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Quantum-Mechanical Nonperturbative Response of Driven Chaotic Mesoscopic Systems

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Consider a time-dependent Hamiltonian $\mathcal{H}(Q,P;x(t))$ with periodic driving $x(t) = A\sin(\Omega t)$. It is assumed that the classical dynamics is chaotic, and that its power spectrum extends over some frequency range $|\omega| < \omega_{\rm cl}$. Both classical and quantum-mechanical (QM) linear response theory (LRT) predict a relatively large response for $\Omega < \omega_{\rm cl}$, and a relatively small response otherwise, independent of the driving amplitude A. We define a nonperturbative regime in the (Ω, A) space, where LRT fails, and demonstrate this failure numerically. For $A > A_{\rm prt}$, where $A_{\rm prt} \propto \hbar$, the system may have a relatively strong response for $\Omega > \omega_{\rm cl}$ due to QM nonperturbative effect.

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The *wall formula* for the calculation of friction in nuclear physics [1] and the *Drude formula* for the calculation of conductance in mesoscopic physics are just two special results of a much more general formulation of "dissipation theory" [2–5]. The general formulation of the "dissipation" problem [4] is as follows: Assume a time-dependent chaotic Hamiltonian $\mathcal{H}(Q,P;x(t))$. For $x =$ const, the energy is constant of the motion. For nonzero $V \equiv \dot{x}$ the energy distribution evolves, and the *average* energy increases with time. This effect is known as *dissipation.* Ohmic dissipation means that the rate of nonconservative energy absorption ("heating") is $d\langle\mathcal{H}\rangle/dt = \mu V^2$, where μ is defined as the dissipation coefficient. In case of periodic driving $x(t) = A\sin(\Omega t)$, one should replace V^2 by the mean square value $\frac{1}{2}(A\Omega)^2$, and the dissipation coefficient $\mu(\Omega)$ becomes frequency dependent.

In the case of the wall formula, (Q,P) is a particle moving inside a chaotic "cavity," and x controls the deformation of the boundary. Ohmic dissipation (in the sense defined above) implies a friction force which is proportional to the velocity, where μ is the "friction coefficient," and μV^2 is the heating rate. A mesoscopic realization of such a system would be a quantum dot whose shape is controlled by electric gates. In case of the mesoscopic Drude formula, (Q,P) is a charged particle moving inside a chaotic "ring," and x is the magnetic flux through the hole in the ring. Ohmic dissipation implies Ohm law, where $V \equiv \dot{x}$ is the electromotive force, μ is the conductance, and μV^2 is the heating rate. For a mesoscopic realization of such a system note that ring geometry is not important. One may consider a simple two-dimensional quantum dot driven by a time-dependent homogeneous perpendicular magnetic field [6].

In the general analysis of the dissipation problem [4,5], one argues that due to the driving there is diffusion in energy space. This diffusion process is biased because of its E dependence, leading to a systematic increase of the average energy. This is the reason for having dissipation. Therefore we find convenient from now on to consider the diffusion coefficient $D_{\rm E}$ as the object of our study. The relation between $d\langle\mathcal{H}\rangle/dt$ and $D_{\rm E}$ constitutes a generalization of the so-called fluctuation-dissipation relation.

Ohmic dissipation is implied if $D_{\rm E} \propto V^2$, or $D_{\rm E} \propto A^2$ in the case of periodic driving. Such behavior can be established within the framework of classical mechanics [2,3] using general classical considerations [7]. The classical formulation of the dissipation problem [5] can be regarded as a systematic scheme that justifies the use of classical "linear response theory" (LRT). The precise conditions for the applicability of the classical LRT result are further discussed in [4,5], and we are going to mention later what we call "the trivial slowness condition." We are interested in the *quantum-mechanical* (QM) theory of dissipation. The traditional derivation [8] of QM LRT leads formally to the

same result as in the classical analysis [5]. Therefore, from now on we no longer distinguish between the "classical" LRT result and the "quantal" LRT results and use just the term "LRT result." As a matter of terminology, it should be noted that the QM formulation of LRT, also known as Kubo-Greenwood formalism, is completely equivalent [5] to the well-known Fermi golden rule (FGR) picture.

