

Complete Quantum Teleportation with a Kerr Nonlinearity

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We present a scheme for the quantum teleportation of the polarization state of a photon employing a cross-Kerr medium. The experimental feasibility of the scheme is discussed and we show that, using the recently demonstrated ultraslow light propagation in cold atomic media, our proposal can be realized with presently available technology.

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Quantum entanglement is a powerful resource at the basis of the extraordinary development of quantum information. Among the most fascinating examples of the possibilities offered by sharing quantum entanglement are quantum teleportation [1], quantum dense coding [2], entanglement swapping [3], quantum cryptography [4], and quantum computation [5]. Quantum teleportation is the “reconstruction,” with 100% success, of an unknown state given to one station (Alice), and performed at another remote station (Bob), on the basis of two bits of classical information sent by Alice to Bob. Perfect teleportation is possible only if the two parties share a maximally entangled state. The most delicate part needed for the effective realization of teleportation is the Bell-state measurement, i.e., the discrimination between the four, maximally entangled, Bell states [6] which has to be performed by Alice and whose result is communicated to Bob through the classical channel. There have been numerous proposals for its realization in different systems [7] and recently successful, pioneering experiments [8–10] have provided convincing experimental proof of principle of the correctness of the teleportation concept.

These experiments differ by the degrees of freedom used as qubits and for the different ways in which the Bell-state measurement is performed. The Innsbruck experiment [8] is the conceptually simplest one, since each qubit is represented by the polarization state of a single photon pulse. In this experiment, however, only two out of the four Bell states can be discriminated, and therefore the success rate cannot be larger than 50% [11]. The Rome experiment [9] employs the entanglement between the spatial and the polarization degrees of freedom of a photon and it is able to distinguish all the corresponding four Bell states completely. However, in this scheme the state to be teleported is generated within the apparatus (it cannot come from the outside), and therefore the scheme cannot be used as a computational primitive in a larger quantum network for further information processing, as it has been recently proposed in Ref. [12]. Finally, the Caltech experiment [10] is conceptually completely different since it implies the teleportation of the state of a continuous degree of freedom [13], the mode of an electromagnetic field, employing the entangled two-mode squeezed states at the output

of a parametric amplifier. In this case, the Bell-state measurement is replaced by two homodyne measurements and a direct comparison with the original quantum teleportation scheme of Ref. [1] cannot be made. Up to now, only coherent states of the electromagnetic field have been successfully teleported using this scheme.

It is therefore desirable to have a scheme for a Bell-state measurement that can be used in the simplest case of the Innsbruck scheme. This would imply the possibility of realizing the first *complete* verification of the original quantum teleportation scheme [1] and also of having a device useful for other quantum protocols, as quantum dense coding [2]. What we need is a device able to discriminate among the four Bell states that can be realized with the polarization-entangled photon pairs produced in Type-II phase matched parametric down-conversion [14], that is

$$|\psi^\pm\rangle = \frac{|V_1, H_2\rangle \pm |H_1, V_2\rangle}{\sqrt{2}} \quad (1a)$$

$$|\phi^\pm\rangle = \frac{|V_1, V_2\rangle \pm |H_1, H_2\rangle}{\sqrt{2}}, \quad (1b)$$

where $|H\rangle$ and $|V\rangle$ denote the horizontally and vertically polarized one-photon states, respectively, and 1, 2 refer to two different spatial modes.

It has been recently shown that it is impossible to perform a complete Bell measurement on two-mode polarization states using only linear passive elements [15] (unless the two photons are entangled in more than 1 degree of freedom [16]), and for this reason schemes involving some effective nonlinearities, such as resonant atomic interactions [17], or the Kerr effect [18], have been proposed. In the present Letter, we propose a scheme for a perfect Bell-state discrimination based on a nonlinear optical effect, the cross-phase modulation taking place in Kerr media. In this respect, our scheme is based on a $\chi^{(3)}$ medium as the “Fock-filter” proposal of Ref. [18]. However, our scheme is different and simpler and, above all, is feasible using available technology, since we shall show that the needed crossed-Kerr nonlinearity can be obtained using the recently demonstrated ultraslow light

propagation [19], achieved via electromagnetic induced transparency (EIT) [20] in ensembles of cold atoms.

