Squeezing and Entanglement of Atomic Beams

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We propose and analyze a scheme for generating entangled atomic beams out of a Bose-Einstein condensate using spin-exchanging collisions. In particular, we show how to create both atomic squeezed states and entangled states of pairs of atoms.

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The recent experimental achievement of Bose-Einstein condensates [1] has raised a lot of interest since it may lead to important applications [2–4]. Some of these applications are based on the analogies between an atomic condensate and a single mode optical field. For example, atoms in the condensate can be outcoupled to produce a coherent atomic beam [5] similar to a laser beam. In this Letter we build on these analogies to show how to produce entanglement between atoms in different internal states similar to the one created for photons with different polarizations by parametric down-conversion [6,7]. In particular, we analyze a physical situation which gives rise to the following: (i) a beam of atoms in a broadband two-mode squeezed state (a continuous variable entangled state) with respect to two internal levels; (ii) a pair of outgoing atoms in an effective maximally entangled state in a (2×2)-dimensional Hilbert space. The physical mechanism responsible for this process is spin-exchanging collisions, where two atoms of the condensate interact to create two correlated atoms in two different internal states.

We consider a Bose-Einstein condensate confined in an optical trap and in some internal level $|0\rangle$. Two atoms in the condensate can collide to create a pair of atoms in two other internal levels

$$2|0\rangle \rightarrow |+1\rangle + |-1\rangle . \quad (1)$$

The situation we have in mind is illustrated in Fig. 1. Levels $|0, \pm 1\rangle$ could correspond to the hyperfine Zeeman levels $|F = 1, M = 0, \pm 1\rangle$ of an alkali atom. In that case, the selection rules would prevent other collisional processes to occur. We assume that the process (1) can be switched on and off by changing some external parameter. For example, the condensate level could be shifted in a time-dependent way by an external field so that energy conservation effectively allows or inhibits the process in Eq. (1). This level shift could be accomplished, for example, by an off-resonant microwave or laser field with an appropriate polarization. We also assume a one-dimensional situation where the trapping potential along the transverse direction is sufficiently tight so that the atomic motion is frozen along the y, z directions. We will choose different trapping potentials along the x direction in order to illustrate our ideas of how to create squeezed atomic states and entangled pairs. In both cases, the potential will be identical for the atomic levels $|\pm 1\rangle$ and such that the atoms in those levels can escape from the trap, in order to facilitate measurements with them without being affected by the atoms in the condensate level $|0\rangle$. For instance, this could be obtained if the optical trap is made by a strong laser along the x direction crossed by a weak laser along y. In this configuration, if the energy shift between the $|0\rangle$ and $|\pm 1\rangle$ levels is larger than the trap depth induced by the weak laser but still much smaller than that by the strong laser, the $|M_F = \pm 1\rangle$ atoms will be free to move along x but will be bound in the other two directions.

This situation is described by the following second quantized Hamiltonian ($\hbar = 1$)

$$H = \sum_{i=\pm 1} \int_{-\infty}^{\infty} \hat{\phi}_i^\dagger(x) \left[-\frac{\partial_{xx}^2}{2m} + V(x) \right] \hat{\phi}_i(x)\, dx + \int_{-\infty}^{\infty} g(x,t) [\hat{\phi}_{+1}^\dagger(x) \hat{\phi}_{-1}^\dagger(x) e^{-i2\mu t} + \text{H.c.}]\, dx , \quad (2)$$

where $\hat{\phi}_{\pm 1}$ are quantized fields describing the atoms in the internal levels $|\pm 1\rangle$ satisfying the commutation relation $[\hat{\phi}_i(x,t), \hat{\phi}_j^\dagger(x',t)] = \delta_{ij}\delta(x - x')$. In Eq. (2), the first term describes the kinetic and potential energy of the atoms in the internal levels $|\pm 1\rangle$. The second term describes the creation (or annihilation) of two atoms in those

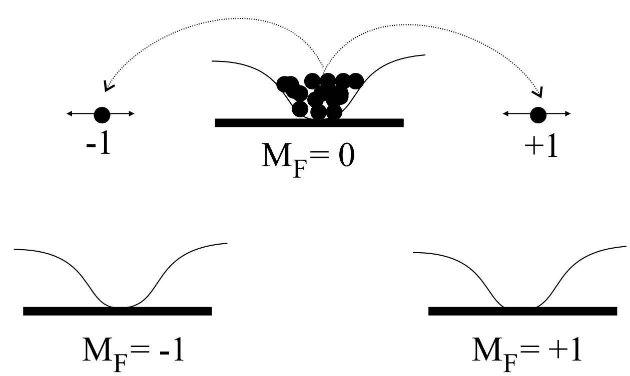

FIG. 1. Atomic configuration: The condensate is in the hyperfine Zeeman level $|F = 1, M = 0\rangle$, which is shifted with respect to the $|F = 1, M = \pm 1\rangle$ states.