So let us assume that the obvious *classical* conditions for the validity of the LRT result are satisfied [9]. Now the question is whether, upon quantization, there are *additional* $\hbar$-dependent conditions for the applicability of the LRT result [9]. In the traditional quantum mechanical literature, as well as in the recent mesoscopic literature, the focus is on the consequences of having finite mean level spacing Δ. This leads to the identification of the QM-adiabatic regime (extremely slow driving) and to the discussion of either the Landau-Zener mechanism [3] or the Debye relaxation absorption mechanism [12] for dissipation, as well as to the discussion of QM resonances. The main observation of [13] is that there is another regime, the nonperturbative regime (see Fig. 1), where QM LRT is not valid. As strange as it sounds, this does not imply a failure of the LRT result. On the contrary, another observation of [13] is that the regime where the classical approximation applies is well contained in the nonperturbative regime, hence the LRT result becomes valid again because of quantal-classical correspondence (QCC) considerations. However, if the system does not have a good classical limit [as in random matrix theory (RMT) models] this "recovery" of the LRT result is not guaranteed. Moreover, as discussed later, QCC consideration cannot exclude the possibility of having a relatively large quantal nonperturbative response whenever the LRT result is small in comparison.

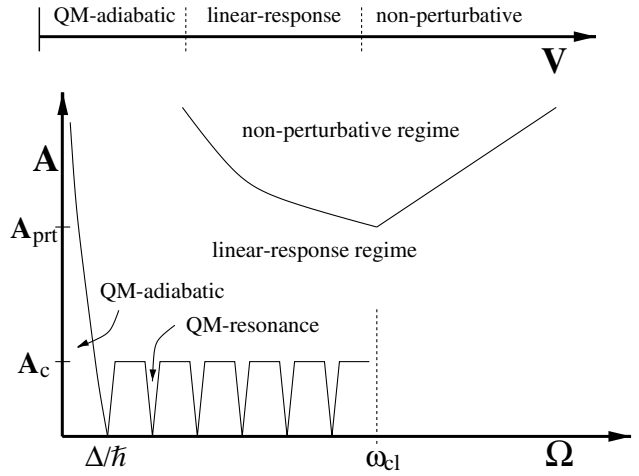

FIG. 1. Upper diagram: The various V regimes in the theory of quantum dissipation for linear driving $x(t) = Vt$. Lower diagram: The various (Ω, A) regimes for periodic driving $x(t) = A\sin(\Omega t)$. Note the analogy with Fig. 5 of Ref. [5] with $x \leftrightarrow A$ and $V \leftrightarrow A\Omega$. The QM-adiabatic regime (including the regime $A < A_c$, but excluding the narrow stripes of QM resonances) is defined by having *vanishing* first-order probability go to other levels. See the text for further explanations and definitions of A_c and $A_{\rm prt}$.

The outline of this Letter is as follows: (i) We extend the theoretical considerations of [13] to the case of periodic driving. (ii) We give a specific example where the LRT result fails because of a quantal nonperturbative effect. (iii) We comment on the issue of localization. (iv) We discuss the role of QCC considerations in the theory. Based on the theoretical considerations, the reader should realize that the existence of the nonperturbative regime is not related to having finite mean level spacing Δ, but rather to having finite bandwidth $\Delta_b = \hbar\omega_{\rm cl}$, where $\omega_{\rm cl}$ is the dropoff frequency of the LRT response. In the context of mesoscopic physics this bandwidth is known as the Thouless energy.

Given $\mathcal{H}(Q, P; x)$ with $x = \text{const}$, we can define a fluctuating quantity $\mathcal{F}(t) = -\partial\mathcal{H}/\partial x$. The autocorrelation function of $\mathcal{F}(t)$ will be denoted by $C(\tau)$. The power spectrum $\tilde{C}(\omega)$ is defined as its Fourier transform. The intensity of fluctuations is defined as $\nu = \tilde{C}(0)$, and it is convenient to define the correlation time as $\tau_{\rm cl} = \tilde{C}(0)/C(0)$. We assume for simplicity of presentation that the single time scale $\tau_{\rm cl}$ completely characterizes the chaotic dynamics of the system: The power spectrum of the chaotic motion is assumed to be continuous, and it is nonvanishing up to the cutoff frequency $\omega_{\rm cl} = 2\pi/\tau_{\rm cl}$. We assume that $\tilde{C}(\omega)$ is vanishingly small for $\omega > \omega_{\rm cl}$.

