

Neutron-Proton Differential Flow as a Probe of Isospin-Dependence of the Nuclear Equation of State

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The neutron-proton differential flow is shown to be a very useful probe of the isospin-dependence of the nuclear equation of state (EOS). This novel approach utilizes constructively both the isospin fractionation and the nuclear collective flow as well as their sensitivities to the isospin-dependence of the nuclear EOS. It also avoids effectively uncertainties associated with other dynamical ingredients of heavy-ion reactions at intermediate energies.

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The rapid advance in experiments with rare isotopes has opened up several new frontiers in nuclear science [1–3]. It is now possible to study in detail structures of nuclei in many unexplored regions of the periodic chart and novel properties of highly isospin asymmetric nuclear matter. Prospects for new physics in nuclear reactions induced by especially exotic neutron-rich nuclei have generated much interest in the nuclear science community. In particular, fast fragmentation beams at the planned Rare Isotope Accelerator (RIA) will provide a unique opportunity to explore features of nuclear matter at extreme isospin asymmetries towards the pure neutron matter. The isospin-dependence of the nuclear equation of state (EOS) is among the most important but very poorly known properties of neutron-rich matter [4]. It is relevant to type II supernova explosions, to neutron-star mergers, and to the stability of neutron stars. It also determines the proton fraction and electron chemical potential of neutron stars at β equilibrium. These quantities consequently determine the cooling rate and neutrino emission flux of protoneutron stars and the possibility of kaon condensation in dense stellar matter [5]. At present, nuclear theories predict vastly different isospin-dependence of the nuclear EOS depending on both the many-body techniques and the bare two-body and/or three-body interactions employed. In this Letter, we show that the neutron-proton differential flow is a very useful probe of the isospin-dependence of the nuclear EOS. This approach utilizes constructively both the isospin fractionation and the nuclear collective as well as their sensitivities to the isospin-dependence of the nuclear EOS. It also avoids effectively uncertainties associated with other dynamical ingredients of heavy-ion reactions at intermediate energies. The experimental measurement of neutron-proton differential flow provides thus a novel means for determining the isospin-dependence of the nuclear EOS.

Various theoretical studies (e.g., [6,7]) have shown that the energy per nucleon $e(\rho, \delta)$ in nuclear matter of density ρ and isospin asymmetry parameter $\delta \equiv (\rho_n - \rho_p)/(\rho_n + \rho_p)$ can be approximated very well by a parabolic function

$$e(\rho, \delta) = e(\rho, 0) + S(\rho)\delta^2, \quad (1)$$

where $e(\rho, 0)$ is the EOS of symmetric nuclear matter. In the above $S(\rho)$ is the symmetry energy

$$S(\rho) = (2^{2/3} - 1)\frac{3}{5}E_F^0 u^{2/3} + S_{\text{pot}}(\rho), \quad (2)$$

where E_F^0 is the Fermi energy at normal nuclear matter density ρ_0 and $S_{\text{pot}}(\rho)$ is the potential contribution to the symmetry energy. Vastly different density-dependences of $S_{\text{pot}}(\rho)$ given by various many-body theories have led to rather divergent predictions on the isospin-dependence of the nuclear EOS. Here we adopt the parametrization of $S_{\text{pot}}(\rho)$ from the study of neutron stars (e.g., [5,8,9])

$$S_{\text{pot}}(\rho) = [S(\rho_0) - (2^{2/3} - 1)\frac{3}{5}E_F^0]F(u), \quad (3)$$

where $u \equiv \rho/\rho_0$ is the reduced density and $S(\rho_0)$ is the symmetry energy at ρ_0 . The latter is known to be in the range of about 27–36 MeV [10–12]. We consider two typical parametrizations of $S_{\text{pot}}(\rho)$ with $F(u) = 2u^2/(1+u)$ and $F(u) = u^{1/2}$, respectively. The isospin-dependence of the nuclear EOS can be characterized by the curvature of the symmetry energy at ρ_0

$$K_{\text{sym}} \equiv 9\rho_0^2 \left. \frac{\partial^2 S(\rho)}{\partial^2 \rho} \right|_{\rho=\rho_0}. \quad (4)$$

