocated in [7]. Finally an evident restriction comes at large p where the hard collision mechanism is expected to be responsible for particle creation. As a result one expects the p distribution to have a power behaved tail.

A realistic form of the p distribution corresponding to a single color string can be extracted from the experimentally measured one in  $p\bar{p}$  soft collisions at 630 and 1800 GeV/c [8]. Assuming that the effects of string interactions are small for  $p\bar{p}$  interactions, we may take that the measured distribution coincides with the one for a single ordinary (Q = 1) string. This distribution has a form

$$w_1(p) = \frac{(k-1)(k-2)}{2\pi p_0^2} \frac{p_0^k}{(p+p_0)^k}.$$
 (2)

4864

© 2000 The American Physical Society

Comparing the distributions at 630 and 1800 GeV and also taking into account the behavior of the minimum bias distributions in all the energy interval from 63 to 1800 GeV [9] we parametrize (for realistic energies)

$$p_0 = 2 \text{ GeV}/c, \qquad k = 19.7 - 0.86 \ln E, \quad (3)$$

where E is the c.m. energy in GeV. With (2) the averages  $\langle p \rangle$  and  $\langle p^2 \rangle$  are given by

$$p_{1} \equiv \langle p \rangle_{1} = p_{0} \frac{2}{k-3},$$

$$\langle p^{2} \rangle_{1} = p_{0}^{2} \frac{6}{(k-3)(k-4)}.$$
(4)

Passing to the string with color Q, to satisfy (1), we change  $p_0^2 \rightarrow Q p_0^2$ , so that our distribution corresponding to the string with color Q is

$$w_{Q}(p) = \frac{(k-1)(k-2)}{2\pi Q p_{0}^{2}} \frac{(p_{0}\sqrt{Q})^{k}}{(p+p_{0}\sqrt{Q})^{k}}.$$
 (5)

We stress that this distribution refers only to the soft part of the spectrum in the central region, which supposedly is generated by the string decay. The observed spectrum also contains a contribution from hard events (with a produced cluster having p > 1.1 GeV/c, in the definition of [8]).

Our picture for the high-energy particle production consists in assuming that in the collision several color strings are created which may overlap in the transverse space. In the overlap area the color fields of the strings add algebraically. Because of the vector character of color charge, the resulting color squared of the overlap area is just a sum of the color squared of the overlap prize strings. Thus in the overlap of *n* strings a new color string is formed corresponding to color  $Q_n = \sqrt{n}$ . The fact that the color of the overlapping strings is proportional to the square root of their number and not to their number has an immediate consequence of damping the multiplicities of the produced particles by a factor of the order of 3 at the LHC energies [10]. Here we study its influence on the *p* distribution.

To find the latter we have to know the distribution in the areas of overlaps of n strings, which gives the weights with which different overlaps contribute to particle production. It has been shown in [10] that in the "thermodynamic limit," that is for an idealized system with a very large total interaction area S and correspondingly large number of strings N, the properties of the system and the overlap distribution in particular are governed by a dimensionless parameter

$$\eta = \sigma_0 \frac{N}{S} = \sigma_0 \rho \,. \tag{6}$$

Here  $\sigma_0$  is an area of a single string and  $\rho$  the string density. Note that at  $\eta > \eta_c \approx 1.12-1.20$  the percolation phase transition occurs, most of the space being occupied by a single cluster formed by many overlapping strings.

It has been shown in [10] that in the thermodynamic limit the distribution in the overlap areas with different n follows the Poisson law with an average value equal to  $\eta$ :

$$\lambda(n) = \frac{S_n}{S} = a(\eta) \frac{\eta^n}{n!}, \qquad (7)$$

with  $a = \exp(-\eta)$ . Here  $S_n$  is the total area in which exactly *n* strings overlap. Equation (7) is valid for n = $0, 1, 2, ..., \lambda(0)$  giving the part of the area in which there are no strings at all. Evidently the latter part does not produce particles. So we are interested only in the relative contribution to particle production of overlaps with n = 1, 2, ... This will be given by (7) with a different normalization factor  $a = 1/(\exp \eta - 1)$ .

