Effects of Spatial and Temporal Smoothing on Stimulated Brillouin Scattering in the Independent-Hot-Spot Model Limit

Ph. Mounaix,[1] L. Divol,[2] S. Hüller,[1] and V. T. Tikhonchuk[3]
[1]*Centre de Physique Théorique, UMR 7644 du CNRS, Ecole Polytechnique, 91128 Palaiseau Cedex, France*
[2]*Commissariat à l'Energie Atomique, F-91680 Bruyères-le-Chatel, France*
[3]*P.N. Lebedev Physics Institute, Russian Academy of Science, Moscow 117924, Russia*
(Received 6 April 2000)

The influence of laser beam smoothing on stimulated Brillouin backscattering (SBBS) is studied analytically in the limit of the independent hot spot model. It is shown that the temporal beam smoothing can reduce the SBBS reflectivity significantly even though the laser bandwidth is *smaller* than the growth rate for the average intensity. The explanation of this reduction effect is given in terms of SBBS growth in the statistically significant hot spots. The minimum laser bandwidth corresponding to an important reduction of the reflectivity is thus determined properly. The dependence of this reduction effect on the acoustic damping for a given laser bandwidth is discussed.

PACS numbers: 52.40.Nk, 52.35.Mw, 52.35.Nx

Much experimental and theoretical work has been devoted over the last two decades to studying the influence of laser beam smoothing on scattering instabilities. In the case of both spatial and temporal smoothing, such as, e.g., smoothing by induced spatial incoherence (ISI) [1], the early theoretical work on this topic [2] was restricted to the regime where the linear growth rate $\langle\gamma_0\rangle$ of the instability for the average laser intensity is much smaller than the laser bandwidth $\Delta\omega_0$. In this limit, it was found that the temporal beam smoothing can reduce the gain factor of the instability by a factor $\sim\langle\gamma_0\rangle/\Delta\omega_0 \ll 1$. By continuity with the coherent limit, it was furthermore conjectured that no important reduction effect should be expected in the complementary domain $\langle\gamma_0\rangle \geq \Delta\omega_0$. This conjecture was widely accepted until the late 1980s when Mostovych *et al.* [3] strongly questioned it by showing experimentally that a significant reduction of the instability could occur even though $\langle\gamma_0\rangle \geq \Delta\omega_0$. Although this effect was observed more than ten years ago, no satisfactory theoretical account of this reduction had been given so far.

The regimes currently of practical interest correspond to the limit $\langle\gamma_0\rangle > \Delta\omega_0$ where the scattering instabilities can grow significantly in many small scale hot spots (or speckles) of finite lifetime $\tau_c \sim \Delta\omega_0^{-1}$ randomly distributed in time and throughout the interaction region. The macroscopic reflectivity of the plasma is thus expected to be mainly determined by the rare high intensity hot spots. In the case of an inhomogeneous plasma, multiple amplifications in successive hot spots is prevented by the resonance mismatch and the instability is properly described by the so-called "independent hot spot model" [4]. This model is characterized by the following: (i) an independent description of the backscattering instability from each single intense hot spot, and (ii) an averaging over the hot spot intensity to obtain the overall (macroscopic) reflectivity. Step (i) of this model, including a proper description of important diffraction effects due to the needlelike hot spot shape, was carried out in Refs. [5] and [6] in the context of the broadband-ISI limit (BISI) in which the spatial location of each intense hot spot does not move significantly during its lifetime. There, the reflectivity of a three-dimensional (3D) cylindrical hot spot of finite lifetime was computed analytically and written in the form of a uniform expression valid for any hot spot intensity. In the present Letter, we use this result to perform step (ii) of the independent hot spot model in the case of stimulated Brillouin backscattering (SBBS). We show that the temporal smoothing can reduce SBBS significantly even in the regime $\langle\gamma_0\rangle > \Delta\omega_0$, and we discuss the dependence of this reduction effect on the acoustic damping for a given laser bandwidth.