Consider the time-dependent case $x(t) = A\sin(\Omega t)$. The LRT result for the diffusion in energy is

$$D_{\rm E} = \tfrac{1}{2}\,\tilde{C}(\Omega) \times \tfrac{1}{2}\,(A\Omega)^2 \tag{1}$$

and we shall use the notation $D_0 = \frac{1}{4}\nu(A\Omega)^2$. The most transparent QM derivation of this result is based on the FGR picture. The energy levels of the systems are E_n, and the mean level spacing is Δ. The Heisenberg time is $t_{\rm H} = 2\pi\hbar/\Delta$. The transitions between levels are determined by the coupling matrix elements $(\partial\mathcal{H}/\partial x)_{nm}$. It is well known [14] that for reasonably small $\hbar$ this matrix is a banded matrix. The bandwidth is $\Delta_b = 2\pi\hbar/\tau_{\rm cl}$, and the variance of the in-band elements is $\sigma^2 = \nu/t_{\rm H}$. It is common to define the QM system using the four parameters $(\Delta, b, \sigma, \hbar)$, where $b = \Delta_b/\Delta$. It is also useful to regard the semiclassical relations $\tau_{\rm cl} = 2\pi\hbar/(b\Delta)$ and $\nu = (2\pi\hbar/\Delta)\sigma^2$ as definitions whenever the classical limit is not explicitly specified [as in RMT models]. The FGR picture implies strong response if and only if $\hbar\Omega < \Delta_b$, leading to $\Omega < \omega_{\rm cl}$ as in the classical case. Using the FGR picture it is straightforward to recover $D_0 = (\pi/2)\,(\hbar/\Delta)\,(\sigma A\Omega)^2$ in agreement with Eq. (1).

The *trivial slowness condition* for the applicability of *classical* LRT [4,5] is $V\tau_{\rm cl} \ll \delta x_c^{\rm cl}$. Here $\delta x_c^{\rm cl}$ is the parametric change that leads to the breakdown of the linearization of $\mathcal{H}(Q, P; x + \delta x)$ with respect to δx. Upon quantization there are two other parametric scales that

become important [5], namely, $\delta x_c^{\rm qm}$ and $\delta x_{\rm prt}$. The former is the parametric change which is required in order to mix neighboring levels, while the latter is the parametric change required in order to mix all the levels within the band. Hence we define

$$A_c \equiv \delta x_c^{\rm qm} = \frac{\Delta}{\sigma} \propto \hbar^{(1+d)/2}, \tag{2}$$

$$A_{\rm prt} \equiv \delta x_{\rm prt} = \sqrt{b}\,\frac{\Delta}{\sigma} = \frac{2\pi\hbar}{\sqrt{\nu\tau_{\rm cl}}}. \tag{3}$$

The above parametric scales, of time-independent first-order perturbation theory (FOPT), manifest themselves also in the time-dependent analysis. FOPT gives the following result for the probability $P_t(n\,|\,m)$ to make a transition from an initial level m to some other level n,

$$\left|\frac{1}{\hbar}\left(\frac{\partial\mathcal{H}}{\partial x}\right)_{nm}\int_0^t x(t')\exp\!\left(i\,\frac{(E_n - E_m)t'}{\hbar}\right)dt'\right|^2 .$$

The total transition probability is $p(t) = \sum_n' P_t(n\,|\,m)$, where the prime implies omission of the $n = m$ term. In the regime $\Omega < \omega_{\rm cl}$, one obtains

$$p(t) = \frac{1}{\hbar^2}\,\nu A^2 \times \begin{cases} (\Omega^2/\tau_{\rm cl})\,\frac{1}{4}\,t^4 & \text{for } 0 < t \ll \tau_{\rm cl} \\ \Omega^2\,\frac{1}{3}\,t^3 & \text{for } \tau_{\rm cl} \ll t \ll 1/\Omega \\ \frac{1}{2}\,t & \text{for } 1/\Omega \ll t \ll t_{\rm H} \end{cases},$$

while in the regime $\Omega > \omega_{\rm cl}$, one obtains

$$p(t) = \frac{1}{\hbar^2}\,\frac{\nu}{\tau_{\rm cl}}\,\frac{A^2}{\Omega^2}\begin{cases}[1-\cos(\Omega t)]^2, & 0 < t \ll \tau_{\rm cl}, \\ 3/2, & \tau_{\rm cl} \ll t \ll t_{\rm H}. \end{cases}$$

