# Self-Duality in Superconductor-Insulator Quantum Phase Transitions

Adriaan M. J. Schakel
*Low Temperature Laboratory, Helsinki University of Technology, P.O. Box 2200, FIN-02015 HUT, Finland*
*and Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, 14195 Berlin, Germany*
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It is argued that close to a Coulomb interacting quantum critical point the interaction between two vortices in a disordered superconducting thin film separated by a distance $r$ changes from logarithmic in the mean-field region to $1/r$ in the region dominated by quantum critical fluctuations. This gives support to the charge-vortex duality picture of the observed reflection symmetry in the current-voltage characteristics on both sides of the transition.

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One of the most intriguing results found in experiments on quantum phase transitions in superconducting films, 2-dimensional Josephson-junction arrays [1], quantum Hall systems [2], and 2-dimensional electron systems [3] is the striking similarity in the current-voltage ($I$-$V$) characteristics on both sides of the transition. By interchanging the $I$ and $V$ axes in one phase, an $I$-$V$ characteristic of that phase at a given value of the applied magnetic field (in superconducting films, 2-dimensional Josephson-junction arrays, and quantum Hall systems) or charge-carrier density (in 2-dimensional electron systems) can be mapped onto an $I$-$V$ characteristic of the other phase at a different value of the magnetic field or charge-carrier density. This reflection symmetry hints at a deep connection between the conduction mechanisms in the two phases that can be understood by invoking a duality transformation [4,5]. Whereas the conducting phase is most succinctly described in terms of charge carriers of the system, the insulating phase is best formulated in terms of vortices, which behave as quantum point particles in these systems. The duality transformation links the two surprisingly similar looking descriptions.

There appears to be, however, one disturbing difference. Whereas charges interact via the usual 3-dimensional $1/r$ Coulomb potential, vortices are believed to interact via a logarithmic potential—at least for distances smaller than the transverse magnetic penetration depth $\lambda_\perp$, which is typically larger than the sample size [6]. This is disturbing because the difference should spoil the experimentally observed reflection symmetry.

It is this fundamental problem we wish to address in this Letter. It will be shown that, close to a Coulomb interacting quantum critical point (CQCP), the interaction between vortices in disordered superconducting films changes from logarithmic in the mean-field region to $1/r$ in the region dominated by quantum critical fluctuations. This conclusion is an exact result, depending only on the presence of a CQCP.

A common characteristic of the systems mentioned is, apart from impurities, the presence of charge carriers confined to move in a 2-dimensional plane. As the $1/r$ Coulomb repulsion between charges is genuine 3-dimensional, we assume this interaction not to be affected by what happens in the film, which constitutes a mere slice of 3-dimensional space. In contrast to this, the interaction between vortices is susceptible to the presence of a CQCP. This is because the vortex interaction is a result of currents around the vortex cores which are confined to the plane.

As a starting point, we take the observation (for a review, see Ref. [7]) that close to a CQCP the electric field $E$ scales with the correlation length $\xi$ as $E \sim \xi_t^{-1}\xi^{-1} \sim \xi^{-(z+1)}$. Here, $\xi_t$ denotes the correlation time, indicating the time period over which the system fluctuates coherently, and $z$ is the dynamic exponent. Thus conductivity measurements [3,8] close to a CQCP collapse onto a single curve when plotted as a function of the dimensionless combination $\delta^{\nu(z+1)}/E$, where $\delta = (K - K_c)/K_c$ measures the distance from the critical point $K_c$, and $\nu$ is the correlation length exponent, $\xi \sim \delta^{-\nu}$. (For a field-controlled transition, $K$ stands for the applied magnetic field, while for a density-controlled transition it stands for the charge-carrier density.) The scaling of the electric field with the correlation length expresses the more fundamental result that the anomalous scaling dimension $d_{\mathbf{A}}$ of the magnetic vector potential $\mathbf{A}$ is unity, $d_{\mathbf{A}} = 1$.