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levels due to a collision between two atoms in the condensate. The condensate is described by a macroscopic wave function and μ is the corresponding chemical potential. The function $g(x,t)$ is proportional to the s-wave scattering length and the condensate density. We have included the time dependence explicitly to account for the depletion of the condensate, as well as to take into account the change of the external parameters which allow one to control the process (1). On the other hand, in writing Eq. (2) we have ignored the collisions among the atoms in levels $|\pm 1\rangle$ as well as the quantum fluctuations of the condensate, which is valid if the number of atoms in the condensate is large and in the other levels is small [8]. There are other collisional terms, like $\hat{\phi}_i^\dagger \hat{\phi}_i \phi_0^* \phi_0$ ($i = \pm 1$), which are allowed by the collisional selection rules and are of the same order in the number of condensate atoms as the second term in the Hamiltonian. However, they can be included as an effective potential for the atomic beam fields. In the following, we understand $V(x)$ as a renormalized potential which includes such collisional terms.

The quadratic Hamiltonian (2) gives rise to the following linear Heisenberg equations:

$$i\partial_t \hat{\phi}_{\pm 1}(x,t) = -\frac{\partial_{xx}^2}{2m}\,\hat{\phi}_{\pm 1}(x,t) + V(x)\hat{\phi}_{\pm 1}(x,t) + g(x,t)\hat{\phi}_{\mp 1}^\dagger(x,t)e^{-i2\mu t}. \tag{3}$$

In the Heisenberg picture, the initial state of the atoms $|\Psi\rangle$ in levels $|\pm 1\rangle$ is the vacuum, since we assume that initially (at time $t \to -\infty$) all the atoms are in the condensate, that is, $\hat{\phi}_{1,2}(x,-\infty)\,|\Psi\rangle = 0$. Thus, the coupled equations (3) describe the generation of atoms in levels $|\pm 1\rangle$ out of the vacuum. Given a potential $V(x)$ and the function $g(x,t)$ one can solve them numerically in the same way as one solves the Bogoliubov–de Gennes equations for the time-dependent excitations of a condensate [8]. Thus, the solutions can be written in terms of time-dependent Bogoliubov transformations, which implies that there will be correlations between the atoms created in levels $|\pm 1\rangle$. This is precisely the physical origin of the entanglement that will be described in the following.

We will now study two limiting situations: (i) *The squeezing limit,* in which the typical time for the atoms to leave the trap is much larger than the one related to the collisional process, so that many atoms accumulate in the levels $|\pm 1\rangle$ before they escape from the trap. (ii) *The qubit entanglement limit,* where the atoms leave the trap before a new pair of atoms is created. The first situation is analogous to the squeezed light generation by nondegenerate parametric down-conversion [6], whereas the second case is similar to the one in which pairs of polarization entangled photons are created [7].

Let us first study the situation of squeezed atomic beams. We consider a simple model for which we can obtain the properties of the atomic correlations analytically. The conclusions drawn from this model are qualitatively valid for more complicated situations, in which one has to rely on numerical calculations. We take a potential $V(x) = 0$ for $x > 0$ and infinite otherwise, so that $\hat{\phi}_{\pm 1}(0,t) = 0$, and assume that the condensate wave function is such that $g(x,t) = g_0(t)$ for $0 \le x \le a$ and zero otherwise. The function $g_0(t)$ is switched on and off in a time of the order γ^{-1}. This time can be related, for example, to the typical depletion time of the condensate. We will consider separately the two spatial regions, (I) $x \ge a$ and (II) $0 < x < a$, and then connect them via the requirement that the field operators and the first derivatives have to be continuous.

