Phenomenological Evidence for Gluon Depletion in pA Collisions

Rudolph C. Hwa

Institute of Theoretical Science and Department of Physics, University of Oregon, Eugene, Oregon 97403-5203

Ján Pišút and Neva Pišútová

Department of Physics, Comenius University, SK-84215, Bratislava, Slovakia

The data of J/ψ suppression at large x_F in pA collisions are used to infer the existence of gluon depletion as the projectile proton traverses the nucleus. The modification of the gluon distribution is studied by use of a convolution equation whose nonperturbative splitting function is determined phenomenologically. The depletion factor at $x_1 = 0.8$ is found to be about 25% at $A = 100$.

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It is conventional in the study of J/ψ production in heavy-ion collisions that the gluon distribution before the hard subprocess of $c\bar{c}$ production is assumed to be the same as in a free nucleon [1–3]. The unconventional view that the gluon distribution can be modified in the nuclear medium due to depletion was suggested in [4]. In this paper we focus on p-A collisions and show that the data [5] on $\alpha(x_F)$ can be used to infer that gluon depletion in the projectile proton is not negligible.

Charmonium absorption in pA collisions has been studied in [6,7] without finding any satisfactory explanation for the x_F dependence of $\alpha(x_F)$. In [8] the effect of energy loss of partons is considered, but that is only one aspect of gluon depletion. Here we pay particular attention to the evolution of the gluon distribution of the projectile as it traverses the nucleus. The approximate absence of dilepton suppression and the consequent implication that the quark distribution is nearly unaltered by the nuclear medium lead some to expect that the gluon distribution would be unaltered also. However, such a view is based on the validity of Q^2 evolution of the parton distribution functions [9]. We adopt the reasonable alternative view that the evolution in a nucleus is different from that of perturbative QCD (pQCD) at high Q^2; indeed, we shall let the data guide us in determining the proper dynamics of the low-Q^2 nonperturbative process.

The Fermilab E866 experiment measured the J/ψ suppression in p-A collisions at 800 GeV/c with a wide coverage of x_F [5]. The result is given in terms of $\alpha(x_F)$, which is defined by the formula

$$R(x_F, A) = \sigma_A(x_F)/A\sigma_N(x_F) = A^{\alpha(x_F)-1}, \qquad (1)$$

where $\sigma_{N,A}$ is the cross section for J/ψ production by a proton on a nucleon (N) or on a nucleus (A). In [5] a parametrization of $\alpha(x_F)$ for J/ψ production is given by

$$\alpha(x_F) = 0.952(1 + 0.023x_F - 0.397x_F^2) \qquad (2)$$

for $-0.1 < x_F < 0.9$. It is our aim here to explore the implication of Eq. (2) on the evolution of the gluon distribution.

Since the semihard subprocess of $g + g \rightarrow c + \bar{c}$ is common for p-N and p-A collisions, they cancel in the ratio $R(x_F, A)$ so the x_F dependence can come from three sources: (a) the ratio of the gluon distribution in the projectile passing through a nucleus to that in a free proton, $G(x_F, A)$, (b) nuclear shadowing of gluons in the target, $N(x_F, A)$, and (c) hadronic absorption of the $c\bar{c}$ states after the semihard subprocess, $H(x_F, A)$. Putting them together, we have

$$R(x_F, A) = G(x_F, A)N(x_F, A)H(x_F, A). \qquad (3)$$

$G(x_F, A)$ and $N(x_F, A)$ are ignored in [6,7]. Since $x_F < 0.25$ in [6], there is not much dependence on x_F to be ascribed to $H(x_F, A)$, but in [7], where the full range of x_F is considered, $H(x_F, A)$ is forced to carry the entire x_F dependence by a fitting procedure, resulting in an unreasonably short octet lifetime. Our approach by including $G(x_F, A)$ and $N(x_F, A)$ in Eq. (3) is therefore complementary to the work of [6,7].