The overall p distribution P(p) is obtained by convoluting the distribution (5) in p for fixed Q and (7) in n and taking into account that for an overlap of n strings  $Q = \sqrt{n}$ :

$$P(p) = \sum_{n=1}^{\infty} \lambda(n) w_{\sqrt{n}}(p) \,. \tag{8}$$

As a consequence of (1) we find

$$\langle p \rangle = p_1 a(\eta) \sum_{n=1}^{\infty} n^{1/4} \frac{\eta^n}{n!} \,. \tag{9}$$

For small  $\eta$  this gives  $\langle p \rangle / p_1 = 1 + 0.094 \eta$ . The behavior of  $\langle p \rangle / p_1$  for  $0.5 < \eta < 4$  can also be well described by a linear dependence  $1 + 0.098 \eta$ .

So even with a fixed average  $p_1$  for a single string, the overall average grows with  $\eta$ . On the other hand,  $\eta$  grows both with the energy E and the atomic number A of the colliding particles [10], so that fusion of strings by itself leads to the growth of the average transverse momentum with E and A. With the distribution (2) the average  $p_1$ also rises with energy from 0.28 GeV/c at 19.4 GeV to 0.43 GeV/c at 5500 GeV. This rise has to be combined with that due to the growth of  $\eta$  for collisions with nuclei. Using our earlier calculations [10] we find the values of  $\eta$  for central *p*-PB and Pb-Pb collisions at c.m. energies 19.4, 200, and 5500 GeV shown in the second and fourth columns of Table I. In the third and fifth columns we present the corresponding values of  $\langle p \rangle$  which follow from Eq. (9) with the distribution (5). Comparing these values with the experiment, one should remember that they refer only to the soft part of the spectra.

The form of the p distribution in p-Pb and Pb-Pb collisions corresponding to Eq. (8) is shown in Fig. 1. With the growth of E and A the high p tail is strongly enhanced.

TABLE I. Dependence of mean transverse momentum on energy.

Energy (GeV)	η	p-Pb $\langle p_T \rangle (\text{GeV}/c)$	η	Pb-Pb $\langle p_T \rangle$ (GeV/c)
19.4	0.53	0.30	1.19	0.32
200	0.60	0.35	1.82	0.39
5500	0.76	0.46	3.54	0.58



FIG. 1. Transverse momentum distributions in the reactions p-Pb and Pb-Pb at different c.m. energies (normalized to unity). Curves 1, 2, and 3 correspond to p-Pb, and curves 4, 5, and 6 to Pb-Pb at 19.4, 200, and 5500 GeV, respectively.

This corresponds to the well-known Cronin effect. In Fig. 2 we present the ratio of the distributions for the p-Pb and Pb-Pb to p-p reactions. They are well compatible with the experimental ones [11].

Now we pass to the FBC in the *p* distributions in the central region. They can be studied from the observation of the average *p* in the backward hemisphere  $\langle p_B \rangle_{p_F}$  for events with given  $p = p_F$  in the forward hemisphere. In the absence of any interaction between color strings (independent color string model) the average *p* in both hemispheres evidently is identical with this average for a single string. Then

$$F(p_F) \equiv \langle p_B \rangle_{p_F} / p_1 = 1.$$
 (10)

Of course, correlations between different strings are imposed not only by their fusion and percolation, but also on purely kinematical grounds, due to energy-momentum



FIG. 2. Ratio of  $p_T$  distributions for the reactions p + Pb and Pb-Pb to p-p at c.m. energies 19.4, 200, and 5500 GeV. For p-Pb the curves go down with energy. For Pb-Pb the slope rises with energy.

conservation and transverse movement of string ends. However, as mentioned above, these latter effects are totally restricted to the fragmentation regions and should become negligible in the midrapidity region at large enough energy. Modulo these kinematical effects, the mere existence of FBC in the central region, that is, the difference of the right-hand side of Eq. (10) from unity, is a clear signature of a dynamical interaction between strings. In our percolation string model function F can be found explicitly.

With two different observables  $p_B$  and  $p_F$  we introduce the corresponding probability  $w(p_F, p_F)$  which generalizes Eq. (5) and shows the probability to find the observed particle with transverse momenta  $p_F$  and  $p_B$ in the forward and backward hemispheres. Technically it can be found from the inclusive cross section  $2Ed^3\sigma/d^3p$ integrated over the angles in the forward or backward hemispheres, respectively, and properly normalized. Our starting point will be an assumption that there is no correlation between emission of particles in the forward and backward hemispheres for a single string:

$$w_Q(p_F, p_B) = w_Q(p_F)w_Q(p_B),$$
 (11)

where each of the functions w on the right-hand side is given by Eq. (5). Then for a single string of color Q

$$\langle p_F \rangle_Q = \langle p_B \rangle_Q = \langle p \rangle_Q = \sqrt{Q} p_1.$$
 (12)