For $x \ge a$ (I) we can write the solutions of Eq. (3) as

$$\hat{\phi}_{\pm 1}(x,t) = \int_0^\infty [\hat{A}_{\pm 1}(\omega)e^{-ik(\omega)x} + \hat{B}_{\pm 1}(\omega)e^{ik(\omega)x}] \times e^{-i\omega t}\,d\omega\,, \tag{4}$$

where $k(\omega) = \sqrt{2m\omega/\hbar}$. Scattering theory assigns the operators $\hat{A}_{\pm 1}(\omega)$ and $\hat{B}_{\pm 1}(\omega)$ a definite physical meaning. They are annihilation operators of particles in incoming and outgoing plane wave modes with velocity $\mp\sqrt{2\hbar\omega/m}$, respectively. The condition $\hat{\phi}_{\pm 1}(x,-\infty)\,|\Psi\rangle = 0$ is then translated into $\hat{A}_{\pm 1}(\omega)\,|\Psi\rangle = 0$. The physical interpretation is that initially there are no input atomic beams, so the input modes should be in the vacuum state. The output modes $\hat{B}_{\pm 1}(\omega)$, determined by the inputs $\hat{A}_{\pm 1}(\omega)$ and the dynamics in the condensate region, are directly related to measurable quantities. The state of the output components can be detected by velocity-selective light imaging [9]. In order to determine the state of the output modes for vacuum inputs, we need to solve the dynamical equation (3) in the condensate region $0 \le x \le a$.

For this purpose, it is convenient to take a Fourier transformation of Eq. (3), obtaining a coupled set of equations for $\hat{\Phi}_{\pm 1}(x,\Delta)$ defined through

$$\hat{\phi}_{\pm 1}(x,t) = e^{-i\mu t}\int_{-\infty}^\infty \hat{\Phi}_{\pm 1}(x,\Delta)e^{\mp i\Delta t}\,d\Delta\,. \tag{5}$$

Because of the fact that $g_0(t)$ is time dependent, the Fourier components $\hat{\Phi}_{+1}(x,\Delta)$ are correlated with $\hat{\Phi}_{-1}(x,\Delta - \omega)$ for a range of frequencies ω. For applications, however, it is desirable to have pure entanglement between *two* measurable modes, which in our case are the output Fourier components. In fact, that can be obtained in the limit $\gamma \ll g_0$, where g_0 is the maximum value reached by $g_0(t)$. As shown below, the bandwidth of $\hat{\Phi}_{\pm 1}(x,\Delta - \omega)$ is roughly determined by the coupling rate g_0, which is much larger than the width of the Fourier transform of $g_0(t)$. This means that the Fourier transform of $g_0(t)$ can be replaced by $g_0\delta(\omega)$. We call this approximation the steady output condition, since it corresponds to the physical condition that the atomic loss in the condensate is negligible before we get a steady output. Imposing the boundary conditions allows us to express the outgoing operators in terms of the ingoing ones as a Bogoliubov transformation

$$\hat{B}_{\pm 1}(\mu \pm \Delta) = \alpha_{\pm 1}(\Delta)\hat{A}_{\pm 1}(\mu \pm \Delta) + \beta_{\pm 1}(\Delta) \times \hat{A}^{\dagger}_{\mp 1}(\mu \mp \Delta), \quad (6)$$

where the coefficients α, β can be determined by solving the corresponding scattering equations. Note that to keep the commutation relations these coefficients satisfy the general requirements $|\alpha_{\pm 1}(\Delta)|^2 - |\beta_{\pm 1}(\Delta)|^2 = 1$ and $\alpha_{+1}(\Delta)\beta_{-1}(\Delta) - \alpha_{-1}(\Delta)\beta_{+1}(\Delta) = 0$. From Eq. (6), we see that the outgoing modes $\hat{B}_{\pm 1}(\mu \pm \Delta)$ are in a pure two-mode squeezed state [6], with the squeezing parameter r_Δ given by

$$\tanh(r_\Delta) = \frac{|\beta_{+1}(\Delta)|}{|\alpha_{+1}(\Delta)|} = \frac{|\beta_{-1}(\Delta)|}{|\alpha_{-1}(\Delta)|}. \quad (7)$$

The dependence of the squeezing r_Δ on the detuning Δ determines the squeezing spectrum. Note that a pure two-mode squeezed state is an ideal continuous variable entangled state, with the entanglement characterized by the squeezing parameter [10]. Continuous variable entangled states have many applications in recent quantum information protocols [11].