We consider the case of a weakly inhomogeneous plasma in which the resonance length for a given SBBS wave triplet is comparable to the hot spot length. In this limit, SBBS in each hot spot can be treated as in a homogeneous plasma, whereas multiple amplifications in successive hot spots can be neglected due to the fact that the light backscattered in a given hot spot is out of resonance in any other hot spot it encounters on its way out of the interaction region. Since the laser intensity between the hot spots is much smaller than the average intensity, we neglect SBBS outside the hot spots and assume that, within each hot spot, it grows from the thermal level of the acoustic noise as given by the standard fluctuation theory [5]. This implies, in particular, that the acoustic damping is large enough for the SBBS ion sound wave to have time to get back to the thermal level between two successive occurrences of a hot spot at a given place. For this condition to be fulfilled, the acoustic damping must typically be greater than a few tenths of ps^{-1} (i.e., the damping normalized to the acoustic frequency must be greater than a few percent) [7]. In order to apply the results of Refs. [5] and [6], we model each of the actual speckles by an effective cylindrical hot spot of length L, lifetime T, and on-axis intensity I. T is defined as the full width at half-maximum of the temporal intensity profile of the actual hot spot. I is defined so that the energies

 0031-9007/00/85(21)/4526(4)$15.00

flowing between $-T/2$ and $T/2$ through the cross section of a cylindrical and a real hot spot are the same [8]. L is defined so that the stationary linear convective gains are the same. Approximating the real hot spot intensity profile near its maximum by a quadratic function of space and time, one obtains [9] $T = \tau_c$ and $I = 5I_{\max}/6$, where τ_c is the coherence time of the laser field [assuming a one-point correlation function of the form $\exp(-t^2/\tau_c^2)$] and $I_{\max}$ is the real hot spot peak intensity. The cylindrical hot spot length and waist in the 3D case are $L = 1.36z_c$ and $w_0 = 0.55\rho_c$ for a square top-hat random phase plate (RPP), and $L = 1.99z_c$ and $w_0 = 0.64\rho_c$ for a circular top-hat RPP. Here ρ_c and z_c are defined by $\rho_c = f\lambda_0$ and $z_c = \pi(1 - n_e/n_c)^{1/2}f^2\lambda_0$, where f is the f number, λ_0 is the laser wavelength in vacuum, and n_e/n_c is the electron density normalized to critical.

The energy backscattered by a hot spot during its lifetime is given by

$$E_{\mathrm{HS}}(I) = \int_0^{\tau_c + 2L/c} P_{\exp}(I,t)\,dt\,, \qquad (1)$$

with $P_{\exp}(I,t) = \min[P^L_{\exp}(I,t), P_{\max}(I)]$. Here $P^L_{\exp}(I,t) = \min[P^M_{\exp}(I,t), \xi P^S_{\exp}(I,t)]$ is the linear backscattered power, where $P^M_{\exp}$ and $P^S_{\exp}$ denote the backscattered power in the modified and standard decay regime, as given by Eq. (34) of Ref. [6] and Eq. (41) of Ref. [5], respectively. The normalization factor ξ is taken such that $\xi = 1$ in the large gain limit and $\xi = 2$ in the small gain limit corresponding to Thomson scattering. The maximum backscattered power $P_{\max}(I) = \min(1,\Delta)I\pi w_0^2$ accounts for the hot spot intensity depletion heuristically [10]. The geometrical factor $\Delta \equiv \Delta\Omega_{\exp}/\Delta\Omega_{sc}$ is the ratio of the solid angle in which the backscattered light is collected to the far field solid angle of the backscattered beam. The overall macroscopic reflectivity R_{SBS} can be obtained from Eq. (1) by summing up the hot spot contributions. One finds

$$R_{\mathrm{SBS}} = \frac{1}{\langle I\rangle S_{\mathrm{int}}T_{\mathrm{int}}}\int_{3\langle I\rangle}^{I_{\max}} E_{\mathrm{HS}}(I)\left|\frac{dM_{\mathrm{ISI}}(I)}{dI}\right| dI\,, \qquad (2)$$

where $\langle I\rangle$ is the average laser intensity, S_{int} is the interaction region cross section, and T_{int} is the pulse duration. The quantity $M_{\mathrm{ISI}}(I)$ denotes the average number of hot spots with intensity greater than I that appear during the interaction in the interaction region. It is proportional to the interaction space-time volume $L_{\mathrm{int}}S_{\mathrm{int}}T_{\mathrm{int}}$, where L_{int} is the interaction length. For not too high I, it can be obtained from the theory of stochastic Gaussian fields [11–13] in $d + 1$ dimensions (d space dimensions + 1 time dimension). The cutoff intensity $I_{\max}$ accounts for the poor sampling of very high intensity hot spots in the finite space-time interaction region for a given laser field realization (i.e., for a given shot). A reasonable estimation of $I_{\max}$ can be obtained from the condition $M_{\mathrm{ISI}}(I_{\max}) = 1$ (typically: $10 \leq I_{\max}/\langle I\rangle \leq 20$). Defining the hot spot density $p(I)$ by $|dM_{\mathrm{ISI}}(I)/dI| = (L_{\mathrm{int}}S_{\mathrm{int}}T_{\mathrm{int}})p(I)$, one has according to Garnier [13]