In both cases for $t > t_{\rm H}$ we have recurrences, and therefore $p(t) \le p(t_{\rm H})$. (One should be more careful near resonances: There, $t_{\rm H}$ should be replaced by $2\pi\hbar/\delta$, where δ is the detuning.) The necessary condition for applicability of FOPT at time t is that $p(t') \ll 1$ for any $t' < t$, which can be written as $p([0,t]) \ll 1$. The necessary condition for the applicability of the FGR picture is $p([0,\tau_{\rm cl}]) \ll 1$. This is the FGR condition [5] which guarantees the separation of time scales $\tau_{\rm cl} \ll \tau_{\rm prt}$. The FOPT break time $\tau_{\rm prt}$ is defined as the maximal t for which $p([0,t]) < 1$. Now we can define a *nonperturbative regime* by the requirement $p([0,\tau_{\rm cl}]) > 1$. It is straightforward to observe that the nonperturbative regime is contained in the region $A > A_{\rm prt}$, where

$$\left(\frac{A_{\rm prt}}{A}\right)\omega_{\rm cl} < \Omega < \left(\frac{A}{A_{\rm prt}}\right)\omega_{\rm cl}. \tag{4}$$

The location of the nonperturbative regime is illustrated in Fig. 1. For completeness of presentation we have also indicated the subregion in $\Omega < \omega_{\rm cl}$ where we have first-order response equal to zero. The condition is $p([0,\infty]) \ll 1$ or, equivalently, $p(t_{\rm H}) \ll 1$. This region contains the QM adiabatic regime, including the region $A < A_c$, but excluding the narrow stripes of resonances.

We have defined the location of the nonperturbative regime, but we have not yet suggested how Eq. (1) should be modified. Using RMT assumptions with regard to $(\partial\mathcal{H}/\partial x)_{nm}$, and inspired by related studies of wave packet dynamics [15], we expect the result

$$D_{\rm E} = (C/\sqrt{v_{\rm PR}}) \times D_0\,, \tag{5}$$

where $v_{\rm PR} = V/(\delta x_{\rm prt}/\tau_{\rm cl})$, and C is a numerical constant. A detailed derivation of (5) will be presented in the future. (The crucial step is to argue that at $\tau_{\rm prt}$ there is a crossover from ballistic behavior to diffusion in the sense of [5].) For periodic driving this result should be averaged over a period leading to $D_{\rm E} \propto A^{2-\alpha}$ with $\alpha = 1/2$.

We wanted to give a numerical example that demonstrates the nonperturbative response effect. Evidently, the simplest is to consider a time-dependent version of Wigner's banded random matrix (WBRM) model,

$$\mathcal{H} = \mathbf{E}_0 + x(t)\mathbf{B}\,, \tag{6}$$

where $\mathbf{E}_0$ is an ordered diagonal matrix, and $\mathbf{B}$ is a banded matrix. This model [16,17] is characterized by the parameters $(\Delta, b, \sigma, \hbar)$ which we have defined previously. For the numerical experiment we have assumed a rectangular band profile such that all of the elements $0 < |n - m| \le b$ are taken from the *same* distribution, and outside the band all of the elements are identically zero. The results of the simulations are summarized in Fig. 2. It should be realized that WBRM model Eq. (6) has a big disadvantage. Namely, unlike the physical examples of the introduction, the statistical properties of the model are not invariant for $x(t) \mapsto x(t) + \text{const}$. One may wonder why we do not use one of the two other popular variations of the Wigner model [10,11], e.g., $\mathcal{H} = \mathbf{E} + (\cos x)\mathbf{B}_1 + (\sin x)\mathbf{B}_2$. The problem is that, for these models, one obtains $\delta x_{\rm prt} \sim \delta x_c^{\rm cl} \sim 2\pi$. Therefore there is no regime where LRT fails because of *quantal* nonperturbative effect [9]. (Such failure requires the generic separation of scales $\delta x_{\rm prt} \ll \delta x_c^{\rm cl}$.) Thus it seems that the only way to make a RMT model x-invariant, is to keep it "perturbative" in nature. The lack of x-invariance in the standard WBRM model complicates the calculation of the period-averaged $D_{\rm E}$ and leads to $D_{\rm E} \propto A^{2-\alpha}$ with α changing gradually from $1/2$ to 1. The numerical analysis of Fig. 2b fits well to $\alpha \sim 3/4$.

