

New Perspective on Cosmic Coincidence Problems

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(Received 5 June 2000)

Cosmological data suggest that we live in an interesting period in the history of the universe when $\rho_\Lambda \sim \rho_M \sim \rho_R$. The occurrence of *any* epoch with such a “triple coincidence” is puzzling, while the question of why we happen to live during this special epoch is the “Why now?” problem. We introduce a framework which makes the triple coincidence inevitable; furthermore, the “Why now?” problem is transformed and greatly ameliorated. The framework assumes that the only relevant mass scales are the electroweak scale M_{EW} , and the Planck scale M_{Pl} and requires $\rho_\Lambda^{1/4} \sim M_{EW}^2/M_{Pl}$ parametrically. Assuming that the true vacuum energy vanishes, we present a simple model, where a false vacuum energy yields a cosmological constant of this form.

PACS numbers: 98.80.Cq

Recent observations using high-redshift type-IA supernovae [1] indicate that a small but nonzero cosmological constant (Λ) is present in the Universe today. Evidence for a finite Λ also comes from the cosmic age, large scale structure, and probably most convincingly from the combination of the cosmic microwave background anisotropy and cluster dynamics (see Ref. [2] for recent reviews). The data suggest a flat universe, $\Omega_{total} = 1$, with about 70% of the energy density coming from Λ and 30% in the form of nonrelativistic matter.

The tiny energy density of the cosmological constant $\rho_\Lambda \sim (2 \text{ meV})^4$ poses the most severe naturalness problem in theoretical physics: Why is $\rho_\Lambda \sim 10^{-120} M_{Pl}^4$ [3]? Moreover, the near equivalence between the two components in the energy density raises a serious question: Why should we observe them to be so nearly equivalent *now*? While a cosmological constant is by definition time independent, the matter energy density is diluted as $1/R^3$ as the Universe expands. Thus, despite evolution of R over many orders of magnitude, we appear to live in an era during which the two energy densities are roughly the same (see Fig. 1). This is the “Why now?” problem. If we believe the data, there appears to be no choice but to believe that we live in a special time in the history of the Universe. This possibility is thought by many to be sufficiently distasteful to warrant disbelieving the data.

One possible partial remedy to this problem would be to assume that the “cosmological constant” is not a constant at all, but has always been comparable to the rest of the energy densities. Such a possibility is realized in some models of quintessence which possess “scaling” behavior (see [4] for a recent attempt along this line). Although such an idea may make the closeness of the effective Λ to the total energy density natural, it does not explain the coincidence that the quintessence field becomes settled with a finite energy density comparable to the matter energy density just now.

Returning to Fig. 1, we notice *two* remarkable features. The first is that there is an era in the history of the Uni-

verse where *all three* forms of energy, in matter, radiation, and Λ , become comparable within a few orders of magnitude. The second is that the universe happens to be in this interesting era *now*. These observations lead to two different questions. The first is the “cosmic triple coincidence” problem: Why should three forms of energy density very nearly cross at any point in the evolution of the universe? The second question is a generalization of the “Why now?” problem: Why is the Universe in this era of triple coincidence now? The first problem is epoch independent: In principle, at any point in the history of the Universe the three forms of energy could be measured and extrapolated in time, and the triple coincidence could be inferred. The existence of a triple coincidence is reminiscent of the unification of gauge coupling constants in grand unified theories, and suggests an underlying theory in which the coincidence has a simple explanation.

