Free-Electron Laser without Inversion: Gain Optimization and Implementation Scheme

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We consider a scheme of two noncollinear wigglers with an intermediate magnetic drift region, constituting a free-electron laser without inversion (FELWI). Two mechanisms of phase shifts in the drift region between the wigglers owing to a series of magnetic lenses can give rise to FELWI: velocity- and angle-dependent shifts. An appropriate combination of these shifts is shown to provide the conditions for amplification without inversion. The phase shifts optimizing the gain are found. A specific scheme for the drift region is suggested.

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The idea of free-electron lasers without inversion (FELWI) was suggested and discussed in a series of papers [1–5]. It is well known that in the usual free-electron laser (FEL) with a magnetic wiggler light amplification is efficient in the vicinity of the resonance frequency $\omega_{\rm res} \approx 2\gamma^2 c k_w$ or, for a given light frequency ω, at a resonance value of the initial relativistic factor of the electron $\gamma_{\rm res} \approx \omega/2ck_w$, where $k_w = 2\pi/\lambda_0$ and λ_0 is the period of the wiggler. Deviations from the resonance conditions are characterized by the resonance detuning

$$\Omega = \omega \frac{v_0 - v_{\rm res}}{c}, \tag{1}$$

where v_0 is the initial electron velocity and $v_{\rm res}$ is the resonance velocity, corresponding to $\gamma_{\rm res}$. In dependence on Ω, the gain G of the usual FEL is an antisymmetric function, such that $G(\Omega) > 0$ for $\Omega > 0$, $G(\Omega) < 0$ for $\Omega < 0$, and $\int G(\Omega)\, d\Omega = 0$.

In the "hot-beam" regime the gain G is averaged over a wide distribution function $f(\varepsilon)$ (where ε is the initial electron energy) to give $\overline{G} \propto f'(\varepsilon_{\rm res}) \propto 1/(\Delta\varepsilon)^2$ in the case of usual FEL and $\overline{G} \propto f(\varepsilon_{\rm res}) \propto 1/(\Delta\varepsilon)$ in the case of FELWI, where $\Delta\varepsilon$ is the width of the distribution $f(\varepsilon)$. Hence, in FELWI amplification can take place at *any position* of $\varepsilon_{\rm res}$ with respect to the mean electron energy $\overline{\varepsilon}$, whereas in the usual FEL amplification takes place only if $\varepsilon_{\rm res}$ is located on a rising slope of $f(\varepsilon)$. This is a clear distinction between amplification without and with inversion. The described different dependence of $\overline{G}$ on $\Delta\varepsilon$ in FEL and FELWI, $\propto 1/\Delta\varepsilon^2$ and $\propto 1/\Delta\varepsilon$, respectively, indicates the main potential advantage of FELWI: in the case of beams with a wide distribution function $f(\varepsilon)$ the gain of FELWI is expected to be larger than that of a normal FEL.

A specific scheme of FELWI suggested and considered in Refs. [1–5] included two identical wigglers and some device between them. This device was assumed to give rise to a velocity-dependent phase shift $\Delta\varphi(v_0)$. The phase of a slow electron motion in FEL is determined as $\varphi = (k_L + k_w)z - \omega t$, where k_L is the wave vector of the wave amplified. In Refs. [1–5] the phase shift received by electrons in the drift region between the two wigglers was assumed to have a stepwise form

$$\Delta\varphi = \begin{cases} \pi - \Omega T, & \Omega < 0, \\ -\Omega T, & \Omega > 0, \end{cases} \tag{2}$$

where T is the time of flight of an electron through a single wiggler. Such a phase shift does provide mostly positive gain $G(\Omega)$, as required for FELWI, but possibilities of its practical implementation are not clear.