There are two types of localization effects that we had to consider in our numerical experiments. The "WBRM model localization" can be avoided by using amplitudes $A < b^{3/2}(\Delta/\sigma)$ in order to guarantee that the instantaneous eigenstates of Eq. (6) are not localized at any time. The "dynamical localization effect," on the other hand, cannot be avoided. It is associated with the periodic nature of the driving. By extending standard argumentation, one observes that the eigenstates of the (one period) Floquet operator have localization length $\xi \times \Delta$, and that the associated break time is $t^* = \xi \times (2\pi/\Omega)$. The two

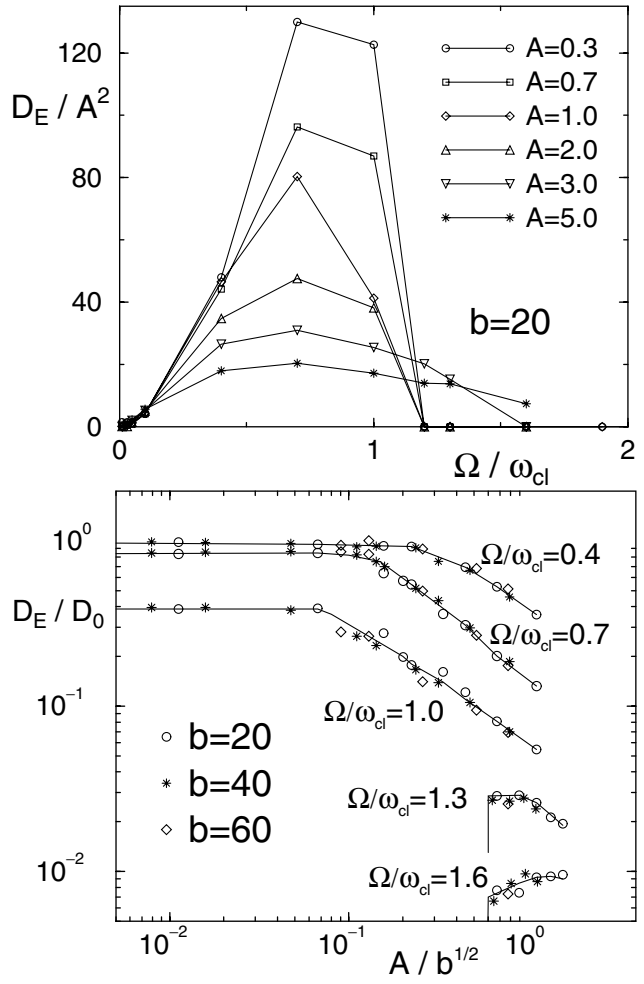

FIG. 2. The response of a quantum-mechanical system is displayed as a function of A and Ω. The evolution is determined by the WBRM model Eq. (6). The units of energy, time, and amplitude are chosen such that $\Delta = 0.5$, $\hbar = 1$, and $\sigma = 1$, respectively. Upper panel: plots of D_E/A^2 versus Ω/ω_{cl} for few values of A. For small ω the plots coincide as expected from Eq. (1). As A becomes larger the deviations from Eq. (1) become more pronounced, and we also get a response for $\Omega > \omega_{cl}$. Lower panel: plots of D_E/D_0 versus $A/\sqrt{b}$ for few values of Ω/ω_{cl}. The LRT result [Eq. (1)] implies $D_E/D_0 = 1$ for $\Omega/\omega_{cl} < 1$. The purpose of the horizontal scaling is to demonstrate that A_{prt} rather than A_c is responsible for the deviation from this LRT expectation. Each "point" in the above plots is determined by a simulation that involves typically 35 realizations of the evolution until we start to see saturation due to dynamical localization effect. The typical time step is $dt = 10^{-4}$. In each step we verify that the normalization is preserved to an accuracy of 0.01%. In order to eliminate finite size effects we have used a self-expanding algorithm. Namely, additional $10b$ sites are added to each edge whenever the probability in the edge sites exceeds 10^{-15}. The diffusion coefficient is determined from the fitting $\delta E(t)^2 = \text{const} \times t^{\beta}$. For subdiffusive behavior ($\beta < 0.86$) we set $D_E = 0$. The preparation of each point in the above plots requires ~4 CPU days on Alpha XP1000 machine.

must be related by $2D_E t^* = (\xi\Delta)^2$ leading (in the LRT regime) to the result $t^* = 2\pi^2(A/A_c)^2 \times t_H$. In all of our numerical experiments the diffusion has been determined for times where dynamical localization is not yet apparent. Another possibility, which we have not used, is to add a small noisy component to the driving, such as to mimic the typical experimental situation of having dephasing time much shorter than t^*.

It is not obvious that the nonperturbative behavior that is implied by RMT assumptions, and applies to RMT models, should apply also to Hamiltonians that possess a well defined classical limit. On the contrary, the same considerations as in [15] can be applied in order to argue that RMT considerations are not compatible with the QCC principle. Here we are going to explain the main idea, and define our expectations.

