Experimental Evidence of Binary Aperiodic Stochastic Resonance

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A novel kind of aperiodic stochastic resonance is experimentally studied in a vertical cavity surface emitting laser. We characterize the response of the system to a random, binary signal as a function of an applied external noise. A maximum in the input-output correlation is found for a nonzero added noise. We present analytic results with a good agreement with the measurements. We also discuss the physical meaning of the phenomenon using simple arguments, and we compare it to stochastic resonance.

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In the past years many evidences of noise-induced ordering have been given in different fields, ranging from biology and geology, to information theory and physics. An important example of such a behavior is represented by stochastic resonance (SR). SR is a specific response of bistable systems to a sinusoidal modulation in the presence of noise. An improvement of the quality of the output signal is observed as the amount of noise is increased, up to an optimal (resonant) value. SR has been introduced in 1981 [1] to explain the periodicity of the continental ice volume in the quaternary era, and it has been the object of extensive investigations, mainly since the experimental evidence in a bistable ring laser [2]. SR has been studied in detail with analogic simulations and with analytic and numeric investigations [3]. Recently, the observation of SR in the dynamical behavior of a vertical cavity surface emitting laser (VCSEL) [4] has provided experimental results of the same quality of model simulations, allowing for the first time the direct verification of many of the predictions of the theory [5].

SR provides a very interesting mechanism to improve the quality of signal amplification in nonlinear systems, especially for the transmission of information through noisy channels. However, while investigations of SR have provided important insights into the physics of noise-induced resonance, the generalization from a sinusoidal input to an arbitrary shaped signal is not straightforward. As an example, the matching condition between the input signal frequency and the Kramers rate, allowing for the highest value of the signal-to-noise ratio, is typical of periodic signals.

The response of nonlinear systems to small, *aperiodic* signals in the presence of noise has been recently addressed. In 1994 Neiman *et al.* [6] have studied the transmission of harmonic noise (i.e., the response of a 2nd-order linear filter to a white, Gaussian noise) through a bistable, symmetric system in the presence of external white noise. The term aperiodic stochastic resonance (ASR) was coined in Ref. [7], referring to the amplification of subthreshold, random signals in the Fitz Hugh–Nagumo model of an excitable system. Since the input signal is broadband, the standard time scale matching condition of SR does not apply: the resonance can be shown by evaluating the cross correlation of the output versus the input signal. Several authors have provided evidence of resonant behavior as a function of the noise in biological systems [8].

In this Letter we experimentally investigate the response of a bistable, optical system to a random, binary (telegraph) signal changing the amount of applied external noise. We show for the first time the experimental evidence of ASR for such a simple stochastic input, which is of particular interest in digital communications. The reproducibility and the possibility to control the parameters make our work qualitatively different from previous observations. As a consequence, our system allows for the first experimental analysis of phenomena which are very important in nature, as suggested by several papers on this subject.

The physical system which allows us to observe and characterize the phenomenon described in this Letter is composed of a VCSEL followed by a polarizer and a detection system (see Ref. [5] for a detailed description of the setup). By sweeping the pump current, the laser can emit in different polarization and transverse profile configurations. The transition between two states is generally characterized by a bistable current region, where noise-induced jumps occur between the two states. The laser dynamics can be reconducted in this case to a van't Hoff-Arrhenius process, with average permanence times usually given by a Kramers law, i.e., exponentially decreasing with the noise intensity [9]. By introducing additional noise in the pump current, the statistics of the jumps can thus be changed. The polarization fluctuations are transformed into light intensity variations by the polarizer and detected by means of a photodiode (400 kHz bandwidth) whose signal is acquired by a digital scope. The signals from a variable intensity, white-noise generator (10 MHz bandwidth) and a pseudorandom binary sequence generator are summed and coupled into the laser input current. The binary sequence is a 16 000-bit word with a bit duration of $T_b = 4$ μs. Its amplitude is 0.27 mA (peak-peak), smaller than the width of the bistable region (0.49 mA). As a consequence, the

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current steps are not large enough to induce a laser state jump without the aid of the noise.