The nuclear shadowing problem has been studied in detail by Eskola *et al.* [10,11], using the deep inelastic scattering data of nuclear targets at high Q^2. On the basis of leading-order pQCD evolution they can determine the parton distributions at any $Q^2 > 2.25$ GeV^2. The results are given in terms of numerical parametrizations (called EKS98 [11]) of the ratio $N_i^A(x, Q^2) = f_{i/A}(x, Q^2)/f_i(x, Q^2)$, where f_i is the parton distribution of flavor i in the free proton and $f_{i/A}$ is that in a proton of a nucleus A. We shall be interested in the ratio for the gluon distributions only at $Q^2 = 10$ GeV^2, corresponding to $c\bar{c}$ production, and denote it by $N(x, A)$. From the numerical output of EKS98 we find that a simple formula can provide a good fit to within 2% error in the range $40 < A < 240$ and $0.01 < x < 0.12$; it is

$$N(x, A) = A^{\beta(x)}, \qquad (4)$$

where

$$\beta(\xi(x)) = \xi(0.0284 + 0.0008\xi - 0.0041\xi^2), \quad (5)$$

with $\xi = 3.912 + \ln x$. Thus the A dependence is minimal at $\xi = 0$, corresponding to $x = 0.02$.

The variable x in Eq. (4) is the gluon momentum fraction in a nucleon in the nucleus, usually referred to as x_2. Both x_F in Eq. (1) and x_2 in Eq. (4) are to be converted to the x_1 variable for the projectile nucleon, using

$$x_F = x_1 - x_2, \qquad x_1 x_2 = \tau \equiv M_{J/\psi}^2/s, \quad (6)$$

so that a part of Eq. (3) can be rewritten as

$$R(x_F, A)/N(x_2, A) = A^{\alpha(x_F(x_1)) - \beta(x_2(x_1)) - 1}. \quad (7)$$

In our approach we treat $H(x_F, A)$ as having negligible dependence on x_F for all x_F. Attempts [6,7] to find that dependence have failed and led to the suggestion of the existence of an unaccounted mechanism responsible for the enhanced suppression in $R(x_F, A)$ at large x_F. In our view that mechanism is gluon depletion. Of course, if the x_F dependence of $H(x_F, A)$ were independently known, its incorporation in our analysis is straightforward. For us here, we identify the x_1 dependence of $G(x_1, A)$ in Eq. (3) with that in Eq. (7), which is completely known, and proceed to the study of the phenomenological implication on gluon depletion.

In the spirit of pQCD evolution, even though the effect of a nuclear target on the projectile gluon distribution is highly nonperturbative, we now propose an evolution equation on the gluon distribution $g(x, z)$, where z is the path length in a nucleus. For the change of $g(x, z)$, as the gluon traverses a distance dz in the nucleus, we write

$$\frac{d}{dz} g(x, z) = \int_x^1 \frac{dx'}{x'} g(x', z) Q\left(\frac{x}{x'}\right), \quad (8)$$

where $Q(x/x')$ describes the gain and loss of gluons in dz, but unlike the splitting function in pQCD, it cannot be calculated in perturbation theory. Equation (8) is similar to the nucleonic evolution equation proposed in [12], except that this is now at the parton level. Instead of guessing the form of $Q(x/x')$, which is unknown, we shall use Eq. (7) to determine it phenomenologically.

To that end, we first define the moments of $g(x, z)$ by

$$g_n(z) = \int_0^1 dx\, x^{n-2} g(x, z). \quad (9)$$

Taking the moments of Eq. (8) then yields

$$dg_n(z)/dz = g_n(z) Q_n, \quad (10)$$

where $Q_n = \int_0^1 dy\, y^{n-2} Q(y)$. It then follows that

$$g_n(z) = g_n(0) e^{zQ_n}, \quad (11)$$

whose exponential form suggests $Q_n < 0$ for the physical process of depletion. The gluon depletion function $D(y, z)$ is defined by

$$g(x, z) = \int_x^1 \frac{dx'}{x'} g(x', 0) D\left(\frac{x}{x'}, z\right), \quad (12)$$

where $g(x', 0)$ is the gluon distribution in a free nucleon. From Eq. (12) we have $g_n(z) = g_n(0) D_n(z)$, where $D_n(z)$ is the moment of $D(y, z)$. Comparison with Eq. (11) gives

$$D_n(z) = e^{zQ_n}. \quad (13)$$

To relate this result to $R(x_F, A)$, we first note that $G(x_F, A)$ in Eq. (3) is, by definition, $G(x_F, A) = g(x_1, A)/g(x_1, 0)$, where x_F is expressed in terms of x_1. It then follows from Eq. (3) that

$$J(x_1, A) \equiv g(x_1, 0)\, R(x_F(x_1), A)/N(x_2(x_1), A)$$
$$= g(x_1, A) H(A). \quad (14)$$