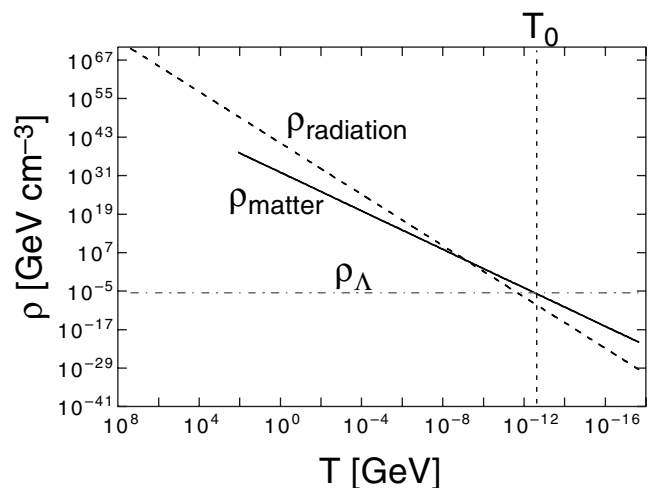


FIG. 1. Evolution of the matter, radiation energy densities as a function of the radiation temperature and the cosmological constant. Note a near triple coincidence of all three at about $T_0 = 2.7 \text{ K}$. We used $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_\Lambda = 0.7$, $\Omega_M = 0.3$.

In this Letter, we propose a solution to the triple coincidence problem. We begin with one of the central observations of particle physics, the special significance played by two mass scales: the reduced Planck scale, $M_{\text{Pl}} \sim 10^{18}$ GeV, and the electroweak scale, $M_{\text{EW}} \sim 10^3$ GeV. Given these two scales, the existence of the triple coincidence of matter, radiation, and the cosmological constant can follow in a completely natural way. In particular, once one assumes that Λ is order $M_{\text{EW}}^2/M_{\text{Pl}}$ (and we will give explicit models in which this arises), this triple coincidence is inevitable, split only by $\mathcal{O}(1)$ coefficients such as α and π . As we will see, this solution to the cosmic coincidence problem also allows for a new understanding of the “Why now?” problem.

Another apparent “coincidence” in cosmology arises when one examines the components that comprise the matter density. Current data favors cold dark matter as the dominant component of Ω_M , but baryons ($\Omega_B \sim 0.05$) and neutrinos ($\Omega_\nu \sim 0.003\text{--}0.15$) are also appreciable. Having argued for the triple coincidence, we outline how the framework can be extended to give a fivefold coincidence: It is natural for Ω_B and Ω_ν to also be $\mathcal{O}(1)$ at the present time.

We begin with the minimal standard model. The physics of electroweak symmetry breaking is well-known to suffer from a hierarchy problem: The energy scale at which the electroweak symmetry is broken, $\sim\text{TeV}$, is 15 orders of magnitude smaller than the Planck scale 10^{18} GeV, which we take to be the fundamental length scale of the Universe. Proposed models in which this hierarchy is explained are substantially more complicated at the electroweak scale than the standard model itself, with a plethora of new particles. Among these, there are typically stable particles which can be produced as thermal relics, such as the lightest supersymmetric particle in models with hidden-sector supersymmetry breaking [5]. It is often remarked that the relic density of a stable particle at the electroweak scale gives the correct order of magnitude for the cold dark matter component of the Universe, which is believed to dominate the matter energy density. This important result is apparently a numerical coincidence, but in our new perspective it will be guaranteed.

It is simple to estimate the relic density solely from the fundamental parameters of the theory. For a stable particle χ of mass m with electroweak interactions, its annihilation cross section is given roughly by $\sigma \sim 1/m^2$. When the temperature falls below the mass m , the expansion rate $H \sim T_f^2/M_{\text{Pl}}$ wins over the annihilation rate $\Gamma \sim \sigma n_\chi$ and the abundance n_χ/s freezes out when $H \sim \Gamma$. After freeze-out, the abundance n_χ/s is constant, so that the matter energy density (dominated by χ) is

$$\rho_M = m_\chi n_\chi \sim \frac{1}{\sigma} \frac{m^3}{M_{\text{Pl}}} \left(\frac{T}{m}\right)^3 \sim \frac{M_{\text{EW}}^2 T^3}{M_{\text{Pl}}}, \quad (1)$$