For the two parametrizations of $S_{\text{pot}}(\rho)$ considered here the K_{sym} parameter is 61 and –69 MeV, respectively. These values are well within the wide range of K_{sym} from about –400 to +466 MeV predicted by many-body theories (e.g., [3,6]). Available data of giant monopole resonances do not give a stringent constraint on the K_{sym} parameter either [13]. The corresponding single particle symmetry potential $V_{\text{asy}}^{n/p}$ is

$$V_{\text{asy}}^{n/p} = \pm 4e_a \frac{u^2}{1+u} \delta + 2e_a \frac{u^2}{(1+u)^2} \delta^2, \quad (5)$$

and

$$V_{\text{asy}}^{n/p} = \pm 2e_a u^{1/2} \delta - \frac{1}{2} e_a u^{1/2} \delta^2, \quad (6)$$

for $K_{\text{sym}} = +61$ and –69 MeV, respectively. In the above e_a is a constant of

$$e_a = S(\rho_0) - (2^{2/3} - 1)\frac{3}{5}E_F^0 \approx 19 \text{ MeV}. \quad (7)$$

The + and (-) signs are for neutrons and protons, respectively. For small isospin asymmetries and densities near ρ_0 the above symmetry potentials reduce to the well-known Lane potential which varies linearly with δ [14].

Our study is based on the isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model (e.g., [15,16]). In this model protons and neutrons are initialized according to their density distributions predicted by the relativistic mean-field (RMF) theory [17]. The isospin-dependent reaction dynamics is included through isospin-dependent nucleon-nucleon scatterings and Pauli blockings, the symmetry potential $V_{\text{asy}}^{n/p}$, and the Coulomb potential V_c^p for protons. A Skyrme-type parametrization is used for the isoscalar potential. For a recent review of the IBUU model, we refer the reader to Ref. [3].

To establish the essence and to reveal the advantage of analyzing the neutron-proton differential flow, we first study individually the isospin fractionation and the isospin-dependence of nucleon collective flow. These phenomena in nuclear reactions induced by neutron-rich nuclei are very interesting in their own rights. Here we concentrate on studying their sensitivities to the isospin-dependence of the nuclear EOS only. The isospin fractionation is an unequal partitioning of the neutron to proton ratio N/Z of unstable asymmetric nuclear matter between low and high density regions. It is energetically favorable for the unstable asymmetric nuclear matter to separate into a neutron-rich low density phase and a neutron-poor high density one [18]. Indications of this phenomenon have been found recently in intermediate energy heavy-ion experiments [19]. We quantify the degree of isospin fractionation by using the ratio of $(N/Z)_{\text{free}}$ to $(N/Z)_{\text{bound}}$. Here $(N/Z)_{\text{free}}$ and $(N/Z)_{\text{bound}}$ are the saturated neutron to proton ratios of nucleons with local densities less (free) and higher (bound) than $\rho_c \equiv 1/8\rho_0$, respectively. The density cutoff ρ_c has often been used in transport models for identifying approximately free nucleons quickly. The specific value of ρ_c used here does not affect our conclusions in this work. Shown in Fig. 1 is the degree of isospin

fractionation as a function of the neutron to proton ratio $(N/Z)_{\text{sys}}$ of the reaction system (left), impact parameter (middle), and beam energy (right), respectively, for reactions between several Sn isotopes. It is seen that the degree of isospin fractionation increases with both the $(N/Z)_{\text{sys}}$ and the impact parameter, but decreases with the beam energy. It is very interesting to see that the degree of isospin fractionation is rather sensitive to the K_{sym} parameter. This sensitivity is very strong, noticing that the change of K_{sym} made here is only about 1/7 of its variation predicted by the many-body theories. The origin of isospin fractionation and its dependence on the K_{sym} parameter can be easily understood from the density dependence of the symmetry energy. Since the repulsive symmetry potential for neutrons increases with density, more neutrons will be repelled from high density regions to low density regions. The opposite is true for protons because of their attractive symmetry potentials. Furthermore, the magnitude of symmetry potentials is higher for $K_{\text{sym}} = -69$ MeV than for $K_{\text{sym}} = +61$ MeV for densities less than about ρ_0 . One thus expects to see a higher degree of isospin fractionation with $K_{\text{sym}} = -69$ MeV as shown here.