Moreover, it was realized [4] that a simple one-dimensional scheme of FELWI cannot provide the gain with the desired features [mostly positive and $\int G(\Omega)\, d\Omega \neq 0$]. The reason is the difference between the electron initial velocity v_0 and its velocity in the drift region v or, in other words, the difference between the initial and drift-region detunings Ω and $\tilde{\Omega}$, where

$$\tilde{\Omega} = \omega \frac{v - v_{\rm res}}{c} = \Omega + \zeta, \tag{3}$$

$$\zeta = \omega \frac{v - v_0}{c} = \frac{\omega}{\gamma^2} \frac{\Delta\varepsilon}{\varepsilon}, \tag{4}$$

and $\Delta\varepsilon$ is the energy gained by an electron after crossing the first wiggler. An addition of the term ζ (4) to the detuning Ω in the expression (3) for $\tilde{\Omega}$ changes the gain of a two-wiggler system with an arbitrary phase shift $\Delta\varphi(\tilde{\Omega})$ in such a way that the integral $\int G(\Omega)\, d\Omega$ becomes zero.

To overcome the deadlock, it was suggested [4] to use a noncollinear (two-dimensional) scheme with the wave vector of light $\mathbf{k}_L$ directed under a small angle $\theta \ll 1$ to the z axis (the wiggler's axis) and lying in the xz plane (in a geometry with the wiggler's magnetic field parallel to $\vec{x}$ and electric field of a light wave parallel to $\vec{y}$). In such a scheme, instead of (2), the phase shift considered in [4] was taken in the form

$$\Delta\varphi = \begin{cases} \pi - \tilde{\Omega} T, & \Omega < 0, \\ -\tilde{\Omega} T, & \Omega > 0. \end{cases} \tag{5}$$

Such a phase shift does provide a FELWI gain with a nonzero integral over the detuning, but its realization looks

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even more difficult than that of the phase shift in the form (2).

In this Letter we discuss different types of phase shifts, linear in the detuning, which optimize the FELWI gain. In principle, linear shifts are much more convenient for practical implementation. In the noncollinear scheme, we discuss two physical mechanisms providing phase shifts depending on $\widetilde{\Omega}$ and on the field-induced angular spreading of electrons in the drift region. We show that a combination of these two mechanisms can be used to compensate the contribution to the total phase shift of the term ζ in $\widetilde{\Omega}$ (3) and optimize the FELWI gain. Finally, a specific scheme with combining magnetic lenses and turning magnets in the drift region is suggested to provide the FELWI gain with the required features.

As a first step, let us ignore the mentioned above difference between Ω (1) and $\widetilde{\Omega}$ (3).

Let us assume that the phase shift $\Delta\varphi$ is a linear function of the detuning Ω,

$$\Delta\varphi = C_1 - C_2 x\,, \tag{6}$$

where $x = \Omega T$ and C_1 and C_2 are constants to be found from the condition that $\Delta\varphi$ (6) maximizes the integral

$$I(C_1, C_2) \equiv \int_{-\infty}^{+\infty} dx\, g(x)\,. \tag{7}$$

In Eq. (7), $g(x)$ is the dimensionless gain of Ref. [4] [Eq. (26)]:

$$g(x) = \frac{2[1 - \cos(x)] + 2\cos[x + \Delta\varphi(x)]}{x^3} - \frac{\cos[\Delta\varphi(x)] + \cos[2x + \Delta\varphi(x)]}{x^3} + \frac{\sin[x + \Delta\varphi(x)] - \sin[2x + \Delta\varphi(x)] - \sin(x)}{x^2}\,. \tag{8}$$

Such an optimization shows that the maximal value of the integral I (7) is reached at $C_1 = \pi/2$ and $C_2 = 1.04$ or

$$\Delta\varphi_{\rm opt} = \pi/2 - 1.04x = \pi/2 - 1.04\Omega T \approx \pi/2 - \Omega T\,. \tag{9}$$

In Fig. 1 the gain $g_{\rm opt}$ corresponding to $\Delta\varphi_{\rm opt}$ (9) is plotted in its dependence on the dimensionless detuning $x = \Omega T$. The maximal integral $I_{\rm opt} = \int g_{\rm opt}(x)\,dx \approx 3.1$ is approximately 3 times larger than the same integral for $g(x)$ found for $\Delta\varphi$ of Eq. (2).