where the factor $(T/m)^3 \sim (R_f/R)^3$ is the usual dilution due to the expansion of the universe after freeze-out. Given this result, it is easy to estimate the temperature at which

matter-radiation equality occurs, simply by equating $\rho_R = T^4$ with ρ_M , and we find

$$T_{\text{eq}} \sim \frac{M_{\text{EW}}^2}{M_{\text{Pl}}}. \quad (2)$$

Now, we would like to understand the presence of a *triple* coincidence, where Λ becomes equal to matter and radiation. If this is not to be a numerical accident, it must be that Λ itself is determined in terms of the fundamental mass scales of the theory as

$$\rho_\Lambda \sim \left(\frac{M_{\text{EW}}^2}{M_{\text{Pl}}}\right)^4. \quad (3)$$

In such a theory, the existence of a triple coincidence between radiation, matter, and Λ is *guaranteed* when the Universe is at the temperature $T_{\text{eq}} \sim M_{\text{EW}}^2/M_{\text{Pl}}$, regardless of the numerical value of M_{EW} . For $M_{\text{EW}} \sim 1$ TeV, $T_{\text{eq}} \sim 10^{-3}$ eV ~ 10 K.

We will shortly present models where Λ is plausibly determined in terms of the fundamental scales in the correct combination. However, before discussing this, it is important to note that our triple coincidence is significantly split by $\mathcal{O}(1)$ dimensionless factors. A correct version of Eq. (1) is [6]

$$\rho_M = \frac{0.756(n+1)x_f^{n+1}}{g^{1/2}\sigma_0 M_{\text{Pl}}} s(T), \quad (4)$$

where g is the effective number of degrees of freedom at the time of the freeze-out, $s(T) = \frac{2\pi^2}{45} g_0 T^3$ is the entropy of the Universe, and $x_f = m/T_f \approx 20$ is a parameter which describes the freeze-out temperature T_f . The annihilation cross section is thermally averaged as $\langle\sigma v\rangle = \sigma_0 x_f^{-n}$ with $n = 0$ ($n = 1$) for S -wave (P -wave) annihilations. These dimensionless factors are not very important. However, the cross section σ_0 is suppressed by weak coupling factors so that $\sigma_0 \sim \pi\alpha^2/M_{\text{EW}}^2$, which makes ρ_M bigger by a factor of $\sim 1/\pi\alpha^2 \sim 10^3$. This enhancement of the matter energy density causes the temperature at the matter-radiation equality to become roughly a factor of 10^3 bigger, $T_{\text{eq}} \sim 1$ eV instead of 10^{-3} eV. The existence of the (approximate) triple coincidence is *not* a numerical accident, however, since it is parametrically guaranteed for any values of M_{EW} and M_{Pl} .

The following picture emerges from the above considerations. Based on rough order of magnitude estimates, the triple coincidence of the matter, radiation energy densities, and Λ can be natural consequences of electroweak physics. Indeed, the evolution of our Universe shows a near triple coincidence (Fig. 1). If the coincidence were perfect, the Universe would be rather boring: before $T_{\text{eq}} \sim M_{\text{EW}}^2/M_{\text{Pl}}$, it is radiation dominated and structure cannot form; after T_{eq} the Universe starts to inflate, leaving a completely empty universe. However the $\mathcal{O}(10^3)$ enhancement in ρ_M causes matter-radiation equality to occur before matter- Λ equality. Thus, as can be seen in Fig. 1, there is a small triangle during which density

fluctuations can grow, resulting in structure. If the cosmological constant is truly a constant, then this interesting period does not last long; the matter energy density soon dilutes to the level of Λ and this interesting period ends. On the other hand, the apparent cosmological constant could be due to an energy density in a quintessence field $\phi(\sim M_{\text{Pl}})$ of mass $m_\phi^2 \sim M_{\text{EW}}^8/M_{\text{Pl}}^6$, which has been prevented from falling by the Hubble friction until the triple coincidence era. In this case, ϕ will fall to the minimum of its potential a few e foldings into the coincidence era, and the universe will return to being matter dominated.