Our “Bell box” is described in Fig. 1 and can be divided into two parts: the left part is composed by three polarization rotators (R , R^\dagger) and by the “quantum phase gate” (QPG) which will be described below, and can be called “the disentangler,” since it realizes the unitary transformation changing each Bell state of Eqs. (1) into one of the four factorized polarization states, i.e.,

$$|\psi^+\rangle \rightarrow |H_1, V_2\rangle, \quad (2a)$$

$$|\psi^-\rangle \rightarrow |V_1, V_2\rangle, \quad (2b)$$

$$|\phi^+\rangle \rightarrow |H_1, H_2\rangle, \quad (2c)$$

$$|\phi^-\rangle \rightarrow |V_1, H_2\rangle. \quad (2d)$$

The right part of the scheme is composed by two polarizing beam splitters (PBSs) and by four detectors with single-photon sensitivity, and simply serves the purpose of detecting the four states of the factorized polarization basis,

$$\begin{aligned} \{|e_1\rangle, |e_2\rangle, |e_3\rangle, |e_4\rangle\} \\ = \{|H_1, H_2\rangle, |H_1, V_2\rangle, |V_1, H_2\rangle, |V_1, V_2\rangle\}, \end{aligned} \quad (3)$$

where $\{|e_i\rangle\}$ are the tensor product of the single-photon polarization basis states,

$$|H_i\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_i \quad |V_i\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_i. \quad (4)$$

Because of the one-to-one correspondence of Eqs. (2), it is clear that the detection of each Bell state corresponds to a different pair of detector clicks, so that they are unambiguously distinguishable. The disentangler, and in particular the QPG, is the most delicate part as concerns the experimental implementation, since it involves a two-qubit operation, i.e., an effective photon-photon interaction. In fact, if \tilde{R}_i is a simple polarization rotation by $\pi/4$ rad for mode i (and \tilde{R}_i^\dagger its inverse), i.e., $|H_i\rangle \rightarrow (|H_i\rangle + |V_i\rangle)/\sqrt{2}$, $|V_i\rangle \rightarrow (|V_i\rangle - |H_i\rangle)/\sqrt{2}$, we have

$$R_1 = \tilde{R}_1 \otimes I_2, \quad R_2 = I_1 \otimes \tilde{R}_2, \quad (5)$$

which can be obtained using a $\lambda/2$ retardation plate at a $\pi/8$ angle. In Eq. (5) I_i is the 2×2 unit matrix for mode

i . The general QPG $P(\varphi)$ is a universal two-qubit gate as long as $\varphi \neq 0$ [21,22], and, in the two-photon polarization basis (3) we are considering here, it can be written as

$$|H_1, H_2\rangle \rightarrow |H_1, H_2\rangle, \quad (6a)$$

$$|H_1, V_2\rangle \rightarrow |H_1, V_2\rangle, \quad (6b)$$

$$|V_1, H_2\rangle \rightarrow |V_1, H_2\rangle, \quad (6c)$$

$$|V_1, V_2\rangle \rightarrow e^{i\varphi}|V_1, V_2\rangle. \quad (6d)$$

The experimental realization of this gate has been reported in Ref. [23], in the case when one qubit is given by the internal state of a trapped ion and the other qubit by its two lowest vibrational states, and recently in Ref. [24] where the two qubits are represented by two circular Rydberg states of a Rb atom and by the two lowest Fock states of a microwave cavity. In the optical case we are interested in, the QPG between two frequency-distinct cavity modes has been experimentally investigated in Ref. [22], using, however, weak coherent states instead of single photon pulses, demonstrating therefore only conditional quantum dynamics and not the full quantum transformation of Eqs. (6). As it can be easily checked, the QPG (6) can be realized using a crossed-Kerr interaction involving the vertically polarized modes only,

$$H_K = \hbar\chi a_{V_1}^\dagger a_{V_1} a_{V_2}^\dagger a_{V_2}, \quad (7)$$

so that the conditional phase shift is $\varphi = \chi t_{\text{int}}$, where t_{int} is the interaction time within the Kerr medium.

