Sudden Hadronization in Relativistic Nuclear Collisions

Johann Rafelski[1] and Jean Letessier[2]

[1]*Department of Physics, University of Arizona, Tucson, Arizona 85721*

[2]*Laboratoire de Physique Théorique et Hautes Energies, Université Paris 7, 2 place Jussieu, F-75251 Cedex 05 Paris, France*

(Received 21 June 2000)

We formulate and study a mechanical instability criterion for sudden hadronization of dense matter fireballs formed in 158*A* GeV Pb-Pb collisions. Considering properties of quark-gluon matter and hadron gas we obtain the phase boundary between these two phases and demonstrate that the required deep quark-gluon-plasma supercooling prior to sudden hadronization has occurred.

PACS numbers: 25.75.–q, 12.38.Mh, 24.10.Pa

Hot and dense hadron matter fireball is formed in central collisions of relativistic 158*A* GeV heavy Pb nuclei with Pb target, comprising a new state of matter [1]. Driven by internal pressure, a fireball expands and ultimately a breakup (hadronization) into final state particles occurs. Early on in the hadron production data analysis it was discovered that strange hadrons emerge from a source in which strange s and antistrange $\bar{s}$ quarks have the same size phase space [2]. This is the case if final state hadrons are directly produced by the deconfined quark matter phase.

The required sudden fireball breakup could arise if a fireball made of the new form of matter significantly supercools, and in this state encounters a strong mechanical instability [3]. Despite extensive ensuing study, the mechanisms determining when and how sudden hadronic particle abundances are formed (chemical freeze-out) have not been fully understood [4–6]. However, there is growing evidence that the fireball breakup occurs over a relatively short period of time [7–9].

We propose and study here a natural mechanical instability criterion ensuing the fireball expansion into the metastable supercooled state. We consider the exploding fireball dynamics in its center of momentum frame of reference. The surface normal vector of exploding fireball is $\vec{n}$, and the local velocity of matter flow $\vec{v}_{\rm c}$. The rate of momentum flow vector $\vec{\mathcal{P}}$ at the surface is obtained from the energy-stress tensor T_{kl} [10]:

$$\vec{\mathcal{P}} = P^{(i)}\vec{n} + (P^{(i)} + \varepsilon^{(i)})\frac{\vec{v}_{\rm c}\vec{v}_{\rm c}\cdot\vec{n}}{1-\vec{v}_{\rm c}^{2}}\,. \qquad (1)$$

The upper index (i) refers to the intrinsic energy density ε and pressure P of matter in the frame of reference, locally at rest, i.e., observed by a comoving observer. We omit the superscript (i) in the following. For the fireball expansion to continue, $\mathcal{P} \equiv |\vec{\mathcal{P}}| > 0$ is required. For $\mathcal{P} \to 0$ at $v_{\rm c} \neq 0$, we have a conflict between the desire of the motion to stop or even reverse, and the continued inertial expansion.

When the flow velocity remains large but $\mathcal{P} \to 0$, the intrinsic pressure P must be negative. As an illustration consider the fireball to be made of a quark-gluon liquid confined by an external vacuum pressure $\mathcal{B}$. The total pressure and energy include the particle (subscript p) and the vacuum properties:

$$P = P_{\rm p} - \mathcal{B}, \qquad \varepsilon = \varepsilon_{\rm p} + \mathcal{B}\,. \qquad (2)$$

Equation (1) with $\vec{\mathcal{P}} = 0$ thus reads

$$\mathcal{B}\vec{n} = P_{\rm p}\vec{n} + (P_{\rm p} + \varepsilon_{\rm p})\frac{\vec{v}_{\rm c}\vec{v}_{\rm c}\cdot\vec{n}}{1-v_{\rm c}^{2}}\,, \qquad (3)$$

and it describes the (equilibrium) condition where the pressure of the expanding quark-gluon fluid is just balanced by the external vacuum pressure.