We would now like to demonstrate that it is natural to expect Λ of order $(M_{\text{EW}}^2/M_{\text{Pl}})^4$. We will start with a particularly simple model which uses false vacua as the origin of the Λ [7]. We do not insist that this model should be the true origin of Λ , but we find it extremely encouraging that such a simple model serves the purpose. We assume supersymmetry, which is broken at the TeV scale by an order parameter chiral superfield $\langle S \rangle = \theta^2 M_{\text{EW}}^2$, so that the electroweak symmetry breaking is indeed the direct consequence of the supersymmetry breaking. We also assume that there is a “hidden sector,” which couples to the supersymmetry breaking only by Planck-scale suppressed operators. This is reminiscent of the conventional hidden-sector supersymmetry breaking models in supergravity, except that we now reverse the roles of the hidden and the observable sectors. In the hidden sector, we have supersymmetric quantum chromodynamics with $SU(n_c)$ gauge group with n_f flavors $Q + \bar{Q}$, and assume that the beta function $3n_c - n_f$ is somewhat small. Once supersymmetry is broken in the observable sector by the order parameter S , it can generate masses for the hidden sector quarks by

$$\int d^4\theta \frac{S^*}{M_{\text{Pl}}} \bar{Q} Q = \int d^2\theta \frac{M_{\text{EW}}^2}{M_{\text{Pl}}} \bar{Q} Q. \quad (5)$$

Therefore, the masses of quarks and squarks are of the order of $m_Q \sim M_{\text{EW}}^2/M_{\text{Pl}}$. Similarly, the gluino λ of the $SU(n_c)$ gauge group also acquires a mass via the operator

$$\int d^2\theta \frac{S}{M_{\text{Pl}}} W_\alpha W^\alpha = \frac{M_{\text{EW}}^2}{M_{\text{Pl}}} \lambda \lambda, \quad (6)$$

and again the mass is of the order of $m_\lambda/g^2 \sim M_{\text{EW}}^2/M_{\text{Pl}}$. For the simplicity of the analysis, we take the gluino mass somewhat smaller than the others. In the absence of the gluino mass, the decoupling of the quarks generates the low energy dynamical scale:

$$\Lambda_{\text{low}}^{3n_c} = \Lambda_{\text{high}}^{3n_c - n_f} m_Q^{n_f}. \quad (7)$$

Note that the low energy dynamical scale is determined essentially by m_Q if $3n_c - n_f \ll n_f$. It is basically that the gauge coupling constant evolves slowly above m_Q , while it grows quickly below m_Q .

The low energy theory, supersymmetric pure Yang-Mills, develops a gluino condensate $\langle \lambda \lambda \rangle$ with superpotential

$$\int d^2\theta \Lambda_{\text{low}}^3 e^{2\pi i k/n_c}. \quad (8)$$

Here, the integer $k = 1, \dots, n_c$ parametrizes the degenerate vacua consistent with the Witten index n_c . The discrete R symmetry Z_{2n_c} is spontaneously broken down to Z_2 by the gluino condensate. In the presence of the (small) gluino mass, we replace the gauge coupling constant as $8\pi^2/g^2 \rightarrow 8\pi^2(1 + \theta^2 m_\lambda)/g^2$ which breaks the R symmetry explicitly to Z_2 . The vacuum energy can be calculated exactly to lowest order in m_λ :

$$V_k = \frac{16\pi^2}{g^2} |m_\lambda \Lambda_{\text{low}}^3| \left(c - \cos \frac{2\pi k + \Theta}{n_c} \right), \quad (9)$$

where Θ is the Θ parameter of $SU(n_c)$. We assume the ground state energy is tuned to zero by choosing the constant c appropriately. However, the system can drop into any of the states labeled by k and in general has the vacuum energy of the order of $m_\lambda \Lambda_{\text{low}}^3 \sim (M_{\text{EW}}^2/M_{\text{Pl}})^4$. Although in this model we assumed low energy supersymmetry breaking, a simple variation can be extended to the case of gravity-mediated supersymmetry breaking [8]. If the Θ parameter is dynamical, i.e., $\Theta = a/M_{\text{Pl}}$ for an axionlike field a , then a has the properties of a quintessence field, beginning a slow roll down its potential only recently and contributing $\rho_a \sim (M_{\text{EW}}^2/M_{\text{Pl}})^4$ [8].