The disentangler of Fig. 1 realizes the transformation (2) when the conditional phase shift is $\varphi = \pi$, as it can be checked in a straightforward way by writing the matrix form of the transformation

$$R_1^\dagger R_2 P(\pi) R_2^\dagger \quad (8)$$

of Fig. 1 in the factorized polarization basis (3), which is just the matrix form of Eqs. (2) in the chosen basis. The proposed Bell box is therefore extremely simple and also robust against detector inefficiencies. This is due to the fact that in our scheme only one photon at most impinges on each of the four detectors. First of all this means that only single photon *sensitivity* and not single photon *resolution* is needed, and in this case solid-state photomultipliers can provide up to 90% efficiency [16]. Moreover, this implies that the detection scheme is reliable, i.e., it always discriminates the correct Bell state, whenever it answers. In the case of detectors with the same efficiency η , our Bell box gives the (always correct) output with probability η^2 , and it does not give any output (only zero or one photon is detected) with probability $1 - \eta^2$.

As we have already remarked, the most difficult part for the experimental implementation of the scheme is the QPG with a conditional phase shift $\varphi = \pi$. In fact, realizing the transformation (6) means having a large cross-phase modulation at the single photon level between two traveling-wave pulses, with negligible absorption, which

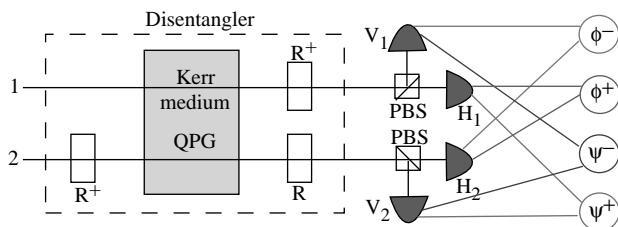


FIG. 1. Scheme of the Bell-state measurement: QPG is the quantum phase gate of Eqs. (6) with $\varphi = \pi$; R (R^\dagger) rotates the polarization by $\pi/4$ ($-\pi/4$), and PBS are polarizing beam splitters. The “disentangler” performs the unitary transformation of Eqs. (2).

is very demanding. For example, in the experiment of Ref. [22], a conditional phase shift $\varphi = 16^\circ$ has been measured, which, however, involved two frequency-distinct *cavity* modes in a high-finesse cavity. However, the recent demonstration of ultraslow light propagation in a cold gas of sodium atoms [19] and with hot Rb atoms [25] opens the way for the realization of significant conditional phase shifts also between two traveling single photon pulses. In fact, the extremely slow group velocity is obtained as a consequence of EIT [20], which, however, as originally suggested by Schmidt and Imamoglu in Ref. [26], can also be used to achieve giant crossed-Kerr nonlinearities. In fact, Harris and Hau [27], developing the suggestions of Ref. [26], showed that, when the ultraslow group velocity is the dominant feature of the problem, nonlinear optical processes between traveling pulses with a low number of photons become feasible.

In particular, in the limit of very small group velocity, and therefore with light pulses compressed to a spatial length much smaller than the medium length, they find a conditional phase shift per photon between two pulses (characterized by frequencies ω_{24} and ω_p in [27]) given by $\varphi = \gamma_{24}\Delta\omega_{24}/(4\gamma_{24}^2 + 4\Delta\omega_{24}^2)$, accompanied by a two-photon absorption $\gamma_{24}^2/(4\gamma_{24}^2 + 4\Delta\omega_{24}^2)$, where $\Delta\omega_{24}$ is the detuning of one of the two pulses and γ_{24} the associated linewidth [see Eq. (10) of Ref. [27]]. For a sufficiently large detuning $\Delta\omega_{24} \gg \gamma_{24}$, two-photon absorption is negligible and we have just the desired result, i.e., a significant conditional phase shift between two traveling single photon pulses without appreciable absorption. Unfortunately, in this same limit, the phase shift becomes $\varphi \approx \gamma_{24}/4\Delta\omega_{24}$ which cannot be too large and close to π , as we have assumed above in the Bell box scheme. This may be a problem because it is possible to see that if the phase φ of the QPG is not equal to π , the scheme of Fig. 1 is no longer perfect, and it does not discriminate the four Bell states with 100% success. However, it should be noted that this is not a theoretical limitation, but only a practical drawback of the specific scheme of Ref. [27]. Furthermore, as mentioned above, the QPG represented by $P(\varphi)$ is a universal two-qubit gate, capable of entangling and disentangling qubits as soon as $\varphi \neq 0$. Moreover, even though different from π , the conditional phase shift φ is a given and measurable property, and it is reasonable to expect that, using the knowledge of the actual value of φ , it is possible to adapt and optimize the teleportation protocol in order to achieve a truly *quantum* teleportation (i.e., that cannot be achieved with only classical means), even in the presence of an imperfect Bell-state measurement. Optimization means that Bob has to suitably modify the four local unitary transformations he has to perform on the received qubit according to the Bell measurement result communicated by Alice. In the optimized protocol, Bob's local unitary transformations will now depend on the phase φ of the QPG and will reduce to those of the original proposal [1] in the ideal case of perfect Bell-state