Taking $\hbar$ to be very small, it is obvious that eventually we shall find ourselves in the regime where $A \gg A_{prt}$. Let us consider the dynamics during a specified time interval $0 < t' < t$. The time t is chosen to be much larger than τ_{cl}. On the basis of QCC considerations, we should be able to make $\hbar$ sufficiently small such that the quantum evolution becomes similar to the classical evolution up to the time t. The classical analysis implies that during this time the stochastic behavior is established. Therefore having detailed QCC during the time t implies that the quantal D_E can be approximated by the classical result. This leads to a contradiction with the RMT prediction Eq. (5) in the domain $\Omega < \omega_{cl}$, but *not* in the domain $\Omega > \omega_{cl}$. We are going to further explain this last point.

Denote the energy dispersion by $\delta E_{qm}(t)$ and the corresponding classical result by $\delta E_{cl}(t)$. For sufficiently small $\hbar$ it should be possible to make a leading order approximation $\delta E_{qm}(t) \approx \delta E_{cl}(t) + \hbar^{\gamma} g(t)$, with $\gamma > 0$. In the $\Omega < \omega_{cl}$ regime the first term in this approximation is dominant. On the other hand, for $\Omega > \omega_{cl}$ the first term gives a vanishingly small result for D_E. Therefore, without any contradiction with QCC considerations, the second term becomes important. Therefore we may have, in principle, an enhanced quantal response for $\Omega > \omega_{cl}$.

In conclusion, we have defined a nonperturbative regime in the (Ω, A) plane, where LRT cannot be trusted. We have demonstrated an actual failure of LRT for a particular (RMT) Hamiltonian. We believe that for generic chaotic systems the RMT mechanism for diffusion competes with the classical mechanism. The actual response of the system is expected to be determined by the predominant mechanism. The study of this conjecture is the theme of our future studies.

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[1] J. Blocki *et al.,* Ann. Phys. (N.Y.) **113**, 330 (1978); S. E. Koonin *et al.,* Nucl. Phys. **A283**, 87 (1977); S. E. Koonin and J. Randrup, Nucl. Phys. **A289**, 475 (1977).

[2] C. Jarzynski, Phys. Rev. E **48**, 4340 (1993); C. Jarzynski, Phys. Rev. Lett. **74**, 2937 (1995).

[3] M. Wilkinson, J. Phys. A **20**, 2415 (1987); **21**, 4021 (1988).
[4] D. Cohen, in *New Directions in Quantum Chaos,* Proceedings of the International School of Physics "Enrico Fermi" Course CXLIII, edited by G. Casati, I. Guarneri, and U. Smilansky (IOS Press, Amsterdam, 2000).
[5] D. Cohen, Ann. Phys. (N.Y.) **283**, 175 (2000).
[6] A. Barnett, D. Cohen, and E. J. Heller, Report No. nlin.CD/0006041.
[7] E. Ott, Phys. Rev. Lett. **42**, 1628 (1979); R. Brown, E. Ott, and C. Grebogi, Phys. Rev. Lett. **59**, 1173 (1987); J. Stat. Phys. **49**, 511 (1987).
[8] Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University Press, New York, 1997), Appendix A.
[9] The most serious attempt to challenge LRT has been done in [10]. However, in view of a later work [11], and as explained in [5], the suppression of diffusion for large V in those studies follows from the violation of the trivial slowness condition $V\tau_{\rm cl} \ll \delta x_c^{\rm cl}$.
[10] M. Wilkinson and E. J. Austin, J. Phys. A **28**, 2277 (1995).
[11] A. Bulgac *et al.,* Phys. Rev. E **54**, 3468 (1996).
[12] R. Landauer and M. Buttiker, Phys. Rev. Lett. **54**, 2049 (1985).
[13] D. Cohen, Phys. Rev. Lett. **82**, 4951 (1999).
[14] M. Feingold and A. Peres, Phys. Rev. A **34**, 591 (1986); M. Feingold *et al.,* Phys. Rev. Lett. **66**, 986 (1991).
[15] D. Cohen *et al.,* Phys. Rev. Lett. **84**, 2052 (2000).
[16] E. Wigner, Ann. Math. **62**, 548 (1955); **65**, 203 (1957).
[17] G. Casati *et al.,* Phys. Rev. E **48**, R1613 (1993); Phys. Lett. A **223**, 430 (1996).