Passing to the system of percolating strings with different overlaps we find the final distribution in  $p_F$  and  $p_B$  as a suitable generalization of (8)

$$P(p_F, p_B) = \sum_{n=1}^{\infty} \lambda(n) w_{\sqrt{n}}(p_F, p_B).$$
(13)

Integrating this expression over  $p_F$  one obtains the distribution in  $p_F$ :

$$P(p_F) = \int d^2 p_B P(p_F, p_B)$$
$$= \sum_{n=1} \lambda(n) w_{\sqrt{n}}(p_F) = P(p).$$
(14)

As expected, it coincides with the overall distribution (8).

The conditional probability to see a particle with momentum  $p_B$  in the backward hemisphere, provided one observes a particle with momentum  $p_F$  in the forward hemisphere, is given by the ratio

$$P(p_B)_{p_F} = \frac{P(p_F, p_B)}{P(p_F)},$$
(15)

so that the average value of any observable  $A(p_F, p_B)$  for a given fixed  $p_F$  is given by

$$\langle A \rangle_{p_F} = P^{-1}(p_F) \int d^2 p_B A(p_F, p_B) P(p_F, p_B).$$
 (16)



FIG. 3. The FBC parameter F-1 [Eq. (17)] as a function of  $p_F$  for the reactions *p*-Pb and Pb-Pb at c.m. energies 19.4, 200, and 5500 GeV.

Taking  $A(p_F, p_B) = p_B$  we obtain for the function F in (10)

$$F(p_F) = \frac{\sum_{n=1} n^{1/4} \lambda(n) w_{\sqrt{n}}(p_F)}{\sum_{n=1} \lambda(n) w_{\sqrt{n}}(p_F)}.$$
 (17)

Its difference from unity measures the FBC in the transverse momentum distributions.

At small  $\eta$  we find

$$F(p_F) = 1 + 0.0669 \eta \frac{[1.189(p + p_0)]^k}{(p + 1.189p_0)^k}.$$
 (18)

So positive correlations follow (as expected), which grow with the momentum from a nonzero value to saturate at a certain larger value depending on the energy (via k).

The behavior of  $F(p_F) - 1$  for the reactions *p*-Pb and Pb-Pb at different energies is illustrated in Fig. 3. The

FBC rise with energy and atomic number, their magnitude well allowing for the experimental observation.

Summarizing, we have shown that interaction of strings predicts sizable FBC in the transverse momenta in the central region, which can be tested in the forthcoming experiments. Also the dependence of the p spectra on E and A is naturally explained for collisions with nuclei.

This work has been done under Contract No. AEN99-0589-C02-02 from CICYT of Spain.

- [1] A. Capella and A. Krzywicki, Phys. Rev. D **29**, 1007 (1984).
- [2] P. Aurenche, F. Bopp, and J. Ranft, Phys. Lett. 147B, 212 (1984).
- [3] M. A. Braun and C. Pajares, Phys. Lett. B 287, 154 (1992); Nucl. Phys. B390, 542 (1993); B390, 549 (1993); N. S. Amelin, M. A. Braun, and C. Pajares, Phys. Lett. B 306, 312 (1993); Z. Phys. C 63, 507 (1994).
- [4] N. Armesto, M. A. Braun, E. G. Ferreiro, and C. Pajares, Phys. Rev. Lett. 77, 3736 (1996).
- [5] J. Schwinger, Phys. Rev. 82, 664 (1951); T. S. Biro, H. B. Nielsen, and J. Knoll, Nucl. Phys. B245, 449 (1984).
- [6] A. Bialas and W. Czyz, Nucl. Phys. B267, 242 (1986).
- [7] A. Bialas, hep-ph/9909417.
- [8] CDF Collaboration, F. Rimondi *et al.*, in Proceedings of the IX International Workshop on Multiparticle Production, Torino, 2000 (to be published).
- [9] UA1 Collaboration, G. Arnison *et al.*, Phys. Lett. **118B**, 167 (1982); CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **61**, 1819 (1988).
- [10] M. A. Braun and C. Pajares, Eur. Phys. J. C16, 349 (2000);
   N. Armesto and C. Pajares, Int. J. Mod. Phys. A 15, 2019 (2000).
- [11] J. Schukraft, in Proceedings of the International Workshop on Quark-Gluon Plasma Signatures, Strasbourg, France, 1990.