The signal detected by the photodiode, for different values of the input noise strength, is shown in Fig. 1. For low noise (Fig. 1a) the laser mainly remains in its initial state, even if a small amplitude modulation is visible. Increasing the noise, some jumps occur (Fig. 1b). For an input noise around 400 $\mathrm{mV_{rms}}$, the output follows very well the input signal (Fig. 1c). For larger noise strengths, the laser dynamics is determined by the noise more than by the input string, with a strong decorrelation between input and output (Figs. 1d and 1e).

This is a clear indication of a noise-induced resonance, which yields the first experimental evidence of ASR in a real system. The particular type of aperiodic input signal allows for an exact theoretical treatment and for the understanding of the phenomenon in terms of simple physical arguments. Moreover, this system is also of great interest for its applications in communication, in computer architecture, and in almost all real digital systems.

To better quantify the observed behavior, we evaluate the cross correlation between the input and the output signal. We plot for each value of the noise the maximum of the normalized correlation

$$C_{\mathrm{IO}} = \max_{\tau}\{\overline{[x_{\mathrm{in}}(t) - \overline{x}_{\mathrm{in}}][x_{\mathrm{out}}(t + \tau) - \overline{x}_{\mathrm{out}}]}\}, \qquad (1)$$

where the overline denotes the time average and the variables x_{in} and x_{out} are rescaled with respect to half the difference between the two stable states. The result is shown in Fig. 2. A well defined peak is present at $D \simeq 350\ \mathrm{mV_{rms}}$, indicating the optimal reproduction of the input signal. For each value of the noise the cross correlation has a maximum for a nonzero time lag $\tau_{\max}$ between input and output. However, we point out that the experimental values of C_{IO} are not significantly changed if a zero time lag is chosen, instead of $\tau_{\max}$. For the sake of simplicity, we will present in the following a theoretical analysis neglecting the time lag. A more detailed analysis will be reported elsewhere.

In Fig. 2 we also show the result obtained from an analytic expression of the correlation function, based on a two-state model, derived using only the experimental Kramers times. The time evolution within a bit length depends on the initial state, and it is ruled by a van't Hoff-Arrhenius law [9]. From the statistical analysis of such an evolution, we get the probability distribution $P_{\mathrm{in=out}}(T)$ of the residence times T spent by the system in the right state (i.e., the one with the highest activation energy, corresponding to the input state). It is given by

$$P_{\mathrm{in=out}}(T) = \frac{1}{2}\Bigg\{\exp\left(-\frac{T}{T_+}\right)\exp\left(-\frac{T_b - T}{T_-}\right) \times \left[\frac{T_b}{T_+T_-}\sqrt{\frac{T_+T_-}{T(T_b - T)}} \times I_1\left(2\sqrt{\frac{T_+T_-}{T(T_b - T)}}\right) + \left(\frac{1}{T_+} + \frac{1}{T_-}\right) \times I_0\left(2\sqrt{\frac{T_+T_-}{T(T_b - T)}}\right)\right] + \delta(T - T_b)\exp\left(-\frac{T_b}{T_+}\right) + \delta(T)\exp\left(-\frac{T_b}{T_-}\right)\Bigg\}, \qquad (2)$$

where I_n are the nth-order modified Bessel functions and T_+ (T_-) is the longer (smaller) residence time. The correlation C_{IO} is given by

$$C_{\mathrm{IO}} = \int_0^{T_b} dT\ \frac{2T - T_b}{T_b}\, P_{\mathrm{in=out}}(T). \qquad (3)$$

The experimental data are well reproduced by the model; the details of the calculations are given elsewhere [10].