Expansion beyond $\mathcal{P} \to 0$ is in general not possible. A surface region of the fireball that reached it but continues to flow outwards must be torn apart. This is a collective instability, and thus the ensuing disintegration of the fireball matter will be very rapid, provided that much of the surface reaches this condition. We adopt the condition $\vec{\mathcal{P}} = 0$ at any surface region to be the instability condition of an expanding hadron matter fireball.

Negative internal pressure $P < 0$ is a requirement. At this stage the fireball must thus be significantly supercooled. The adiabatic transfer of internal heat into the accelerating flow of matter provides the mechanism which leads on the scale of $\tau = 2 \times 10^{-23}$ s to the development of this "deep" supercooling.

It is possible to determine experimentally if the condition $P < 0$ has been reached. Namely, the Gibbs-Duham relation for a unit volume,

$$P = T\sigma + \mu_b\nu_b - \varepsilon\,, \qquad (4)$$

relates the pressure, to entropy density $\sigma = S/V$, energy density $\varepsilon = E/V$, and baryon density $\nu_b = b/V$, V is the volume, T is the temperature, and μ_b the baryochemical potential. Dividing by ε we obtain

$$\frac{PV}{E} = \frac{T_{\rm h}}{E/S} + \frac{\mu_b}{E/b} - 1\,. \qquad (5)$$

The microscopic processes governing the fireball breakup determine how the quantities entering the right-hand side of Eq. (5) are changed as hadrons emerge. Understanding this we can determine if the intrinsic fireball pressure prior to breakup has been negative.

The energy E and baryon content b of the fireball are conserved. Entropy S is conserved when the gluon content of a quark-gluon-plasma (QGP) fireball is transformed into quark pairs in the entropy conserving process $G + G \to q + \bar{q}$. Similarly, when quarks and antiquarks recombine into hadrons, entropy is conserved in the range of parameters of interest here. Thus also E/b and S/b are conserved across the hadronization condition. The sudden hadronization process also maintains the temperature T and baryochemical potential μ_b across the phase boundary. What changes are the chemical occupancy parameters. As gluons convert into quark pairs and hadrons $\gamma_g \to 0$ but the number occupancy of light valence quark pairs increases $\gamma_q > \gamma_{q_0} \simeq 1$ increases significantly, along with the number occupancy of strange quark pairs $\gamma_s > \gamma_{s_0} \simeq 1$.

The sudden hadronization picture differs from, e.g., the droplet-driven reequilibration transformation [11,12], in which chemical equilibrium of valence quark pair abundances is maintained. Instead a change (reheating) of statistical parameters T, μ_b occurs, along with a possible formation of a mixed phase required for volume expansion. In such reequilibration hadronization picture, entropy increase can also occur [13]. To draw the line between these two hadronization pictures (equilibrium and sudden), we need to determine the quark pair occupancy parameters γ_i. Our study of final state hadron abundances strongly favors $\gamma_q \simeq \exp(m_\pi/2T) > 1$, $\gamma_s \simeq \gamma_q$ [14].

Evaluating Eq. (5) using the results of our data analysis, we indeed obtain $P_f < 0$. The magnitude of $|P_f|$ can vary between a few percent (in terms of energy density E_f/V), up to 20% for the latest published result [14]. The precise value, which arises from several cancellations of larger numbers, is sensitive to the strategy of how the currently available experimental data are described, e.g., if strangeness conservation is implemented, and if so, if differentially at each rapidity, or as an overall conservation law; how many high mass resonances can be excited in hadronization process, etc....

Importantly, we have not been able to obtain a scheme of hadron production analysis which describes the data with χ^2/d.o.f. < 1 and would not imply $P < 0$ for the hadronizing fireball matter. On the other hand, if we do force the hadronizing particles to be in chemical equilibrium, we find χ^2/d.o.f. > 2.5, d.o.f. $= 10$ in our analysis which agrees for this limit with [15], and in this case we find $P > 0$.