$$p(I) = \frac{C(d)}{w_0^{d-1}L\tau_c}\left(\frac{I}{\pi\langle I\rangle}\right)^{(d+1)/2}\frac{e^{-I/\langle I\rangle}}{\langle I\rangle}\,, \qquad (3)$$

where $C(2) = 20/(3\pi\sqrt{2})$ and $C(3) = (2/\pi + 3/4)$. In the case of two spatial dimensions ($d = 2$), we performed simulations by numerically generating the spatiotemporal intensity distribution of a BISI beam and we checked that Eq. (3) gives the proper hot spot statistics. In the following we will use this hot spot density to evaluate R_{SBS}. Inserting Eq. (3) with $d = 3$ in Eq. (2), one obtains

$$R_{\mathrm{SBS}} = \left(\frac{L_{\mathrm{int}}}{L}\right)\frac{(2/\pi + 3/4)}{\pi^2 w_0^2\langle I\rangle\tau_c}\int_3^{u_{\max}} E_{\mathrm{HS}}(u\langle I\rangle)u^2e^{-u}\,du\,, \qquad (4)$$

where $u \equiv I/\langle I\rangle$. The remainder of this letter is devoted to the discussion and physical interpretation of the results obtained from Eq. (4).

Figure 1 shows R_{SBS} as a function of τ_c for typical interaction parameters (see Fig. 1 caption). One can see a significant reduction of the reflectivity for τ_c as large as 4 ps, which corresponds to $\langle\gamma_0\rangle\tau_c \simeq 10$. This theoretical result confirms the experimental ones of Ref. [3] and contradicts the conjecture expressed in [2], saying that an important reduction of the reflectivity should occur for $\langle\gamma_0\rangle\tau_c \ll 1$ only. The key to understanding our result is to compare the hot spot lifetime τ_c with the longest saturation time t^*_{sat} of SBBS in the hot spots. For a given hot spot, SBBS saturates either convectively at $t = t^{\mathrm{conv}}_{\mathrm{sat}}(u)$ or nonlinearly at $t = t^{NL}_{\mathrm{sat}}(u)$, depending on the hot spot (normalized) intensity u. Here $t^{\mathrm{conv}}_{\mathrm{sat}}(u)$ is given by [5] $t^{\mathrm{conv}}_{\mathrm{sat}}(u) = (L/c)[2 + u(\langle\gamma_0\rangle/\nu_S)^2]$, where ν_S is the linear damping of the ion acoustic wave, and $t^{NL}_{\mathrm{sat}}(u)$ is the time at which the backscattered power is equal to the incident power in the hot spot [14]. The longest saturation

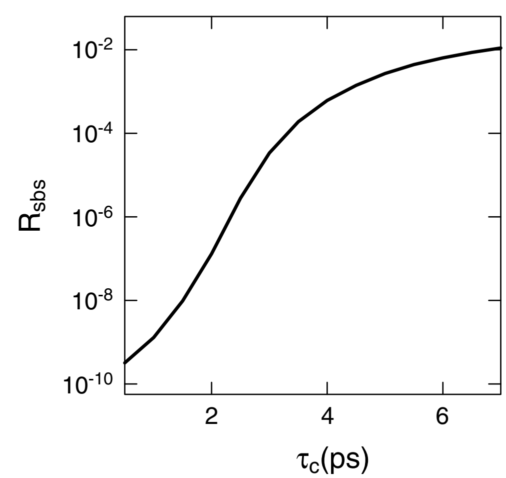

FIG. 1. R_{SBS} as a function of τ_c for a square top-hat RPP with a $f/8$ optics at 3ω. The plasma and laser parameters are $T_e = 3$ keV, $T_i = 1$ keV, $n_e/n_c = 0.1$, $\nu_S/\omega_S = 0.09$, $\langle I\rangle = 2 \times 10^{15}$ W/cm^2, $\langle\gamma_0\rangle = 2.63$ ps^{-1}, and $u_{\max} = 20$.