To better understand the physical meaning of our measurements, let us consider the limit cases of low and high noise. For low noise, the system response during each bit is strongly influenced by the final output state in the previous bit. Indeed, the same output state can be maintained for a time duration corresponding to several bits (see Figs. 1a and 1b). As a result, the output signal is strongly decorrelated from the input. To quantify the above interpretation, we plot in Fig. 3 the correlation length θ_{out} of the output signal, defined as $\theta = 2\int_0^{\infty} C(t)\,dt$, where C is the normalized autocorrelation. The input correlation length is thus $\theta_{\mathrm{in}} = T_b$. For noise strengths lower than about 300 $\mathrm{mV_{rms}}$, θ_{out} is longer than the T_b, providing evidence of a memory effect. As a consequence, the system is likely to remain in its initial state. At resonance (about 350 $\mathrm{mV_{rms}}$), θ_{out} approaches T_b, as expected: the noise is strong enough to supersede the memory effect. Increasing the noise power, the correlation length decreases as the residence times become smaller than T_b: fast fluctuations are found in the response (see Figs. 1d and 1e), leading again to the decorrelation of the output signal versus the input.

A simple estimation of the input-output correlation C_{IO} can be given in the two limit cases of low and high noise. For low noise, we consider that at most a single jump towards the right output can happen within a bit: this first jump is necessary to lose the memory of the previous state. The correlation can therefore be evaluated as the sum of two contributions, corresponding to the system starting, respectively, in the right and the wrong states

$$C_{\mathrm{IO}} \simeq \frac{1}{2} + \frac{1}{2}\left[\int_0^{T_b}\frac{T_b - 2t}{T_bT_-}\exp\left(-\frac{t}{T_-}\right)dt - \exp\left(-\frac{T_b}{T_-}\right)\right] = \frac{T_-}{T_b}\left[\exp\left(-\frac{T_b}{T_-}\right) - 1\right] + 1\,. \qquad (4)$$

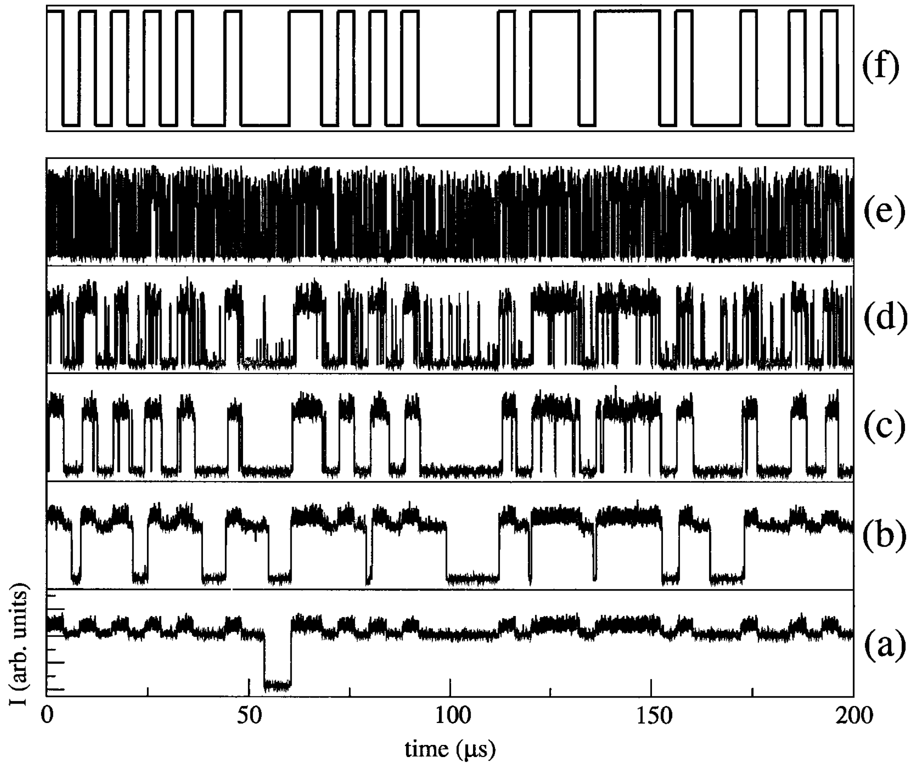

FIG. 1. Polarized laser intensity I for different input noise intensities, with a 50 MHz acquisition bandwidth. (a) 100, (b) 200, (c) 400, (d) 600, and (e) 1200 mV_{rms}. In (f) the input pattern is shown (with an arbitrary vertical scale). A sequence of 50 bits (bit duration: 4 μs) is displayed.