In relating A to the average path length L of the projectile p through the nucleus, we use $L = 3R_A/2 = 1.8A^{1/3}$ fm. We then set $z = L/2$ for the average distance traversed at the point of $c\bar{c}$ production. Thus when referring to the last expression of Eq. (14), we write $J(x_1, A) = g(x_1, z(A)) H(z(A))$, where $g(x_1, z)$ is to be identified with that in Eq. (12). Note that the A dependence of the middle expression in Eq. (14) is, on account of Eq. (7), in terms of $\ln A$, whereas that of the last expression is in terms of z, or $A^{1/3}$. Since it is known that $\ln A \approx A^{1/3}$ for $60 < A < 240$, we shall consider the consequences of Eq. (14) only for A in that range. We suggest that a revised form of presenting the data, different from that in Eq. (1), should be tried in the future.

Taking the moments of $J(x_1, A)$, we get using Eq. (11)

$$\ln J_n(A) - \ln g_n(0) = zQ_n + \ln H(z). \quad (15)$$

To determine Q_n, it is necessary to use as an input the gluon distribution $g(x_1, 0)$ in a free proton at $Q^2 = 10$ GeV^2. We adopt the simple canonical form

$$g(x_1, 0) = g_0 (1 - x_1)^5, \quad (16)$$

where the constant g_0 is canceled in Eq. (15) due to the definition of $J(x_1, A)$. In our calculation we set $g_0 = 1$. Indeed, the accuracy of $g(x_1, 0)$ is unimportant, since it enters Eqs. (14) and (15) in ways that render the result insensitive to its precise form. On the basis of Eqs. (7) and (16), $J(x_1, A)$ is therefore known. The left-hand side (LHS) of Eq. (15) can then be computed except for a caveat. To calculate the moments of $J(x_1, A)$, it is necessary to compute $\int_0^1 dx_1\, x_1^{n-2} J(x_1, A)$. However, x_1 cannot be less than τ in order to keep $x_2 \leq 1$ [see Eq. (6)]. Furthermore, Eq. (7) does not provide reliable information on $J(x_1, A)$ at small x_1, since the parametrizations of $\alpha(x_F)$ and $\beta(x_2)$ are for the variables in ranges that exclude the $x_1 \to \tau$ limit. Fortunately, that part of the integration in x_1 can be suppressed

by considering $n \geq 3$. The part of the integration in the interval $0 < x_1 < \tau$ amounts to only about 2% contribution even at $n = 2$ (if naive extrapolation is used), so its inaccuracy will be neglected. Physically, it is the data at high x_F that we emphasize in our analysis, and that corresponds to the high-n moments of $J(x_1, A)$.

For convenience, let us denote the LHS of Eq. (15) by $K_n(z)$, i.e., $K_n(z) \equiv \ln[J_n(z(A))/g_n(0)]$. For sample cases of $A = 100$ and 200, they are shown as discrete points in Fig. 1 for $3 \leq n \leq 20$. Instead of performing an inverse Mellin transform on $K_n(z)$, our procedure is to fit $K_n(z)$ by

a simple formula that can yield $Q(y)$ by inspection. The fitted curves shown by the solid and dashed lines in Fig. 1 are obtained by use of the formula

$$K_n = -k_0 + \frac{k_1}{n} - \frac{k_2}{n+1} + \frac{k_3}{n+2}. \quad (17)$$

Using k_i and k_i' to denote the values for the cases $A = 100$ and 200, respectively, we have

$$k_0 = 1.592, \quad k_1 = 23.42, \quad k_2 = 97.66, \quad k_3 = 89.17,$$
$$k_0' = 1.831, \quad k_1' = 27.43, \quad k_2' = 113.97, \quad k_3' = 103.80.$$

Because of Eq. (15), the n dependence of K_n prescribes the n dependence of Q_n. Let us therefore write

$$Q_n = -q_0 + \frac{q_1}{n} - \frac{q_2}{n+1} + \frac{q_3}{n+2}. \quad (18)$$

Since Eq. (15) is to be used only for $A > 60$, we evaluate it at $A = 100$ and 200, and take the difference. Denoting z by z_1 and z_2, respectively, for the two A values, and with $\Delta k_i = k_i' - k_i, \Delta z = z_2 - z_1$, we have

$$\Delta k_0 = q_0 \Delta z - \ln \frac{H(z_2)}{H(z_1)}, \qquad \Delta k_i = q_i \Delta z \quad (i \neq 0). \quad (19)$$