discrimination $\varphi = \pi$. We expect that, $\forall \varphi \neq 0$, the average fidelity of the teleported state will be always larger than $2/3$, as it must be for any truly quantum teleportation of a qubit state [28].

Let us therefore consider a generic one-photon state $|\psi\rangle_1 = \alpha|H_1\rangle + \beta|V_1\rangle$, which is given to Alice and has to be teleported to Bob, and let us assume that Alice and Bob share the Bell state $|\psi^+\rangle_{23} = (|H_2V_3\rangle + |V_2H_3\rangle)/\sqrt{2}$, so that the input state for the teleportation process is $|\psi\rangle_1 \otimes |\psi^+\rangle_{23}$. Alice is provided with the "imperfect" Bell box with a QPG $P(\varphi)$, so that the disentangler of Fig. 1 will now be described by the transformation $R_1^\dagger R_2 P(\varphi) R_2^\dagger$. It is easy to check that, when $\varphi \neq \pi$, the four Bell states are no longer completely disentangled and therefore no longer discriminated with 100% success.

Alice has to perform the Bell-state measurement on modes 1 and 2, and the resulting joint state of the three modes just before the photodetections is

$$|\tilde{\psi}\rangle_{123} = \sum_{i=1}^4 |e_i\rangle_{12} \hat{G}_i(\varphi) |\psi\rangle_3, \quad (9)$$

where $|e_i\rangle_{12}$ are the factorized basis states (3) and

$$\hat{G}_1(\varphi) = \frac{1}{2} \begin{pmatrix} 0 & -ie^{i\varphi/2} \sin \frac{\varphi}{2} \\ 1 & e^{i\varphi/2} \cos \frac{\varphi}{2} \end{pmatrix}, \quad (10a)$$

$$\hat{G}_2(\varphi) = \frac{1}{2} \begin{pmatrix} 1 & e^{i\varphi/2} \cos \frac{\varphi}{2} \\ 0 & -ie^{i\varphi/2} \sin \frac{\varphi}{2} \end{pmatrix}, \quad (10b)$$

$$\hat{G}_3(\varphi) = \frac{1}{2} \begin{pmatrix} 0 & -ie^{i\varphi/2} \sin \frac{\varphi}{2} \\ -1 & e^{i\varphi/2} \cos \frac{\varphi}{2} \end{pmatrix}, \quad (10c)$$

$$\hat{G}_4(\varphi) = \frac{1}{2} \begin{pmatrix} -1 & e^{i\varphi/2} \cos \frac{\varphi}{2} \\ 0 & -ie^{i\varphi/2} \sin \frac{\varphi}{2} \end{pmatrix}. \quad (10d)$$

When the photons are detected, Alice sends the results through the classical channel to Bob. Bob is left with the photon of mode 3, and applies a local unitary transformation $\hat{U}_i(\varphi)$ in correspondence to the i th result of the Bell-state measurement. As a consequence, the output state of the teleportation process is

$$\rho_{\text{out}} = \sum_{i=1}^4 \hat{U}_i(\varphi) \hat{G}_i(\varphi) |\psi\rangle_3 \langle \psi| \hat{G}_i(\varphi)^\dagger \hat{U}_i(\varphi)^\dagger. \quad (11)$$