time $t^*_{\rm sat}$ is obtained straightforwardly from $t^*_{\rm sat} = t^{\rm conv}_{\rm sat}(u^*_{\rm sat})$ with $u^*_{\rm sat} = \min(u^*, u_{\max})$, where u^* is the solution to $t^{NL}_{\rm sat}(u^*) = t^{\rm conv}_{\rm sat}(u^*)$. The quantity $I^*_{\rm sat} \equiv u^*_{\rm sat}\langle I\rangle$ is the intensity of the hot spots in which it takes the longest time for SBBS to saturate. If the inequality $\tau_c \ll t^*_{\rm sat}$ holds, there are very few hot spots in which SBBS has time to saturate, which leads to a reduction of the reflectivity. In the opposite limit $\tau_c \geq t^*_{\rm sat}$, all the hot spots live long enough for SBBS to saturate at its maximum level corresponding to the case of purely spatial smoothing, so that the reflectivity is not reduced by the temporal smoothing. Consequently, we claim that the proper ordering associated with a reduction of the reflectivity in the independent hot spot model limit is

$$\tau_c/t^*_{\rm sat} \ll 1\,, \tag{5}$$

and *not* $\langle\gamma_0\rangle\tau_c \ll 1$. Since one typically has $\langle\gamma_0\rangle^{-1} \ll t^*_{\rm sat}$, one can observe a significant reduction of the reflectivity even though $\langle\gamma_0\rangle\tau_c > 1$. It is important to notice that this reduction mechanism can be effective only if the contribution of the unsaturated hot spots is actually significant compared with that of the saturated ones. It can be checked that this condition is fulfilled provided that the unsaturated hot spot intensity is greater than $3\langle I\rangle$, i.e., if these hot spots can be regarded as high intensity hot spots. Since the intensity range of the unsaturated hot spots spreads around $I^*_{\rm sat}$, one is led to the inequality

$$I^*_{\rm sat} > 3\langle I\rangle\,, \tag{6}$$

which must be added to the condition (5). Figure 2 shows $t^*_{\rm sat}$ (solid line) and $I^*_{\rm sat}$ (dashed line) as a function of the normalized acoustic damping ν_S/ω_S, where ω_S is the acoustic (angular) frequency. For the parameters of Fig. 1 and a typical value of $\tau_c \sim 3$ ps, one has $t^*_{\rm sat} \simeq 15$ ps and $\langle\gamma_0\rangle^{-1} \ll \tau_c \ll t^*_{\rm sat}$. It is worth noticing that the extra condition (6) can play an important role in experimental conditions currently of practical interest. For instance, in the Au blowoff plasma of typical NOVA hohlraums with a $f/8$ optics at 3ω, the acoustic wave damping is so low that one has $t^*_{\rm sat} > 10$ ps and $I^*_{\rm sat} < \langle I\rangle$. In this case, even though $\tau_c < 10$ ps [which meets Eq. (5)], one expects the temporal smoothing to have almost no effect on the high intensity hot spot contribution to the reflectivity as Eq. (6) is not fulfilled. In this regime, the backscattered light comes mainly from low intensity hot spots with $I \sim \langle I\rangle$ the contribution of which cannot be estimated in the frame of the independent hot spot model.

Figure 3 shows $R_{\rm SBS}$ as a function of the normalized acoustic damping for three different values of the average hot spot convective gain $\langle G\rangle_{\rm HS} \equiv 2\langle\gamma_0\rangle^2 L/c\nu_S$. Although the convective gain factor is constant along each curve, increasing the normalized damping from 0.05 to 0.25 makes the reflectivity increase by 4 orders of magnitude for $\langle G\rangle_{\rm HS} = 3.5$–$5$ and by 2 orders of magnitude for $\langle G\rangle_{\rm HS} = 10$. Similar curves were obtained numerically by Berger *et al.* [15] who explained this variation of the reflectivity in terms of renormalization of the acoustic damping appearing in $\langle G\rangle_{\rm HS}$ by the laser bandwidth. Such an explanation, which is typical of the perturbative approach in the regime $\langle\gamma_0\rangle\tau_c \ll 1$, is not correct in the independent hot spot model limit where $\langle\gamma_0\rangle\tau_c \geq 1$. Note that, with the parameters of Fig. 3, one has $\langle\gamma_0\rangle\tau_c \geq 5$, 6, and 8.5 for $\langle G\rangle_{\rm HS} = 3.5$, 5, and 10, respectively. The proper explanation of the variation of $R_{\rm SBS}$ as a function of ν_S/ω_S is as follows. For ν_S/ω_S small enough, one has $\tau_c \ll t^*_{\rm sat}$ (cf. Fig. 2). The reduction effect of the laser bandwidth is very effective and the reflectivity is determined by the hot spots which are still in a weakly damped growth phase at the end of their life. The contribution of these hot spots increases with $\langle I\rangle$ and does not