In the above model, we assumed that an unknown mechanism canceled the vacuum energy at the global minimum of the potential. It may be that the true mechanism for cancelling the cosmological constant has the feature that only *part* of the vacuum energy is cancelled. Indeed, Ref. [9] proposes to eliminate the purely standard model loop contributions to the cosmological constant, by embedding the standard model fields on a 3-brane in large extra dimensions. In these models, the effective four-dimensional (4D) cosmological constant is a power series expansion in powers of the standard model vacuum energy $V_{\text{SM}} \sim M_{\text{EW}}^4$ of the form

$$\rho_\Lambda = \sum_{n=0}^{\infty} c_n \frac{V_{\text{SM}}^{n+1}}{M_*^{4n}}, \quad (10)$$

where M_* is the higher-dimensional Planck scale related to the 4D Planck scale as $M_{\text{Pl}}^2 = M_*^6/V_{\text{SM}}$, and the coefficient $c_0 = 0$ naturally. Unfortunately, the terms with $n = 1, 2$ are still too large, but $n = 3$ gives the perfect combination $\rho_{\Lambda,4} \sim (M_{\text{EW}}^2/M_{\text{Pl}})^4$. These models demonstrate that an incomplete cancellation mechanism for truly solving the cosmological constant problem may yield a Λ appropriately determined by M_{EW} and M_{Pl} .

What about neutrino and baryon energy densities? The fact that the neutrino energy density is comparable to the rest has a natural explanation if the neutrino mass arises from the seesaw mechanism. In terms of the two fundamental scales M_{EW} and M_{Pl} , the seesaw mechanism gives $m_\nu \sim M_{\text{EW}}^2/M_{\text{Pl}}$. Then we find

$$\rho_\nu \sim m_\nu T_0^3 \sim \frac{M_{\text{EW}}^2}{M_{\text{Pl}}} T_0^3 \sim \frac{M_{\text{EW}}^8}{M_{\text{Pl}}^4}, \quad (11)$$

again of the same order of magnitude as ρ_M , ρ_R , and ρ_Λ . One can also find models of baryogenesis where the baryon asymmetry is generated naturally as $n_B/s \sim 10^5 M_{EW}/M_{Pl}$, where $10^5 \sim (2\pi/\alpha)^2$ [8]. Then the baryon energy density is

$$\rho_B \sim 10^5 \frac{M_{EW}}{M_{Pl}} T_0^3 m_p \sim 10^5 \frac{m_p}{M_{EW}} \frac{M_{EW}^8}{M_{Pl}^4}, \quad (12)$$

which is also parametrically the same combination of M_{EW} and M_{Pl} if one regards $m_p/M_{EW} \sim 10^{-3}$ as an $O(1)$ coefficient.

Given our solution to the triple coincidence problem, we gain a significant insight into the ‘‘Why now?’’ problem. For this purpose, we need to define the problem more precisely. One aspect of the ‘‘Why now?’’ problem is that the time when the structure starts to form t_{str} and the time of matter- Λ equality t_Λ are close to each other. Within our solution to the triple coincidence problem, this means

$$t_{str} \sim (\pi\alpha)^2 \frac{M_{Pl}^3}{M_{EW}^4} \left(\frac{\delta\rho}{\rho}\right)_p^{-3/2} \sim t_\Lambda \sim \frac{M_{Pl}^3}{M_{EW}^4}. \quad (13)$$

Here, $(\delta\rho/\rho)_p \sim 10^{-5}$ is the primordial density fluctuation generated during inflation. Note that both time scales have the same parametric dependences on M_{Pl} and M_{EW} , which makes this near equality natural. This is essentially the same as the statement that the triple coincidence is natural in our framework. In fact, the equality is surprisingly good, given that our solution by itself does not fix unknown $O(1)$ coefficients.