To simplify the expression for the squeezing parameter r_Δ, we assume $\mu \gg g_0$. This can be achieved in practice since μ is adjustable by changing the shift of level $|0\rangle$. In this case, the squeezing parameter can be written in the following simple form:

$$r_\Delta = |\text{arctanh}\{\tanh(2\theta)\sin[(k_+ - k_-)a]\}|, \quad (8)$$

where $\theta = \text{arctanh}[[(\Delta/g_0)^2 + 1]^{1/2} - \Delta/g_0]$, and $k_\pm = [2m\mu/\hbar \pm (2mg_0/\hbar)[(\Delta/g_0)^2 + 1]^{1/2}]^{1/2}$. The solution (8) reveals some interesting properties of this interaction. First, let us consider a vanishing detuning $\Delta = 0$, that is, we look at the squeezing r_0 between the output modes $\hat{B}_{+1}(\mu)$ and $\hat{B}_{-1}(\mu)$. The parameter r_0 is given by $r_0 = |\text{arctanh}[\sin(g_0\bar{t})]|$, where $\bar{t} = 2a/\sqrt{2\hbar\mu/m}$ is the transmission time of the input atomic beam with velocity $\sqrt{2\hbar\mu/m}$ in the region $0 \le x \le a$. If the dimensionless interaction coefficient $\kappa = g_0\bar{t} = \pi/2$, we have infinite squeezing and infinite output atomic flux. This simply means that the approximation of negligible atomic loss for the condensate has broken down at this point. So, similar to the nondegenerate parametric down-conversion in the optical case [6], our system has a working threshold given by $\kappa = \pi/2 + n\pi$. The system should operate not very close to the threshold to get steady output of entangled atomic beams.

Next, let us look at the squeezing spectrum. The squeezing r_Δ versus the dimensionless detuning Δ/g_0 and the interaction coefficient $\kappa = g_0\bar{t}$ is shown in Fig. 2. From the figure, we see that we have a broadband two-mode squeezed state with the squeezing bandwidth roughly determined by g_0. The steady output condition requires $g_0 \gg \gamma$. In our case, the atomic loss is mainly caused by the output coupling. From Eqs. (4) and (8), the loss rate can be estimated as $\gamma \sim 2g_0 \sinh^2|r_0|/N_0$, where N_0 is the total atom number in the condensate. Even with a high peak squeezing r_0, the steady output condition can still be easily attained. It is also interesting to note that one can control the transmission time $\bar{t}$ by changing the level shift to get a large peak squeezing r_0, and we know that the steady output condition does not put a stringent requirement on the obtainable value of r_0. Thus, in this system in principle one can get a much larger squeezing and therefore a much larger entanglement than in the optical system. As an example, a conservative estimate gives $g_0 \sim 20$ kHz, $a \sim 3\ \mu$m, $\bar{v} = \sqrt{2\hbar\mu/m} \sim 9$ cm/s, corresponding to an output flux about 680 atoms/ms, and we have a very large squeezing $r_0 \sim 2$, which is not yet obtainable in current optical systems. The advantage of large obtainable squeezing in this system is due to the fact that we have a strong nonlinear interaction caused by the collisions with the condensate.

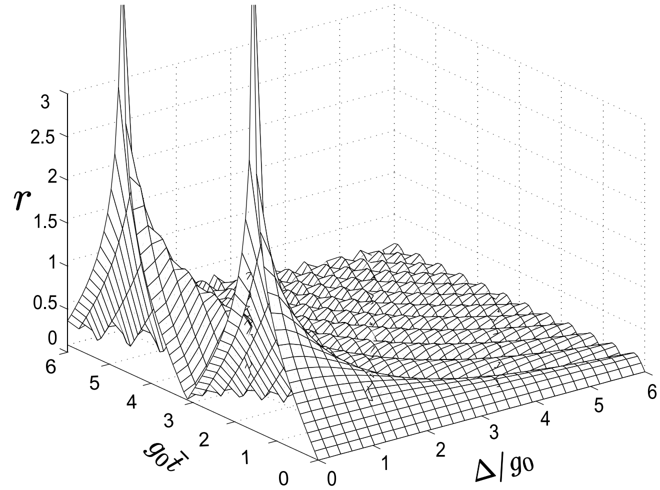

FIG. 2. Squeezing parameter r versus dimensionless detuning Δ/g_0 and interaction coefficient $\kappa = g_0\bar{t}$.