The analysis of collective flow [20] has been very useful in studying various properties of nuclear matter (e.g., [21, 22]). How useful is it for studying the isospin-dependence of the nuclear EOS? To answer this question we study the average in-plane transverse momentum $\langle p_x/N \rangle$ of free nucleons as a function of their reduced rapidity y/y_{cms} . Shown in Fig. 2 is such an analysis for the reaction of $^{124}\text{Sn} + ^{124}\text{Sn}$ at a beam energy of 50 MeV/nucleon and an impact parameter of 5 fm. To make the analysis more sensitive to the symmetry potential, the incident energy is chosen to be near the balance energy of the reaction system according to the systematics of balance energies [23]. Our calculations are performed with the K_{sym} parameter of +61 MeV (upper window) and -69 MeV (lower window), respectively. The difference between neutron and proton flow is found to depend strongly on the K_{sym} parameter. Thus, interesting information on the isospin-dependence

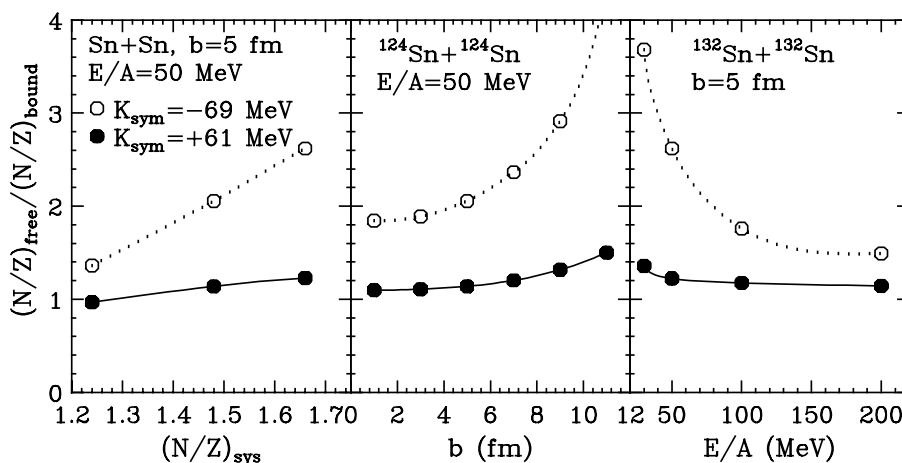


FIG. 1. Isospin fractionation as a function of (N/Z) of the reaction system (left), impact parameter (middle), and beam energy (right) in Sn + Sn reactions.