Understanding in detail the breakup condition $\mathcal{P} \to 0$ requires that we model the shape and direction of flow in the late stage of fireball evolution, obviously not an easy task. However, considering $\vec{n} \cdot \vec{\mathcal{P}} \to 0$, we find the constraint

$$\frac{-PV/E}{1 + PV/E} = \kappa \frac{v_c^2}{1 - v_c^2}, \qquad \kappa = (\vec{v}_c \cdot \vec{n})^2/v_c^2. \quad (6)$$

For an exactly symmetrical, spherical expansion the two vectors $\vec{v}_c$ and $\vec{n}$ are everywhere parallel, thus $\kappa \to 1$. However, in 158A GeV Pb-Pb reactions the longitudinal flow is considerably greater than the transverse flow [16], and we note $\kappa \to 0$ for a longitudinally evolving cylindrical fireball. For the Pb-Pb collisions considered here, our analysis suggests $0.1 < \kappa < 0.6$.

We now substitute, in Eq. (6), the fireball matter properties employing the Gibbs-Duham relation, Eq. (4), and arrive at

$$\frac{E}{S} = \left(T_h + \frac{\mu_b}{S/b}\right)\left\{1 + \kappa \frac{v_c^2}{1 - v_c^2}\right\}. \quad (7)$$

Equation (7) establishes a general constraint characterizing the fireball breakup condition.

The solid line, in Fig. 1, shows the behavior of $v_c(T_h)$ constraint arising from Eq. (7) for the example $E/S = 0.184 \pm 0.05$ GeV (error range shown by dotted lines), $\kappa = 0.6$. Outside of the region bounded by the solid line (i.e., for greater T_h and v_c), the flow expansion can occur as the internal particle pressure is greater than the confining pressure. Also shown in Fig. 1 is the hadron production analysis result [14] and its statistical error, the systematic error is of the same magnitude. The agreement of theory and experiment results from the choice of nonspherical flow with the specific freeze-out shape described by the average value $\kappa = 0.6$; see Eq. (6). Figure 1 illustrates the great sensitivity to the analysis on the freeze-out constraint. The dashed horizontal line, in Fig. 1, is the velocity of sound of the interacting quark-gluon liquid, which barely differs from $1/\sqrt{3}$ [17].

We have so far not used in the discussion any key specific property of the equations of state of the matter filling the fireball. However, our results imply that the matter inside the fireball is deeply supercooled. Can this be the deeply supercooled liquid of quarks and gluons? In fact,

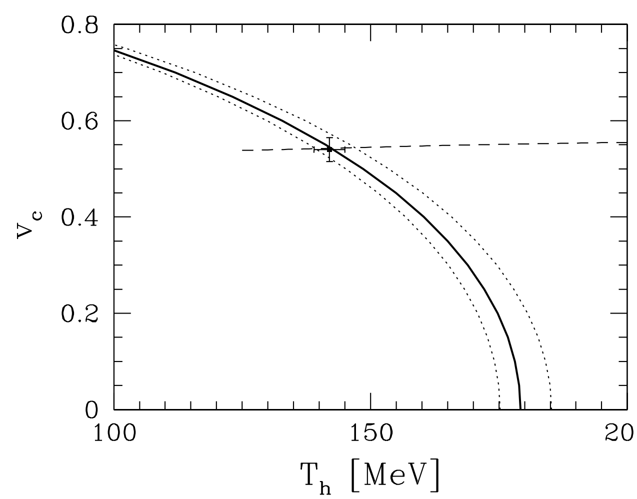

FIG. 1. Fireball velocity as a function of the breakup temperature constraint for the case $E/S = 0.185 \pm 0.005$ GeV, with $S/b = 42$ and $\mu_b = 0.2$ GeV; the dotted lines describe the uncertainty in the determination of E/S. Dashed line: velocity of sound of relativistic quark-gluon liquid. Also shown is the hadron production analysis result [14].