We emphasize that despite the simple model for the potential and condensate shape, we expect that all these features will be present for more realistic models. In fact, if the potential is asymptotically flat we can write Eq. (4) in that region, so that under the steady output condition we will obtain Eq. (6) with different coefficients α and β. For demonstration of these features, we suggest a three-step experiment. First, one can demonstrate the existence of a threshold by controlling the velocity of the output beams. For a certain velocity of the beams, the spin relaxation rate of the condensate will increase dramatically, and that gives the threshold value. Second, one can measure the squeezing spectrum by the velocity selective light imaging [9]. Finally, one can demonstrate the entanglement between the atomic beams by atomic homodyne detection [10]. This can be achieved by an atomic beam splitter and a local oscillator provided by an atom laser, which can be outcoupled from the same condensate.

Now, we will study the situation in which pairs of atoms are created sequentially. We will show that, as in the case of photons [7,12], if we postselect the measurement results, the corresponding internal state of the atoms is effectively maximally entangled. For this purpose, we assume the condensate is located at the region $-a \leq x \leq a$, and the potential $V(x)$ is symmetric and independent of the internal states so that the $|\pm 1\rangle$ atoms have the same probability of going to the regions $x < -a$ and $x > a$. Two atomic detectors are placed, one on the left ($x < -a$) and one on the right ($x > a$) of the condensate. The coupling $g(x,t)$ in the Hamiltonian (2) is assumed to be sufficiently small such that there are at most one pair of atoms generated in each detection interval. Using perturbation theory (in the Schrödinger picture) we can obtain from the Hamiltonian (2) the effective atomic state for each detection interval:

$$|\Psi(t)\rangle = \int f(x,y,t)\hat{\phi}_{+1}^{\dagger}(x)\hat{\phi}_{-1}^{\dagger}(y)\,dx\,dy\,|\text{vac}\rangle, \quad (9)$$

where $f(x,y,t)$ can be easily calculated, and we have neglected the vacuum component since it has no influence on the measurement results. After a time t_0, the atomic pair leaks out of the condensate, and the wave function $f(x,y) \equiv f(x,y,t_0) \approx 0$ for $-a \leq x,y \leq a$. We can decompose the wave function in the form $f(x,y) = f_{LR}(x,y) + f_{RL}(x,y) + f_{LL}(x,y) + f_{RR}(x,y)$, where $f_{LR}(x,y)$ is defined to be equal to $f(x,y)$ if $x < -a$ and $y > a$, and to be zero elsewhere. Other components are defined in a similar way. So the state $|\Psi(t_0)\rangle$ is decomposed into four components, with definite physical meaning for each component. For instance, the component $f_{LL}(x,y)$ represents both of the atoms on the left side. Now we project the state onto the subspace where there is one atom at each side. This projection can be easily achieved in experiments by postselections of the measurement results, similar to many optical experiments involving spontaneous parametric down-conversion [7,12]. After the projection, we have only two components in the effective state $|\Psi_{\text{eff}}\rangle$ (the state selected by the measurement). The potential is independent of the internal states of the atoms, so we have $f_{LR}(x,y) = f_{RL}(y,x)$. With this condition, the effective state has the form (not normalized)

$$|\Psi_{\text{eff}}\rangle = \int f_{LR}(x,y)[\hat{\phi}_{+1}^{\dagger}(x)\hat{\phi}_{-1}^{\dagger}(y) + \hat{\phi}_{-1}^{\dagger}(x)\hat{\phi}_{+1}^{\dagger}(y)]\,dx\,dy\,|\text{vac}\rangle, \quad (10)$$

which can also be written as $|+1,-1\rangle_{LR} + |-1,+1\rangle_{LR}$ with a different notation. Thus with postselection of the measurement results, we get an effective maximally entangled state between the atomic pair, and this state should have many applications in the field of quantum information, such as measurement of Bell inequalities with massive particles [13].

In summary, we have analyzed a scheme for generating both entangled atomic beams and entangled atomic pairs. We have shown that we can get pure continuous entangled state with a large entanglement and effective maximal qubit entanglement with postselection. The generated pure atomic entanglement can be directly used in the demonstration of many interesting quantum information protocols [14].

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Note added.—Upon completion of this work, we have become aware of the very recent preprint by Pu and Meystre [15], which discusses similar ideas on creating squeezed atomic states.

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