Bogoyavlenskij Replies: The hypothesis formulated in [1], and which in [2] is referred to as "Parker's hypothesis," concerns invariant properties of small perturbations $\epsilon b_j(x,y,z)$ near a z-invariant plasma equilibrium magnetic field $\mathbf{B}(x,y)$. Parker's hypothesis is formulated under three important conditions [1]: (1) "the local perturbation to the field is small compared to the total field," p. 361; (2) the length of the flux tube L is "large compared to the characteristic transverse scale of variation ℓ of the field," p. 362; (3) "the magnetic field is analytic in its deviation ϵ from the invariant field $B_i(x,y)$," p. 378.

Parker claims that a counterexample to Parker's hypothesis was provided by Rosner and Knobloch in 1982. However, the example of [3] involves two plasma equilibrium magnetic fields $\mathbf{B}_0(x,y)$ and $\mathbf{B}_1(y,z)$ where the first is z-invariant and the second, x-invariant. They treat $\mathbf{B}_1(y,z)$ as a perturbation of $\mathbf{B}_0(x,y)$ and notice that $\mathbf{B}_1(y,z)$ is not z-invariant. But such a perturbation adds an infinite magnetic energy in any layer $c_1 < z < c_2$, so it is not small. Nor does it satisfy Parker's condition (2). Moreover, the only exact solutions presented in [3] have singularities: "$\mathbf{B}_0(x,y) = (x^2 + y^2)^{-1}(-y,x,0)$, $\mathbf{B}_1(y,z) = (y^2 + z^2)^{-1}(0,-z,y)$, (3.10)." Hence, the case treated in [3] is different from the one treated in [1].

Similarly, the work by Van Ballegooijen [4] cannot really be considered to supply a counterexample. Using an expansion parameter different from [1], Van Ballegooijen constructs the force-free perturbations, $p =$ const, of a constant uniform magnetic field B_0 which depend on z. The lowest order equation [4] is equivalent to the time dependent two dimensional vorticity equation. However the complete solution in [4] is presented in the form of an infinite power series obtained by subsequent resolving of a more complex system of partial differential equations. Whether this power series is well behaved in $\mathbb{R}^3$ and whether it satisfies Parker's condition (2) is not studied. No exact solutions are obtained in [4] and the author writes: "Our conclusions do *not* apply to systems with field lines that are not tied to a boundary. Examples of such systems are the toroidal fields used in fusion machines (e.g., tokamaks)," p. 426. The plasma equilibria derived in [2] are exactly of this type, with toroidal magnetic surfaces and with $p \neq$ const.

Later, in [5], the generalizations of Parker's hypothesis for MHD [6] are reviewed and the statement is called "Parker's theorem." The authors of [7] (1993) continue this characterization of Parker's hypothesis as an established fact when they write: "It is well known that all well-behaved MHD equilibria extending to all space need to be translationally symmetric," p. 2158.

The exact global plasma equilibria derived in [2] model the astrophysical jets and behave ergodically in variable z. They have finite magnetic energy in any layer $c_1 < z < c_2$ and satisfy all three of Parker's conditions. These exact solutions prove without intersection with [3,4], that much more complex topologies of magnetic surfaces than suggested in [4] *do* appear in the arbitrarily small analytical perturbations of the z-invariant equilibria.

In [8], Parker returns to his statement made in [1] that any bounded solution to the equation

$$\frac{\partial}{\partial x}\frac{1}{B^2}\frac{\partial \Psi}{\partial x} + \frac{\partial}{\partial y}\frac{1}{B^2}\frac{\partial \Psi}{\partial y} + \frac{\partial}{\partial z}\frac{1}{B^2}\frac{\partial \Psi}{\partial z} = 0 \quad (1)$$

is constant if $B = B(x,y)$ is nonvanishing everywhere. The statement is used as a key argument in the proof [1] of Parker's hypothesis and also in the proof [6] of its generalization for MHD. The use of this statement [1] is a logical error. Indeed, let us consider one concrete example of [2]:

$$B(x,y) = [1 + (ax + by)^2]^{-1/2}, \qquad \Psi(x,y) = \tan^{-1}(ax + by), \quad (2)$$

where $\tan^{-1}(z)$ is the inverse function for $\tan(z)$ and $a, b =$ const. Function $B(x,y)$ (2) *does* satisfy Parker's condition because it is "*nonvanishing* throughout the entire space, $-\infty < x, y < +\infty$" [8]. Function $\Psi(x,y)$ (2) satisfies Eq. (1). It is bounded, $|\Psi(x,y)| < \pi/2$, and it is *nonconstant.* Hence Parker's statement [1], repeated in [8], is a logical mistake.

The exact plasma equilibria obtained in [2] form continuous families parametrized by an integer N and $2N$ arbitrary real parameters $\beta > 0$, a_N, a_k, b_k, $k = 1,\ldots, N-1$. Varying these parameters, one gets bifurcations of topological structures of magnetic surfaces which form systems of nested tori and nested cylindrical surfaces; see Fig. 1 of [2]. In [9], we present the exact helically symmetric plasma equilibria with analogous properties. All these solutions are smooth and well-behaved and have no tangential discontinuities and no current sheets. They form families of plasma equilibria which do not obey the magnetostatic theorem developed in [10].

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