Another aspect of the ‘‘Why now?’’ problem is why we do not live at a time far beyond t_{str} . This is an old mystery present even if the cosmological constant were zero and not our main concern here. Nonetheless, it is interesting to discuss it briefly. Once structure forms, life as we know it is limited to the time before which all of the available hydrogen has been burnt up in stars. The time scale for this is given by

$$t_{burn} \sim 10^2 \frac{\pi\alpha^2 8\pi M_{Pl}^2}{m_p m_e^2} \sim \frac{M_{Pl}^2}{M_{EW}^3} \frac{M_{EW}}{m_p h_e^2}, \quad (14)$$

where h_e is the electron Yukawa coupling. It is an old puzzle why $t_{burn} \sim t_{str}$. Having expressed t_{str} in terms of the fundamental energy scales, the dependence on M_{Pl} and M_{EW} suggests that t_{burn} is in general much shorter than t_{str} . This would actually strengthen our solution to the ‘‘Why now?’’ problem since it would leave little room between structure formation at t_{str} and the time when all nuclear fuel is used up at $t_{str} + t_{burn}$; then $t_{str} \sim t_\Lambda$ would be enough to solve the puzzle. For a wide range of dimensionless parameters m_p/M_{EW} and h_e , this near equality remains true. In reality, h_e is so small that $t_{burn} \sim 100t_{str}$. The fact that t_{str} and t_{burn} are nearly equal in orders of magnitude is a purely numerical coincidence beyond the scope of our current understanding. Note, however, that the only

requirement for solving the ‘‘Why now?’’ problem is that the two dimensionless quantities m_p/M_{EW} and h_e are not too small.

If m_p/M_{EW} and/or h_e were smaller than observed, stars could still exist much later than $t_{str} \sim t_\Lambda$. However, it is intriguing to note that cosmology cannot be done long after the matter- Λ equality. Structures at cosmological distances go beyond the horizon, the cosmic microwave background is exponentially redshifted and nuclei become highly processed. Even if some life forms remained, they might not be able to perform cosmological observations to discover the big bang.

In summary, we have introduced a new perspective on the cosmic coincidence problems. We begin with the assumption that there are only two fundamental energy scales in the problem, the Planck scale $M_{Pl} \sim 10^{18}$ GeV and the electroweak scale $M_{EW} \sim 1$ TeV. Electroweak scale physics produces a cold dark matter relic, while the cosmological constant is also tied to the electroweak physics as $\rho_\Lambda^{1/4} \sim M_{EW}^2/M_{Pl}$. We then find that a triple coincidence among the matter, radiation, and Λ energy densities is a necessary consequence. Putting $O(1)$ coefficients back into the discussion, the matter energy density is enhanced by a factor of $1/\pi\alpha^2 \sim 10^3$, and a cosmological window opens in the evolution of the Universe between the matter-radiation equality and Λ dominance, during which structure forms. We presented a simple model in which the cosmological constant is indeed generated at $\rho_\Lambda^{1/4} \sim M_{EW}^2/M_{Pl}$. Moreover, we pointed out that the coincidence of the neutrino as well as the baryon energy density can also be simply understood. This framework allows us to understand why the time scale for structure formation and matter- Λ equality are comparable in orders of magnitude, solving the aspect of the ‘‘Why now?’’ problem involving the cosmological constant.

This work was supported in part by the DOE Contract No. DE-AC03-76SF00098 and by NSF Grant No. PHY-95-14797.

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