In addition, because the magnetic vector potential always appears in the gauge-invariant combination $\nabla - q\mathbf{A}$, the anomalous scaling dimension of the electric charge $q$ of the charge carriers times the vector potential is unity too, $d_{q\mathbf{A}} = 1$. Writing the anomalous scaling dimension of the vector potential as a sum $d_{\mathbf{A}} = d_{\mathbf{A}}^0 + \frac{1}{2}\eta_{\mathbf{A}}$ of its canonical scaling dimension $d_{\mathbf{A}}^0 = \frac{1}{2}(d + z - 2)$, obtained by simple power counting, and (half) the critical exponent $\eta_{\mathbf{A}}$, describing how the correlation function decays at the critical point, we conclude that $d_q = d_q^0 - \frac{1}{2}\eta_{\mathbf{A}}$. Here, $d_q^0 = 1 - d_{\mathbf{A}}^0$ stands for the canonical scaling dimension of the electric charge. Now, for a $1/r$ Coulomb potential, the charge scales as $q^2 \sim \xi^{1-z}$ independent of the number $d$ of space dimensions [9]. Combined with the previous result, this fixes the value of the critical point decay exponent $\eta_{\mathbf{A}}$ in

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terms of the number of space dimensions and the dynamic exponent:

$$\eta_{\mathbf{A}} = 5 - d - 2z. \tag{1}$$

In Ref. [9] it was further argued that in the presence of disorder the electric charge is finite at a CQCP, so that $z = 1$. This prediction was first confirmed for disordered superconducting films [10], and subsequently also for 2-dimensional Josephson-junction arrays [1], quantum Hall systems [11], and 2-dimensional electron systems [3]. With $z = 1$, the value of the critical-point decay exponent becomes $\eta_{\mathbf{A}} = 1$ in $d = 2$. As we will now demonstrate, this leads to a qualitative change in the interaction potential between two vortices from logarithmic in the mean-field region, where $\eta_{\mathbf{A}} = 0$, to $1/r$ in the vicinity of the CQCP, where $\eta_{\mathbf{A}} = 1$.

To set the stage, let us first consider a bulk superconductor with two static vortices directed along the $x_3$ axis and separated a distance $r$. For our purposes, the effective phase-only [12] Hamiltonian $\mathcal{H}_{\text{eff}} = (\rho_s/2m^2) \times (\nabla\varphi - q\mathbf{A})^2$ in terms of the phase $\varphi$ of the superconducting order parameter—the so-called Anderson-Bogoliubov mode—suffices (for reviews, see Ref. [13]). Here, $\rho_s$ is the superconducting mass density, which scales as $\rho_s \sim \xi^{2-(d+z)}$ [14], and $m$ is the mass of the charge carriers. The interaction potential can be extracted from the magnetic part of the effective action $S_{\text{mag}}$. Written as a functional integral over the the magnetic vector potential, it is given in the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ by

$$e^{iS_{\text{mag}}} = \int \mathrm{D}\mathbf{A}\, e^{i\int dt\, d^3x\, [-(1/2)(\nabla\times\mathbf{A} - \mathbf{B}^{\mathrm{P}})^2 - (1/2)\lambda^{-2}\mathbf{A}^2]}, \tag{2a}$$

with $\lambda$ the magnetic penetration depth, which is related to $\rho_s$ via $\lambda^{-2} = q^2\rho_s/m^2$. The mass term is generated through the Anderson-Higgs mechanism by integrating out the phase mode $\varphi$. The so-called plastic field $\mathbf{B}^{\mathrm{P}}$ [15],

$$B_i^{\mathrm{P}} = -\Phi_0 \sum_\alpha \int_{C_\alpha} dx_i^\alpha\, \delta(\mathbf{x} - \mathbf{x}^\alpha), \tag{2b}$$

with $\Phi_0 = 2\pi/q$ the magnetic flux quantum in units where the speed of light and Planck's constant $\hbar$ is set to unity, describes the two vortices located along the lines $C_\alpha$ ($\alpha = 1, 2$).

Note that since the anomalous scaling dimension of the magnetic vector potential is unity, the dimension of the Maxwell term is 4, implying that in $d = 2$ it is an irrelevant operator in the renormalization-group sense. This term is, however, important when considering the interaction between vortices.