Since the output state has to reproduce the unknown input state $|\psi\rangle$ as much as possible, it is evident that to optimize the local unitary transformations $\hat{U}_i(\varphi)$, one should "invert" $\hat{G}_i(\varphi)$. The best strategy is suggested by the use of the polar decomposition of the matrices $\hat{G}_i(\varphi)$,

$$\hat{G}_i(\varphi) = \hat{T}_i(\varphi) \hat{R}_i(\varphi), \quad (12)$$

where $\hat{R}_i(\varphi) = \sqrt{\hat{G}_i(\varphi)^\dagger \hat{G}_i(\varphi)}$ is Hermitian and $\hat{T}_i(\varphi)$ unitary, so that Bob's optimal local unitary transformations will be

$$\hat{U}_i(\varphi) = \hat{T}_i(\varphi)^{-1} = \hat{R}_i(\varphi) \hat{G}_i(\varphi)^{-1}. \quad (13)$$

Using Eqs. (10), (12), and (13), one finds the following Bob's optimal unitary transformations:

$$\hat{U}_1(\varphi) = \begin{pmatrix} -i \cos \frac{\pi+\varphi}{4} & \sin \frac{\pi+\varphi}{4} \\ ie^{-i\varphi/2} \sin \frac{\pi+\varphi}{4} & e^{-i\varphi/2} \cos \frac{\pi+\varphi}{4} \end{pmatrix}, \quad (14a)$$

$$\hat{U}_2(\varphi) = \begin{pmatrix} \cos \frac{\pi-\varphi}{4} & -i \sin \frac{\pi-\varphi}{4} \\ e^{-i\varphi/2} \sin \frac{\pi-\varphi}{4} & ie^{-i\varphi/2} \cos \frac{\pi-\varphi}{4} \end{pmatrix}, \quad (14b)$$

$$\hat{U}_3(\varphi) = \begin{pmatrix} i \cos \frac{\pi+\varphi}{4} & -\sin \frac{\pi+\varphi}{4} \\ ie^{-i\varphi/2} \sin \frac{\pi+\varphi}{4} & e^{-i\varphi/2} \cos \frac{\pi+\varphi}{4} \end{pmatrix}, \quad (14c)$$

$$\hat{U}_4(\varphi) = \begin{pmatrix} -\cos \frac{\pi-\varphi}{4} & i \sin \frac{\pi-\varphi}{4} \\ e^{-i\varphi/2} \sin \frac{\pi-\varphi}{4} & ie^{-i\varphi/2} \cos \frac{\pi-\varphi}{4} \end{pmatrix}, \quad (14d)$$

which (once the conditional phase shift is known) can be easily implemented using appropriate birefringent plates and polarization rotators. It can be checked that, in the special case $\varphi = \pi$, the above optimized teleportation protocol coincides with the original one [1], since one has $\hat{U}_1(\pi) = \sigma_x$, $\hat{U}_2(\pi) = 1$, $\hat{U}_3(\pi) = -i\sigma_y$, and $\hat{U}_4(\pi) = -\sigma_z$.

Finally, we have to check that the proposed teleportation protocol, even though no longer with 100% success when $\varphi \neq \pi$, always implies the realization of a true quantum teleportation, that cannot be achieved with only classical means. This amounts to check that the average fidelity of the output state is larger than $2/3$ for $0 < \varphi < 2\pi$. For pure qubit states, the average fidelity F_{av} is defined as

$$F_{\text{av}} = \frac{1}{4\pi} \int d\Omega \langle \psi | \rho_{\text{out}} | \psi \rangle, \quad (15)$$

where the integral is over the Bloch sphere and $|\psi\rangle$ is the generic input state. Using Eqs. (11) and (13) one has

$$\langle \psi | \rho_{\text{out}} | \psi \rangle = \sum_{i=1}^4 |\langle \psi | \hat{R}_i(\varphi) | \psi \rangle|^2, \quad (16)$$

so that, using the explicit expressions for $\hat{R}_i(\varphi)$ that can be obtained from Eqs. (10), and performing the average over the Bloch sphere, one finally finds

$$F_{\text{av}}(\varphi) = \frac{2}{3} + \frac{1}{3} \sin \frac{\varphi}{2}, \quad (17)$$

which is larger than the upper classical bound $F_{\text{av}} = 2/3$ [28] for $0 < \varphi < 2\pi$, as expected.