Owing to the difference between Ω (1) and $\widetilde{\Omega}$ (3), the realizable velocity-dependent phase shift can be linear in $\widetilde{\Omega}$ rather than Ω,

$$\Delta\varphi \to \widetilde{\Delta\varphi} = C_1 - C_2\widetilde{\Omega}T = \Delta\varphi - C_2\zeta T\,, \tag{10}$$

where, as previously, $\Delta\varphi$ is given by Eq. (6) and ζ is given by Eq. (4). The arising correction to $\Delta\varphi$, proportional to ζ, gives rise to a correction to the gain $g(x)$ (8). In the weak-field approximation this correction can be calculated explicitly to give

$$\tilde{g}(x) - g(x) = -\frac{C_2}{2x^2}\{2\sin[\Delta\varphi(x) + x] - \sin[\Delta\varphi(x) + 2x] - \sin[\Delta\varphi(x)]\}\,. \tag{11}$$

It can be shown both analytically and numerically that

$$\tilde{I}(C_1, C_2) = \int_{-\infty}^{+\infty} dx\, \tilde{g}(x) \equiv 0 \tag{12}$$

at any C_1 and C_2 in Eqs. (6) and (11).

The correction to the phase shift $\Delta\varphi$ arising from the difference between the electron initial velocity and its velocity in the drift region modifies the gain of a collinear two-wiggler FEL in such a way that the total square under the curve $\tilde{g}(x)$ becomes zero. This cancellation occurs for any choice of parameters determining the velocity-dependent phase shift in the drift region. The gain $\tilde{g}(x)$ is similar to that of an optical-klystron-type FEL rather than to FELWI. To return to FELWI, we have to find a way to compensate somehow the contribution to the gain from the second (additional) term on the right-hand sides of Eqs. (10) for $\widetilde{\Delta\varphi}$. It is shown below that this goal can be reached in a noncollinear scheme of a two-wiggler FEL.

In the noncollinear scheme of FEL an electron initially moving along the z axis acquires a transverse velocity $v_x(t)$ obeying the equation

$$\dot{v}_x = \frac{\theta}{\gamma mc}\,\dot{\varepsilon} \tag{13}$$

[to be compared with Eq. (14) of Ref. [4]]. In accordance with the definition of the parameter ζ (4), Eq. (13) yields the following expression for the velocity $v_x(T)$ acquired by an electron at the exit from the first wiggler:

$$v_x^{(1)}(T) = \frac{\theta}{2k_w}\,\zeta\,. \tag{14}$$

This result shows that at the exit from the first wiggler an initially perfectly collimated beam of electrons acquires a finite angular width, which can be characterized by the angle α between the velocity $\mathbf{v}(T)$ and the z axis,

$$\alpha \approx \frac{v_x^{(1)}}{v_0} = \frac{\theta}{2ck_w}\,\zeta\,. \tag{15}$$

Hence, the parameter ζ (4) determines both the difference between the detunings $\widetilde{\Omega}$ and Ω (3) and the field-induced angular divergence of the electron beam at the exit from the

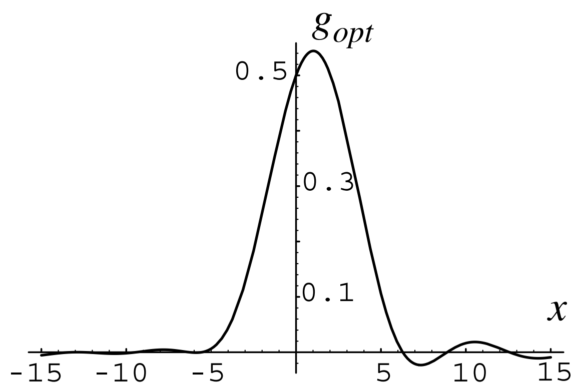

FIG. 1. Optimal gain vs detuning.

first wiggler (15). This double role of the parameter ζ (4) is the key point in an estimate of realizability of FELWI. In principle, the gain close to the optimal one [Fig. 1 and Eq. (8)] can be achieved if a device in the drift region is capable of providing a phase shift,

$$\Delta\varphi_{\rm tot} = \widetilde{\Delta\varphi} + \Delta\varphi', \tag{16}$$

where $\widetilde{\Delta\varphi}$ is given by Eq. (10) and $\Delta\varphi'$ is proportional to the angle α (15) under which electrons move at the exit from the first wiggler,