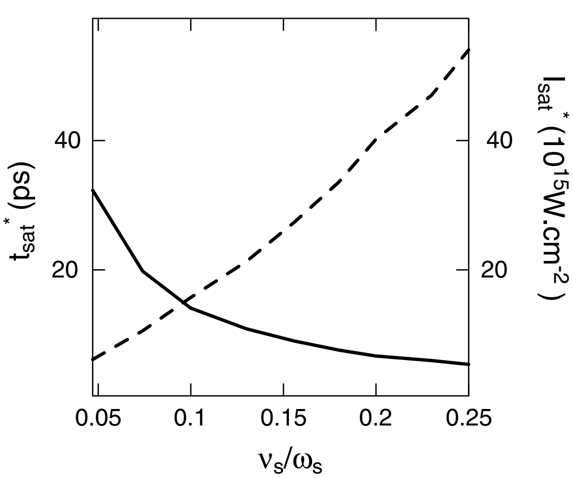

FIG. 2. $t^*_{\rm sat}$ (solid line) and $I^*_{\rm sat}$ (dashed line) as a function of the normalized acoustic damping for a square top-hat RPP with a $f/8$ optics at 3ω. The plasma and laser parameters are $T_e = 3$ keV, 0.7 keV $\leq T_i \leq 2.5$ keV, $n_e/n_c = 0.1$, $\tau_c = 3$ ps, and $u_{\max}$ such that $u_{\max} \geq u^*$ (i.e., $u^*_{\rm sat} = u^*$).

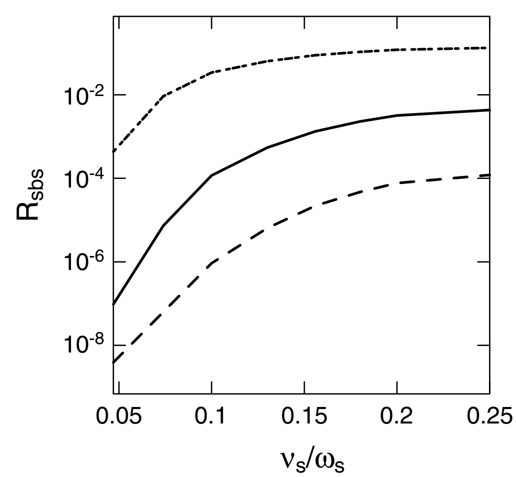

FIG. 3. $R_{\rm SBS}$ as a function of the normalized acoustic damping for three different values of the average hot spot convective gain: $\langle G\rangle_{\rm HS} = 3.5$ (dashed line), $\langle G\rangle_{\rm HS} = 5$ (solid line), $\langle G\rangle_{\rm HS} = 10$ (dot-dashed line). The plasma and laser parameters are the same as in Fig. 2 and $u_{\max} = 20$.

depend significantly on the damping (except through the noise term, which corresponds to a weak dependence). As keeping $\langle G\rangle_{\rm HS}$ fixed yields $\langle I\rangle \sim \nu_S$, it follows that $R_{\rm SBS}$ must increase with ν_S/ω_S at $\langle G\rangle_{\rm HS}$ fixed, as can be seen in Fig. 3. As ν_S/ω_S increases, $t^*_{\rm sat}$ decreases (cf. Fig. 2). It means that the contribution of the saturated hot spots gets more and more important, making the reduction effect of the laser bandwidth less and less effective. For a given ν_S/ω_S, it can also be seen that the reflectivity is closer to its saturation level at high average intensity. This can be attributed to the fact that the condition (6) is less well fulfilled at higher average intensity: the relative contribution of the saturated hot spots to that of the unsaturated ones increases with $\langle I\rangle$ (keeping the other parameters fixed), which weakens the reduction effect of the laser bandwidth. Once $t^*_{\rm sat} \ll \tau_c$, all the hot spots have enough time to saturate well before the end of their life and the reflectivity tends to that of a purely spatially smoothed laser beam. Thus, for large enough ν_S/ω_S and keeping $\langle G\rangle_{\rm HS}$ fixed, one expects $R_{\rm SBS}$ to depend weakly on the damping (through the noise term only), which is in agreement with the results shown in Fig. 3.