a study of QGP equations of state employing properties of QCD interactions and thermal QCD [17], fine-tuned to agree with the properties of lattice QCD results [18], suggests that. We extend this study to consider the phase boundary. The thin solid line in the T, μ_b plane in Fig. 2 shows where the pressure of the quark-gluon liquid equals the equilibrated hadron gas pressure. The hadron gas behavior is obtained evaluating and summing the contributions of all known hadronic resonances considered to be point particles. When we allow for the finite volume of hadrons [19], we find that the hadron pressure is slightly reduced, leading to some (5 MeV) reduction in the equilibrium transition temperature, as is shown by the dashed line in Fig. 2. For vanishing baryochemical potential, we note in Fig. 2 that the equilibrium phase transition temperature is $T_{\rm pt} \simeq 172$ MeV, and when finite hadron size is allowed, $T_{\rm fp} \simeq 166$ MeV, The scale in temperature we discuss is the result of comparison with lattice gauge results. Within the lattice calculations [18], it arises from the comparison with the string tension.

The dotted lines, in Fig. 2, correspond to the condition Eq, (3) using the shape parameter $\kappa = 0.6$, Eq. (6), for (from right to left) $v_c^2 = 0$, 1/10, 1/6, 1/5, 1/4, and 1/3. The last dotted line corresponds thus to an expansion flow with the velocity of sound of relativistic noninteracting massless gas. The thick solid line corresponds to an expansion with $v_c = 0.54$. The hadron analysis result is also shown [14]. Comparing in Fig. 2 the thin solid and dashed lines with the thick line, we recognize the deep supercooling as required for the explosive fireball disintegration. The supercooled zero pressure $P = 0$ QGP temperature is at $T_{\rm sc} = 157$ MeV (see the intercept of the first dashed line to the right in Fig. 2), and an expanding fireball can deeply supercool to $T_{\rm dsc} \simeq 147$ MeV (see the intercept of the thick solid line) before the mechanical instability occurs.

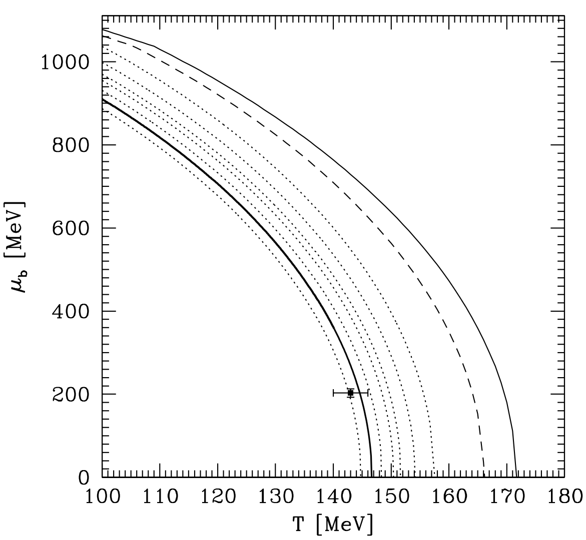

FIG. 2. Thin solid and dashed lines: equilibrium phase transition from hadron gas to QGP liquid without and with excluded volume correction, respectively. Dotted: breakup condition at shape parameter $\kappa = 0.6$, for expansion velocity $v_c^2 = 0$, 1/10, 1/6, 1/5, 1/4, and 1/3, and thick line for $v_c = 0.54$. The experimental point denotes the chemical nonequilibrium freeze-out analysis result [14].

Deep supercooling requires a first order phase transition, and this in turn implies the presence of latent heat $\mathcal{B}$. Physical consistency then requires the presence of external (negative) vacuum pressure $-\mathcal{B}$. More precisely, the vacuum contribution to the physical properties of deconfined matter can be derived from $\ln\mathcal{Z}_{\rm vac} \equiv -\mathcal{B}V\beta$:

$$P_{\rm vac} = \frac{T}{V}\ln\mathcal{Z}_{\rm vac} = -\mathcal{B}\,, \qquad (8)$$

$$\varepsilon_{\rm vac} = -\frac{\partial \ln\mathcal{Z}_{\rm vac}}{V\partial\beta} = \mathcal{B}\left\{1 + \frac{\partial \ln\mathcal{B}/\mathcal{B}_0}{\partial \ln\beta/\beta_0}\right\}. \qquad (9)$$

The temperature, $T = 1/\beta$, dependence of the vacuum pressure has been considered within the model of the color-magnetic vacuum structure [20,21]. Near the phase transformation condition, the variation of $\mathcal{B}$ with β is minimal (see Fig. 2 in [20]), and thus the logarithmically small last term in Eq. (9) can be, in principle, ignored.