For the hadron absorption factor $H(z)$ we write it in the canonical exponential form [13], $H(z) = \exp(-\rho \sigma z)$, where $\rho^{-1} = (4/3)\pi(1.2)^3$ fm³, $z = 0.9A^{1/3}$ fm, and σ is the absorption cross section. Putting these in Eq. (19), we get (with $\Delta z = 1.086$ fm)

$$q_0 + \rho \sigma = 0.22, \qquad q_1 = 3.68, \qquad (20)$$
$$q_2 = 15.01, \qquad q_3 = 13.47$$

in units of fm⁻¹.
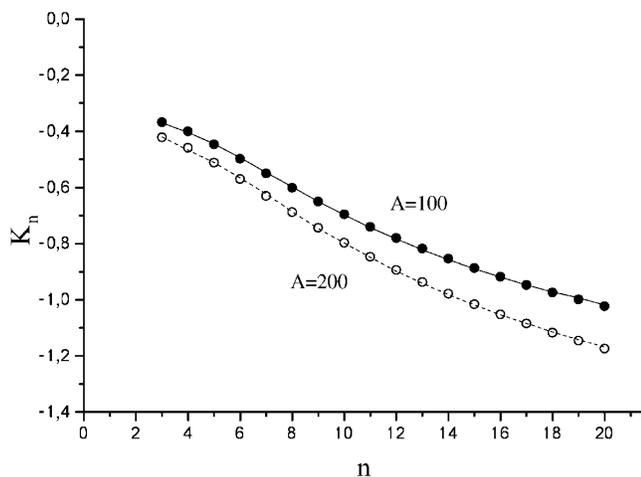
FIG. 1. K_n: The curves are fitted results using Eq. (17).

There is a reason why q_0 and $\rho \sigma$ enter Eq. (20) as a sum. To appreciate the physics involved, we first note that Eq. (18) implies directly

$$Q(y) = -q_0 \delta(1 - y) + q_1 y - q_2 y^2 + q_3 y^3. \quad (21)$$

The first and third terms on the right-hand side (RHS) above are the loss terms (i.e., gluon depletion), while the second and last terms represent gain (i.e., gluon regeneration). If $Q(y)$ consisted of only the first term, then using it in Eq. (8) would give $dg(x, z)/dz = -q_0 g(x, z)$, whose solution is of the same exponential form as that of absorption. With both depletion and absorption present, the exponents lead to a sum, as in Eq. (20). Our $Q(y)$ is, however, more complicated. The $-q_2 y^2$ term gives rise to depletion that depends on the shape of $g(x, z)$, while the $q_1 y + q_3 y^3$ terms generate new gluons at x from all the gluons at $x' > x$.

Since Q_n decreases monotonically with n, we require $Q_3 < 0$, and exclude Q_2 from this consideration because of its inaccuracy discussed earlier. Combining Eqs. (18) and (20), we get $\rho \sigma < q_0 + \rho \sigma - q_1/3 + q_2/4 - q_3/5 = 0.05$ fm⁻¹. We thus set $q_0 = 0.17$ fm⁻¹.

Since it is not easy to see directly from Q_n or $Q(y)$ the magnitude of the effect of gluon depletion and regeneration, we can calculate $g(x_1, z)$, not from Eq. (12), but by fitting the calculated $g_n(z)$ in Eq. (11), using the formula