For high noise, many jumps occur within a bit; thus $C_{IO} \simeq n(T_+ - T_-)/T_b$ where $n = T_b/(T_+ + T_-)$ is the (average) number of jumps, giving

$$C_{IO} \simeq \frac{T_+ - T_-}{T_+ + T_-}. \tag{5}$$

The plots of Eqs. (4) and (5), evaluated using the experimental residence times, are reported in Fig. 4, showing a very good agreement with the experimental data.

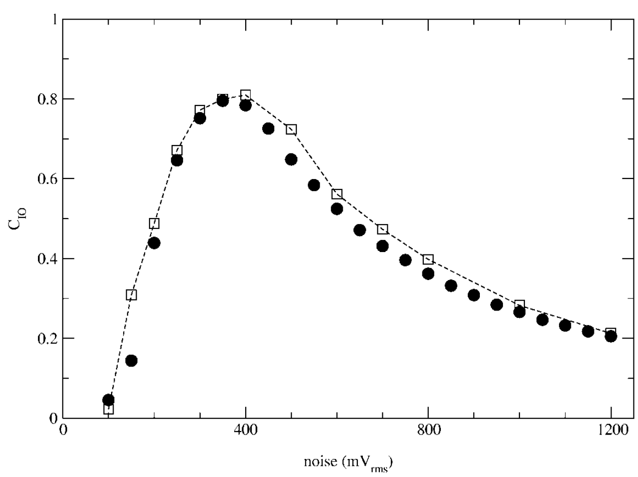

FIG. 2. Normalized cross correlation between the input and the output signals. Dots: experimental data. Squares: analytic results of Eq. (3) using the experimental Kramers times. The dashed line is a guide for the eye.

While the correlation is not significantly affected by the time lag, it is, however, interesting to study the behavior of τ_{max} as a function of input noise. Such a dependence is well established in SR (see, e.g., Ref. [3]). In that case, given an input signal of the form $A\cos(\Omega t)$, the two-state model in the limit of small modulation amplitude gives to first order in A a term proportional to $\cos(\Omega t - \phi)$. The output is dephased by $\phi = \arctan(\Omega/2r_K)$, where r_K is the Kramers rate. In our case, an approximate expression for τ_{max} can be deduced from ϕ by the following considerations. The probability to have an input sequence of

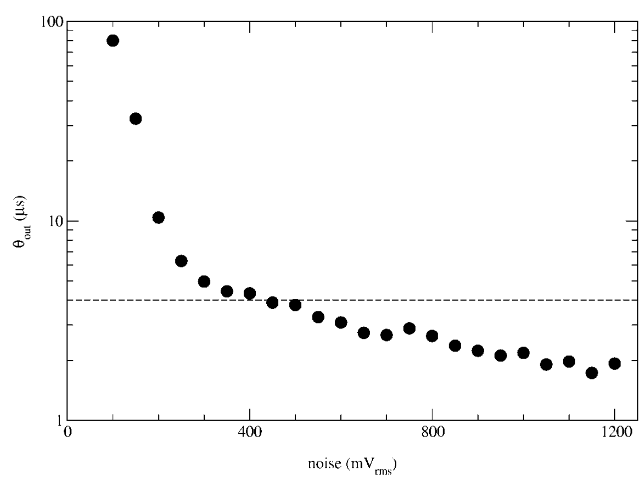

FIG. 3. Correlation length θ_{out} of the output signal. The horizontal, dashed line marks the bit duration.