We now combine the theoretical properties of the QGP equations of state with the dynamical fireball properties in order to constrain $\mathcal{B}$. Reviewing Eq. (5), we obtain

$$-\frac{PV}{E}\,\varepsilon_{\rm QGP} + P_{\rm p} = \mathcal{B}\,. \qquad (10)$$

To evaluate $\mathcal{B}$, we note that lattice results for $\varepsilon_{\rm QGP}$ are well represented by $\varepsilon_{\rm QGP} = aT^4$, with $a \simeq 11$, value extrapolated for the number of light quark flavors being $n_f = 2.5$ at the hadronization point [17]. We obtain, for the fireball formed in Pb-Pb reactions,

$$0.2 \times 11T_{\rm h}^4 \simeq 0.17\ {\rm GeV/fm}^3 \leq \mathcal{B}\,.$$

Is our picture of fireball evolution compelling? We found that particle production occurred at a condition of negative pressure expected in a deeply supercooled state and have shown internal consistency with (strange) hadron production analysis involving chemical nonequilibrium. Moreover, these chemical freeze-out conditions agree with thermal analysis [7], allowing the conjecture that the explosive quark-gluon fireball breakup forms final state hadrons, which do not undergo further reequilibration. However, we noted that the chemical equilibrium reaction picture differs from ours only in terms of its statistical significance (χ^2/d.o.f. > 2.5 [15]). It produces a chemical freeze-out temperature of $T = 168$ MeV, just the value we found for an equilibrium phase transition implicit in the assumption of chemical equilibrium. The higher chemical freeze-out temperature produces a greater population of excited hadronic states. Their decays deform hadron spectra, and this allows for a second evolution stage with a thermal freeze-out temperature at or below 120 MeV.

Strongly in favor of the here described sudden QGP hadronization resulting in the chemical nonequilibrium reaction picture is the presence of a hadron multiplicity excess, related to an entropy excess [22]. This is seen both as multiplicity per baryon, and as the increase of multiplicity comparing *pA* to *AA* reactions. We could not describe this effect by admitting other physical models such as, in medium, change of hadron masses. We have found that invariably the statistical significance of the analysis decreases as we modify individual hadron properties in an *ad hoc* fashion, while maintaining a chemical equilibrium of hadron abundances. Our experience shows that the only theoretical description of hadron production data that works (χ^2/d.o.f. < 1) requires an excess of valence quark pair abundance, irrespective of the detailed strategy of data analysis. This agrees well with the dynamical study of nuclear collisions within the ultrarelativistic quantum molecular dynamics model that concludes that at CERN energies the chemical nonequilibrium is required to characterize the numerical results in terms of a statistical model [23].

Also in favor of our result is the conclusion of Csörgő and Csernai [3], who required the following as verification for the presence of a deeply supercooled state of matter and sudden hadronization: (i) short duration and relatively short mean proper time of particle emission, now seen in particle correlations [7]; (ii) clean strangeness signal of QGP [24]; (iii) universality of produced particle spectra which are the remarkable features of strange particle production [25]; and (iv) no mass shift of the phi meson; despite an extensive search such a shift has not been found by the NA49 collaboration [26].

In summary, we have introduced a constraint, Eq. (7), which relates the physical and statistical properties of the hadronic fireball at the point of sudden breakup. We obtained this result from mechanical stability consideration, employing only properties of the energy-stress tensor of matter, and the Gibbs-Duham relation, Eq. (4). We showed that this constraint is consistent with analysis results obtained considering the experimental particle production data for Pb-Pb collisions at 158*A* GeV. We further studied the behavior of the phase transition between hadron gas and quark-gluon liquid, and have determined the magnitude of the deep supercooling occurring in the fireball expansion. Employing a lattice-QCD based estimate on the number of degrees of freedom in the energy density of the QCD thermal matter, we obtained a constraint on the magnitude of latent heat/vacuum pressure $\mathcal{B} \geq 0.17\ \mathrm{GeV/fm^3}$. We conclude that in theoretical study of the data as well as for reasons of principle the deciding factor about the sudden nature of the phase transformation is the absence of chemical equilibrium.