$$\Delta\varphi' = A\alpha = A\,\frac{\theta}{2ck_w}\,\zeta\,. \tag{17}$$

If the proportionality constant in this equation equals

$$A \approx 2ck_w T/\theta = 4\pi N/\theta \tag{18}$$

(where $N = L/\lambda_0$ is the number of periods in a wiggler), the terms proportional to ζ in $\widetilde{\Delta\varphi}$ and $\Delta\varphi'$ compensate each other to give

$$\Delta\varphi_{\rm tot} \approx \Delta\varphi_{\rm opt} \tag{19}$$

and, correspondingly, $g_{\rm tot}(x) = g(x)$ (8). In such a case the gain of all the system coincides with that of FELWI with ignored difference between the detuning in the drift region $\tilde\Omega$ and the initial detuning Ω [Fig. 1 and Eq. (8)]. In the following a construction is described which, in principle, looks capable of satisfying the formulated conditions.

The scheme under discussion is shown in Fig. 2. The device includes a focusing magnetic lens (FL) and three symmetrically located turning magnets TM1–TM5. Turning magnets TM1 and TM5 are assumed to be made of a couple of magnetic plates of different polarity, with the magnetic field directed parallel or antiparallel the y axis. Two angular parameters α_0 and α_1 characterizing turning magnets TM1 and TM5 are the angle α_0 between the plane of magnetic plates and the x axis and the turn angle α_1, i.e., the angle between electron trajectories before and after crossing the turning magnet.

The most important parts of the device are indicated in Fig. 2 as A, B, C, and D (plus the mirror reflection of the regions B, C, and D after TM3). Under proper conditions, the phase shifts $\widetilde{\Delta\varphi}$ (10) and $\Delta\varphi'$ (17) are provided by the regions $A + B + C$ and $C + D$, correspondingly.

As mentioned above, the beam exiting from the wiggler w_1 has an angular divergence α (15). The focusing lens FL transforms this angular divergence into a distribution of partial beams parallel to the z axis over the transverse coordinate x with $x = l_0\alpha$. Two partial beams shown in Fig. 2 in region B correspond to $\alpha = 0$ and $\alpha \neq 0$. Each of these partial beams consists of electrons with various velocities v. As α and x are small, the dispersion of the focusing lens FL can be ignored whereas the dispersion of turning magnets is rather important. For example, for the turning magnet TM1 the dispersion determines the dependence of the turn angle $\alpha_1(v)$ on the electron velocity v, and it is estimated as

$$\alpha_1(v) = \alpha_1\left(1 - \gamma^2\frac{v - v_{\rm res}}{c}\right). \tag{20}$$

A divergence $\Delta\alpha$ arising after the turning magnet TM1 owing to its dispersion is shown in region C of Fig. 2. Actually, the structure of the beams in region C can be considered as a superposition of diverging partial beams with different velocities and parallel partial beams with equal velocities. Both the turn of parallel beams by the turning magnet TM1 and the arising v-dependent divergence give rise to differences of the corresponding electron routes and, hence, to phase shifts. The phase shift arising owing to the angular divergence in the regions C and D depends on v and corresponds to $\widetilde{\Delta\varphi}$ (10), whereas the phase shift arising owing to the turn of parallel beams with different x coordinates in the region B depends on the angular divergence α (15) at the exit from the first wiggler and corresponds to $\Delta\varphi'$ (17).

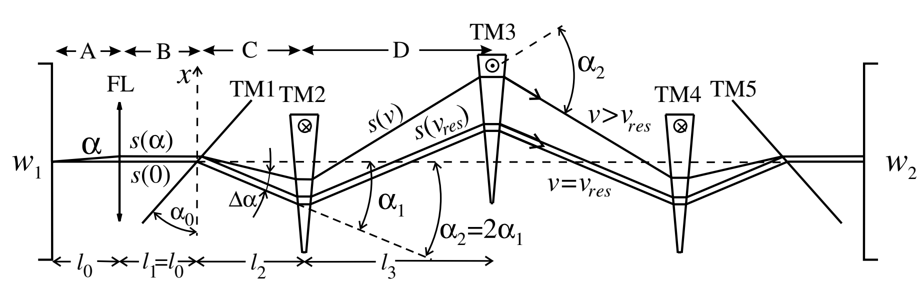

FIG. 2. A scheme of a device between the two wigglers w_1 and w_2.