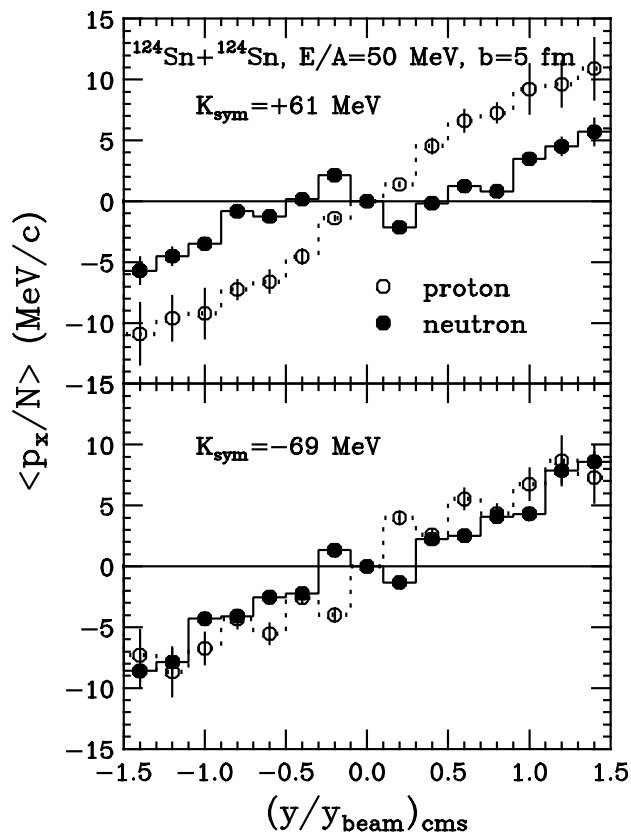


FIG. 2. The average transverse momentum per nucleon in the reaction plane for neutrons and protons as a function of reduced rapidity with the K_{sym} parameter of +61 MeV (upper window) and -69 MeV (lower window), respectively.

of the nuclear EOS can also be obtained from analyzing the collective flow of nucleons. In the case of $K_{\text{sym}} = +61$ MeV the repulsive Coulomb potential dominates over the negative symmetry potential for generating the proton flow. The net sum of Coulomb and symmetry potentials for protons is actually higher than the positive symmetry potential for neutrons. The proton flow is thus higher than the neutron flow. In particular, near the midrapidity, neutrons are still flowing to the negative direction because of the attractive mean field acting on them while protons have already started flowing in the positive sense. By changing the K_{sym} parameter from +61 to -69 MeV the magnitude of symmetry potentials is increased but the Coulomb potential remains the same. Thus the difference between neutron and proton flow is reduced as one expects and is shown in the lower window of Fig. 2.

It is well known that collective flow is also sensitive to both the curvature K_0 of the isoscalar part of the nuclear EOS and the in-medium nucleon-nucleon cross sections (e.g., [23,24]). Both quantities are still associated with some uncertainties despite the remarkable progress made in the last two decades. Also the symmetry potential is normally rather small compared to the isoscalar one. Therefore, to study the isospin-dependence of the nuclear EOS one has to select delicate observables that minimize influences of the isoscalar potential but maximize effects of the

symmetry potential. It would be ideal if these observables can also avoid effects of other dynamical ingredients of the reaction dynamics. We introduce here such an observable, i.e., the *neutron-proton differential flow*

$$F_{n-p}(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} p_{x_i} \tau_i, \quad (8)$$

where $N(y)$ is the total number of free nucleons at rapidity y and $\tau_i = 1$ and -1 for neutrons and protons, respectively. The neutron to proton ratio in $N(y)$ is determined by the degree of isospin fractionation. In essence, this isospin-averaged nucleon flow combines constructively contributions of the symmetry potential to the collective flow of both neutrons and protons. Simultaneously, it reduces significantly influences of the isoscalar potential and the in-medium nucleon-nucleon cross sections as they both act identically on neutrons and protons. Therefore, it uses efficiently the isospin effects shown in both the isospin fractionation and the nucleon collective flow. Shown in Fig. 3 is the neutron-proton differential flow as a function of the reduced rapidity for collisions between three Sn isotopes at a beam energy of 50 MeV/nucleon. A clear signal of the isospin-dependence of the nuclear EOS is seen at both the target and projectile rapidities. The signal becomes stronger with the increasing isospin asymmetry of the reaction system as one expects. Moreover, we found that a 40% variation of the K_0 parameter or a change by a factor of 2 of the in-medium nucleon-nucleon cross sections results in a less than 7% change in the values of $F_{n-p}(y)$. The neutron-proton differential flow is thus a very useful probe of the isospin dependence of the nuclear EOS. We have also performed a study on the excitation function of the neutron-proton differential flow from $E_{\text{beam}}/A = 30$ to 400 MeV for the Sn + Sn reactions at several impact parameters. The signal exists clearly in the whole energy range and is the strongest close to the lower energy end. For a given reaction system involving neutron-rich nuclei, what is the best beam energy to learn the most about the isospin-dependence of the nuclear EOS? We suggest that one selects the incident energy to be close to the balance energy of a reaction system with similar mass but along the β stability valley. It is well known that the transverse flow disappears at the balance energy where the attractive interactions balance the repulsive scatterings [25]. At this energy the nonisospin related background is the smallest for studying the isospin-dependence of the nuclear EOS with the neutron-proton differential flow. It is useful to point out here that the well-established systematics of balance energies [26] can serve as a guide in selecting the best beam energies for the purposes discussed above.