We thank T. Csörgő for constructive comments. This work was supported in part by a grant from the U.S. Department of Energy, DE-FG03-95ER40937. Laboratoire de Physique Théorique et Hautes Energies, LPTHE, at University Paris 6 and 7 is supported by CNRS as Unité Mixte de Recherche, UMR7589.

[1] "Evidence for a New State of Matter: An Assessment of the Results from the CERN Lead Beam Program," compilation by U. Heinz and M. Jacob of the consensus view of experiment spokesmen. E-print nucl-th/0002042.
[2] J. Rafelski, Phys. Lett. B **262**, 333 (1991); J. Letessier and J. Rafelski, Acta Phys. Pol. B **30**, 153 (1999).
[3] T. Csörgő and L. P. Csernai, Phys. Lett. B **333**, 494 (1994).
[4] L. P. Csernai and I. N. Mishustin, Phys. Rev. Lett. **74**, 5005 (1995).
[5] T. S. Biró, P. Lévai, and J. Zimányi, Phys. Rev. C **59**, 1574 (1999).
[6] I. N. Mishustin, Phys. Rev. Lett. **82**, 4779 (1999).
[7] A. Ster, T. Csörgő, and B. Lörstad, Nucl. Phys. **A661**, 419c (1999).
[8] E. E. Zabrodin, L. V. Bravina, L. P. Csernai, H. Stöcker, and W. Greiner, Phys. Lett. B **423**, 373 (1998).
[9] Saeed-uddin, Phys. Lett. B **406**, 123 (1997).
[10] L. D. Landau and E. M. Lifschitz, *Fluid Mechanics* (Butterworth-Heinemann, Oxford, 1999); S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
[11] L. P. Csernai and J. I. Kapusta, Phys. Rev. D **46**, 1379 (1992).
[12] A. Dumitru, C. Spieles, H. Stocker, and C. Greiner, Phys. Rev. C **56**, 2202 (1997).
[13] L. P. Csernai and J. I. Kapusta, Phys. Rev. Lett. **69**, 737 (1992).
[14] J. Letessier and J. Rafelski, Int. J. Mod. Phys. E **9**, 107 (2000), and references therein.
[15] P. Braun-Munzinger, I. Heppe, and J. Stachel, Phys. Lett. B **465**, 15 (1999).
[16] NA49 Collaboration, H. Appelshäuser *et al.,* Phys. Rev. Lett. **82**, 2471 (1999).
[17] S. Hamieh, J. Letessier, and J. Rafelski, Phys. Rev. C **62**, 064901 (2000).
[18] F. Karsch, E. Laermann, and A. Peikert, Phys. Lett. B **478**, 447 (2000).
[19] R. Hagedorn and J. Rafelski, Phys. Lett. **97B**, 136 (1980).
[20] B. Müller and J. Rafelski, Phys. Lett. **101B**, 111 (1981).
[21] J. I. Kapusta, Nucl. Phys. **190B**, 425 (1981).
[22] J. Letessier *et al.,* Phys. Rev. Lett. **70**, 3530 (1993); Phys. Rev. D **51**, 3408 (1995).
[23] L. V. Bravina *et al.,* Phys. Lett. B **459**, 660 (1999).
[24] WA97 Collaboration, F. Antinori *et al.,* Nucl. Phys. **A663**, 717 (2000).
[25] WA97 Collaboration, F. Antinori *et al.,* Eur. Phys. J. C **14**, 633 (2000).
[26] NA49 Collaboration, R. A. Barton *et al.,* J. Phys. G (to be published).