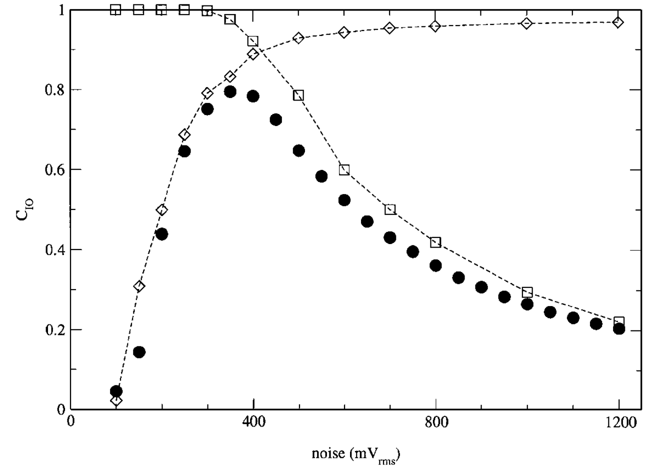

FIG. 4. Normalized input-output cross correlation. Dots: experimental data. The asymptotic results for low noise [Eq. (4), diamonds] and for high noise [Eq. (5), squares] are shown using the experimental Kramers rates for the calculations. The dashed lines are guides for the eye.

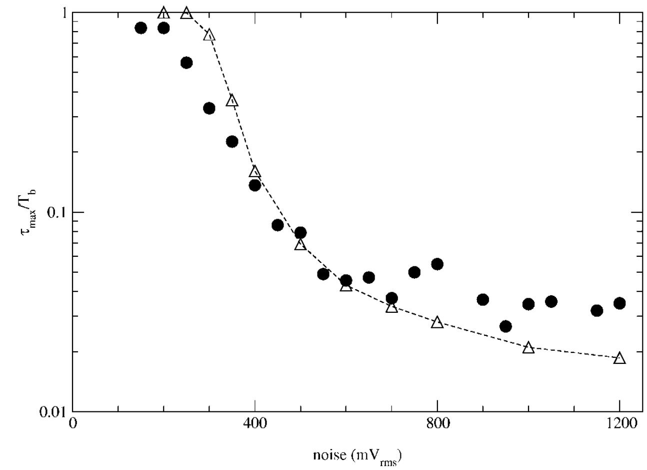

FIG. 5. Time shift between the input and the output signal, as deduced by the abscissa of the cross-correlation maximum. Dots: experimental measurements. Triangles: values deduced from Eq. (6), using the experimental Kramers times. The dashed line is a guide for the eye.

length nT_b is $p_n = 2^{-n}$. The average permanence time of the input signal, in one of the two levels, is therefore $2T_b$. The system is thus modulated with an average period $T = 4T_b$, corresponding to a frequency $\Omega = \pi/2T_b$; accordingly, the time lag is

$$\tau_{\max} = \frac{2T_b}{\pi} \arctan\left(\frac{\pi}{4r_K T_b}\right). \quad (6)$$

We show in Fig. 5 the experimental time lags (dots), together with the curve obtained from Eq. (6), using the experimental Kramers rate r_K. Since we have no free parameters, the agreement is quite good, suggesting that the synchronization is a global phenomenon resulting from an average effect of the input signal on the system. Moreover, we have measured the time response of the polarization to a current variation by evaluating the width of the hysteresis cycle changing the frequency of a current sweep. The characteristic response time is around 100 ns, thus explaining the discrepancy between the theory and the experimental results in Fig. 5 for high noise.

In conclusion, we report on the experimental evidence of ASR for a binary input signal in a real system. The resonant behavior occurs when the added noise is strong enough to remove memory effects. For higher noise, as the residence times in the two states are asymptotically equal, the bit-averaged output signal becomes independent from the input signal value. The phenomenon presented in this Letter is of great importance not only in the field of digital data transmission, but also from the point of view of fundamental physics. Indeed, the study of the resonant conditions for a dichotomic signal provides a useful tool to study, e.g., the transition to ergodic behavior and phase synchronization in a simple system. These subjects are very promising and are currently under investigation.

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