In conclusion, we have presented a physical implementation for the quantum teleportation of the polarization state of single photons, such as those produced in spontaneous parametric down-conversion, based on a crossed-Kerr nonlinearity. In the ideal case, the scheme provides a perfect Bell-state discrimination, and it could be imple-

mented using the giant nonlinearities already demonstrated in atomic gases exploiting EIT [19,25].

Note added in proof.—After submission, we have become aware of Ref. [29] which shows that a conditional phase shift φ close to π could be achieved at single photon level if *both* light pulses are subject to EIT and propagate with slow but equal group velocities. This fact makes us more confident on the feasibility of the proposed scheme.

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- [1] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
 - [2] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
 - [3] M. Zukowski *et al.*, Phys. Rev. Lett. **71**, 4287 (1993); J.-W. Pan *et al.*, Phys. Rev. Lett. **80**, 3891 (1998).
 - [4] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
 - [5] D. P. DiVincenzo, Proc. R. Soc. London A **454**, 261 (1998).
 - [6] S. L. Braunstein *et al.*, Phys. Rev. Lett. **68**, 3259 (1992).
 - [7] L. Davidovich *et al.*, Phys. Rev. A **50**, R895 (1994); J. I. Cirac and A. S. Parkins, *ibid.*, **50**, R4441 (1994).
 - [8] D. Bouwmeester *et al.*, Nature (London) **390**, 575 (1997).
 - [9] D. Boschi *et al.*, Phys. Rev. Lett. **80**, 1121 (1998).
 - [10] A. Furusawa *et al.*, Science **282**, 706 (1998).
 - [11] S. L. Braunstein and H. J. Kimble, Nature (London) **394**, 47 (1998).
 - [12] D. Gottesman and I. L. Chuang, Nature (London) **402**, 390 (1999).
 - [13] L. Vaidman, Phys. Rev. A **49**, 1473 (1994); S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
 - [14] P. G. Kwiat *et al.*, Phys. Rev. Lett. **75**, 4337 (1995).
 - [15] N. Lütkenhaus *et al.*, Phys. Rev. A **59**, 3295 (1999).
 - [16] P. G. Kwiat and H. Weinfurter, Phys. Rev. A **58**, R2623 (1998).
 - [17] M. O. Scully *et al.*, Phys. Rev. Lett. **83**, 4433 (1999).
 - [18] M. G. A. Paris *et al.*, LANL e-print quant-ph/9911036.
 - [19] L. V. Hau *et al.*, Nature (London) **397**, 594 (1999).
 - [20] E. Arimondo, in *Progress in Optics XXXV*, edited by E. Wolf (Elsevier, Amsterdam, 1996); S. E. Harris, Phys. Today **50**, 36 (1997); M. O. Scully and M. S. Zubairy, Quantum Optics (Cambridge University Press, Cambridge, United Kingdom, 1997).
 - [21] S. Lloyd, Phys. Rev. Lett. **75**, 346 (1995).
 - [22] Q. A. Turchette *et al.*, Phys. Rev. Lett. **75**, 4710 (1995).
 - [23] C. Monroe *et al.*, Phys. Rev. Lett. **75**, 4714 (1995).
 - [24] A. Rauschenbeutel *et al.*, Phys. Rev. Lett. **83**, 5166 (1999).
 - [25] M. M. Kash *et al.*, Phys. Rev. Lett. **82**, 5229 (1999).
 - [26] H. Schmidt and A. Imamoğlu, Opt. Lett. **21**, 1936 (1996).
 - [27] S. E. Harris and L. V. Hau, Phys. Rev. Lett. **82**, 4611 (1999).
 - [28] H. Barnum, Ph.D. thesis, University of New Mexico, 1998.
 - [29] M. D. Lukin and A. Imamoğlu, Phys. Rev. Lett. **84**, 1419 (2000).