One expects our theory to be of practical interest for, e.g., gas-filled hohlraum experiments in future large laser facilities (like the Laser MégaJoule in France and the National Ignition Facility in the United States). In such experiments, the plastic window that confines the gas gives rise to an underdense plasma near the entrance holes, with $n_e/n_c \sim 0.03$ and $T_e \sim 3$ keV typically. In this plasma the SBBS convective gain at average intensity is not too high ($\langle G\rangle_{\rm HS} \sim 4$ for $\langle I\rangle \sim 3 \times 10^{15}$ W/cm^2), and the amplification length due to velocity and density inhomogeneities is comparable to the hot spot length, which validates the use of the independent hot spot model. Without temporal smoothing, the low acoustic damping ($\nu_S/\omega_S \sim 0.06$) could result in a significant SBBS reflectivity, coming mainly from the intense hot spots. As the convective saturation time at $3\langle I\rangle$ is around 6 ps, one can already say that using a typical temporal smoothing scheme with $\tau_c \sim 3$ ps will reduce the reflectivity even though $\langle\gamma_0\rangle\tau_c \sim 3 > 1$.

In this Letter we have studied the influence of laser beam smoothing on SBBS reflectivity analytically. Considering the limit of the independent hot spot model, we have shown that the temporal beam smoothing can reduce the SBBS reflectivity significantly even though the laser bandwidth is *smaller* than the growth rate for the average intensity. In the context of experiments in future large laser facilities, this result shows that the efficiency of temporal smoothing in reducing SBBS should be considered carefully as it may be much better than naively expected. For the first time, this work provides the theoretical tools for estimating when temporal smoothing will be effective in reducing SBBS beyond the well known perturbative regime where the laser bandwidth is larger than the average growth rate.

The authors thank H. A. Rose, D. Pesme, and G. Laval for many fruitful discussions they had with them on this topic. A part of this work was enabled thanks to the computing facilities of IDRIS, Orsay (France).

[1] R. H. Lehmberg, A. J. Schmitt, and S. E. Bodner, J. Appl. Phys. **62**, 2680 (1987).
[2] G. Laval *et al.,* Phys. Fluids **20**, 2049 (1977), and references therein.
[3] A. N. Mostovych *et al.,* Phys. Rev. Lett. **59**, 1193 (1987).
[4] H. A. Rose and D. F. DuBois, Phys. Rev. Lett. **72**, 2883 (1994).
[5] L. Divol and Ph. Mounaix, Phys. Plasmas **6**, 4037 (1999).
[6] L. Divol and Ph. Mounaix, Phys. Plasmas **6**, 4049 (1999).
[7] This estimation corresponds to a small dispersion of the hot spot recurrence time around its average value. In the opposite case of a significant dispersion, SBBS may grow from a secularly growing nonthermal level produced by the successive generations of hot spots. Our theory can be extended to this case by regarding the noise level as a free time-dependent parameter which should be determined either numerically (from, e.g., multidimensional particle-in-cell simulations) or experimentally (from, e.g., time-resolved Thomson scattering measurements).
[8] Our reasons for imposing this condition have been motivated by the one-dimensional numerical simulations discussed in Section V of H. A. Baldis *et al.,* Phys. Fluids B **5**, 3319 (1993).
[9] L. Divol, Ph.D. thesis, Ecole Polytechnique, 1999.
[10] In the case of a short lifetime hot spot with $\tau_c \le L/c$, the total backscattered energy computed from Eq. (1) can be greater than the total incident energy in the hot spot. In this case, one must add to Eq. (1) the condition that the backscattered energy remains less than (or equal to) the incident hot spot energy.
[11] J. Adler, *The Geometry of Random Fields* (Wiley, New York, 1981). The application to the field statistics of spatially smoothed laser beam can be found in the two following references.
[12] H. A. Rose and D. F. DuBois, Phys. Fluids B **5**, 590 (1993).
[13] J. Garnier, Phys. Plasmas **6**, 1601 (1999).
[14] Practically, at $t = t^{NL}_{\rm sat}(u)$, $P_{\rm exp}(I,t)$ is given by $P_{\max}(I)$.
[15] R. L. Berger *et al.,* Phys. Rev. Lett. **75**, 1078 (1995).