In conclusion, a novel means for determining the isospin-dependence of the nuclear equation of state, i.e., the analysis of neutron-proton differential flow, is introduced for the first time. This approach uses constructively both the isospin fractionation and the nuclear collective flow as well as their sensitivities to the isospin-dependence of the nuclear EOS. It also avoids effectively uncertainties

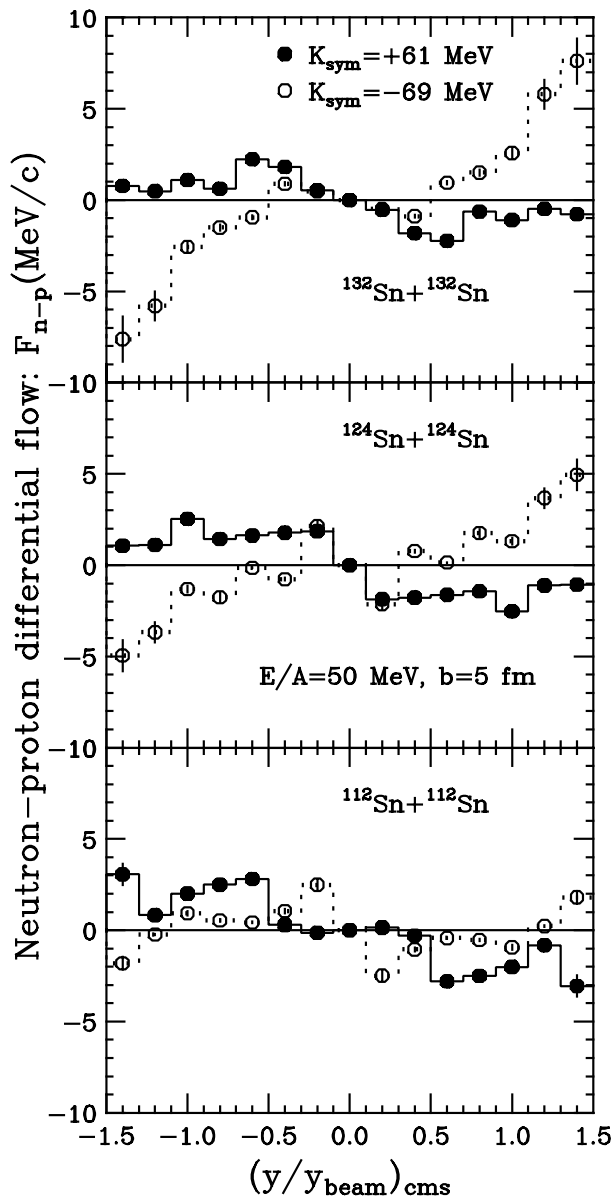


FIG. 3. Neutron-proton differential flow for the reaction of $^{112}\text{Sn} + ^{112}\text{Sn}$, $^{124}\text{Sn} + ^{124}\text{Sn}$, and $^{132}\text{Sn} + ^{132}\text{Sn}$ at a beam energy of 50 MeV/nucleon, an impact parameter of 5 fm and the K_{sym} parameter of +61 MeV (filled circles) and -69 MeV (open circles), respectively.

associated with other dynamical ingredients of heavy-ion reactions at intermediate energies.

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