$$g_n(z) = \sum_{i=1}^{3} a_i(z) B(n - 1, 5 + i), \quad (22)$$

where $B(a, b)$ is the beta function. Then the result yields directly

$$g(x_1, z) = \sum_{i=1}^{3} a_i(z)(1 - x_1)^{4+i}. \quad (23)$$

For $A = 100(200)$, i.e., $z = z_1(z_2)$, we have $a_1 = 0.58(0.485), a_2 = 0.92(1.118)$, and $a_3 = -0.47(-0.56)$ for $g_0 = 1$ in Eq. (16). The result for $G(x_1, z) = g(x_1, z)/g(x_1, 0)$ is then
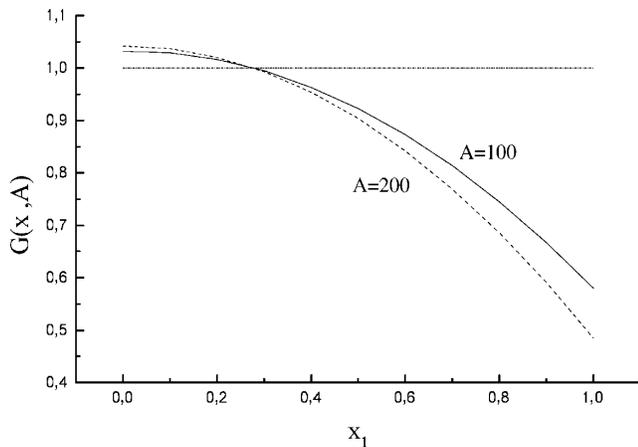
FIG. 2. $G(x_1, z)$: showing the effects of gluon depletion.

FIG. 3. $\alpha(x_F)$ vs x_F: The solid line is our result compared to the data from [3].

$$G(x_1, z) = a_1(z) + a_2(z)(1 - x_1) + a_3(z)(1 - x_1)^2, \tag{24}$$

which is shown in Fig. 2 for two values of A. It is now evident that gluon depletion suppresses the gluon distribution at medium and high x_1, but the unavoidable gluon regeneration enhances the distribution at low x_1. The crossover occurs at $x_1 \approx 0.28$.

Let us now exhibit our result for $\alpha(x_F)$, which is shown in Fig. 3. The line is obtained by the use of Eq. (24) in Eq. (3) and $\sigma = 6.5$ mb in $H(z)$. Only one line is shown for both $A = 100$ and 200, their difference being negligible in the plot. Since our method of using the moments cannot be extended to $n = 2$ due to the problems mentioned after Eq. (16), there is some inaccuracy inherent in our analysis. Thus the fit cannot be expected to be perfect. Our model can reproduce the general trend of the x_F dependence, but not the detail structure, for which more terms in Eqs. (17) and (18) would be needed. The overall suppression is achieved by the use of a phenomenological value of σ, rather than the bound based on the technical assumption of $Q_3 < 0$.

Our analysis has been based on the assumption that $H(z, A)$ is independent of x_F. If and when that x_F dependence can be determined independently, the effect can easily be incorporated in our analysis to modify our numerical result. Since that dependence is not likely to be strong [6–8], the modification would be minor. Our study shows that the J/ψ suppression observed at large x_F in pA collisions [5] strongly suggests the presence of gluon depletion in the beam proton at high x_1. The significance of this finding goes beyond the J/ψ suppression problem itself, since it would revise the conventional thinking concerning the role of partons in nuclear collisions.

Since the gluon distribution is enhanced for $x_1 \lesssim 0.28$, the J/ψ suppression observed in the $x_F \approx 0$ region in the heavy-ion collisions at CERN-SPS cannot be due to the
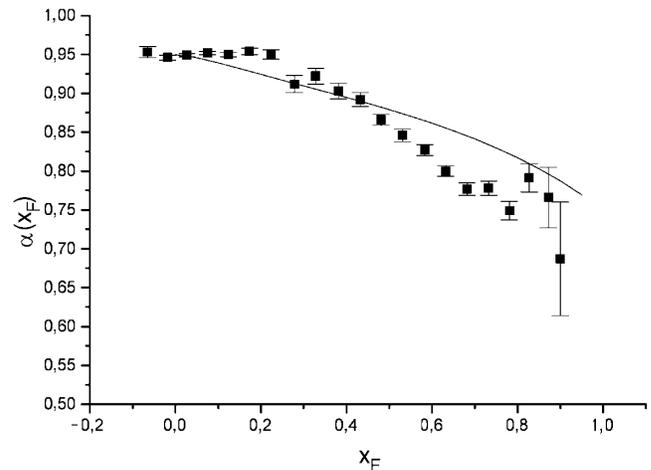
gluon depletion effect. The same would be true at RHIC. However, we expect a significant increase in suppression at large x_F due to gluon depletion, not to color deconfinement. We further speculate at this point that the gluon enhancement at low x may be responsible, at least in part, for the strangeness and dilepton enhancement already observed in heavy-ion collisions.

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