The difference of routes $\Delta s(\alpha) = s(\alpha) - s(0)$ of parallel partial beams arising in regions A, B, and C (plus the mirror reflection of B and C) is determined by the geometry of Fig. 2:

$$\Delta s(\alpha) = s(\alpha) - s(0) \approx -l_0 \alpha \tan(\alpha_0) \frac{\alpha_1^2}{2}. \quad (21)$$

The corresponding phase shift $\Delta\varphi(\alpha) \approx -2k_L \Delta s$ has the form (17) with

$$A = k_L l_0 \tan(\alpha_0) \alpha_1^2. \quad (22)$$

The main condition of the previous section (18) is satisfied if

$$\frac{L}{l_0} = \theta \frac{k_L}{2k_w} \tan(\alpha_0)\alpha_1^2 \approx \frac{\gamma^2\theta}{1 + \gamma^2\theta^2} \tan(\alpha_0)\alpha_1^2. \quad (23)$$

As the distance between two wigglers should not be too big ($l_0 \leq 10$ cm), the right-hand side of Eq. (23) must be large (≥ 30). This is possible if, for example, $\gamma\theta \sim 1$, $\gamma \geq 10^2$, $\pi/2 - \alpha_0 \sim 0.1$, and $\alpha_1 \sim 1/3$.

The turning magnet TM2 (as well as TM3 and TM4) is supposed to be made of magnetic plates of varying (x-dependent) widths, which provide an inhomogeneous magnetic field inside the magnets and x-dependent turn angle

$$\alpha_2(x) = \alpha_2(0)\left(1 + \frac{x}{b} - \gamma^2 \frac{v - v_{\rm res}}{c}\right), \quad (24)$$

where $\alpha_2(0) = 2\alpha_1$, b is the "inhomogeneity" parameter of the turning magnet, and $x \equiv x(v)$ is the vertical position of a partial beam with a given velocity v at the plane of TM2:

$$x(v) - x(v_{\rm res}) \approx l_2 \alpha_1 \gamma^2 \frac{v - v_{\rm res}}{c}. \quad (25)$$

The difference of routes $\Delta s(v) = s(v) - s(v_{\rm res})$ of electrons with $v \neq v_{\rm res}$ and $v = v_{\rm res}$ acquired in the regions C and D (plus their mirror reflection) is calculated directly from the geometry of Fig. 2 to give

$$\Delta\varphi'' = -2\omega \frac{\Delta s(v)}{c} \approx -2\alpha_1^3 \frac{l_2 l_3}{cb} \gamma^2 \tilde{\Omega}. \quad (26)$$

If the coefficient in front of $\tilde{\Omega}$ on the right-hand side of Eq. (26) is close to $-T$,

$$2 \frac{l_2 l_3}{b} \gamma^2 \tilde{\Omega} \approx cT \approx L, \quad (27)$$

the phase shift (26) is similar to $\widetilde{\Delta\varphi}$ (10) except for a missing term $\pi/2$. The latter can be acquired at the last part of the electron's trajectory, between the turning magnet TM5 and the second wiggler w_2, to give $\Delta\varphi'' = \widetilde{\Delta\varphi}$. Altogether, this makes the $\Delta\varphi_{\rm tot} = \Delta\varphi_{\rm opt}$ (9), and $g_{\rm tot}(x) = g_{\rm opt}(x)$.

At $\gamma \sim 10^2$ and $b \sim 1$, the condition (27) can be fulfilled at l_2 and l_3 on the order of a few centimeters.

The considerations described above show that, in principle, the creation of a free-electron laser without inversion is possible. From the physical point of view, the main ingredients of the realization of such a system are the noncollinear geometry and the angular divergence of the electron beam at the exit from the first wiggler. The next step consists of a drift region which provides a superposition of two kinds of phase shifts, which are proportional to the detuning from resonance in the drift region and to the field-induced angular divergence, respectively. With values of these two shifts appropriately adjusted, the total shift appears to be a linear function of the initial detuning from resonance and close to the optimal one. The suggested implementation scheme seems to be quite feasible, although in real experiments some elements of this scheme can possibly be modified or substituted by more convenient ones. Further theoretical investigations are planned, among them the description of spontaneous emission in the systems under consideration, and a quantum description of the FELWI [3].

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