To facilitate the calculation in the case of a superconducting film below, we linearize the first term in Eq. (2a) by introducing an auxiliary field $\tilde{\mathbf{h}}$ via a Hubbard-Stratonovich transformation to obtain the combination $i(\nabla \times \mathbf{A} - \mathbf{B}^{\mathrm{P}}) \cdot \tilde{\mathbf{h}} - \frac{1}{2}\tilde{\mathbf{h}}^2$. After integrating out the magnetic vector potential, we arrive at a form appropriate for a dual description in terms of magnetic vortices rather than electric charges [16]:

$$e^{iS_{\text{mag}}} = \int \mathrm{D}\tilde{\mathbf{h}}\, e^{i\int dt\, d^3x\, [-(1/2)\lambda^2(\nabla\times\tilde{\mathbf{h}})^2 - (1/2)\tilde{\mathbf{h}}^2 - i\tilde{\mathbf{h}}\cdot\mathbf{B}^{\mathrm{P}}]}. \tag{3}$$

Physically, $\tilde{\mathbf{h}}$ represents ($i$ times) the fluctuating local induction; it satisfies the condition $\nabla \cdot \tilde{\mathbf{h}} = 0$. The vortices couple with a coupling constant $g = \Phi_0/\lambda$ independent of the electric charge to $\tilde{\mathbf{h}}$. Observe the close similarity between the original (2a) and the dual form (3). This becomes even more so when an external electric current $\mathbf{j}^{\mathrm{P}}$ is coupled to the $\mathbf{A}$ field by including a term $-\mathbf{A} \cdot \mathbf{j}^{\mathrm{P}}$ in Eq. (2a), and $\mathbf{B}^{\mathrm{P}}$ describing the vortices is set to zero there.

Integrating out the local induction, one obtains the well-known Biot-Savart law for the interaction potential $S_{\text{mag}} = -\int dt\, V$ between two static vortices in a bulk superconductor [17],

$$\begin{aligned} V(r) &= \frac{1}{2\lambda^2} \int d^3x\, d^3y\, B_i^{\mathrm{P}}(\mathbf{x}) G(\mathbf{x} - \mathbf{y}) B_i^{\mathrm{P}}(\mathbf{y}) \\ &= \frac{g^2}{4\pi} \int_{C_1} \int_{C_2} d\mathbf{l}^1 \cdot d\mathbf{l}^2\, \frac{e^{-R/\lambda}}{R} \\ &= -\frac{g^2}{2\pi} L[\ln(r/2\lambda) + \gamma] + \mathcal{O}(r/\lambda)^2, \end{aligned} \tag{4}$$

where we ignored the self-interaction. In Eq. (4), $G(\mathbf{x})$ is the correlation function whose Fourier transform reads $G(\mathbf{k}) = 1/(\mathbf{k}^2 + \lambda^{-2})$, $R$ denotes the distance between the differential lengths $d\mathbf{l}^1$ and $d\mathbf{l}^2$, $L$ is the length of each of the two vortices, and $\gamma$ is Euler's constant. For distances smaller than the magnetic penetration depth, which is the length scale for variations in the current and the magnetic field, the interaction is logarithmic as in a superfluid. If the system size is smaller than $\lambda$, it will replace $\lambda$ as infrared cutoff in the logarithm, and there will be no reference to the electric charge anymore.

To describe magnetic vortices in a film of thickness $w$ [18], the bulk result (3) has to be adjusted in two ways to account for the fact that both the vortices and the screening currents, which produce the second term in (3), are confined to the plane. This is achieved by including a Dirac delta function $w\delta(x_3)$ in the second and third terms. Instead of Eq. (3), one then arrives at the interaction potential [17,18]

$$\begin{aligned} V_\perp(r) &= \frac{1}{2\lambda_\perp} \int d^2x_\perp\, d^2y_\perp\, B_\perp^{\mathrm{P}}(\mathbf{x}_\perp) \\ &\quad \times G_\perp(\mathbf{x}_\perp - \mathbf{y}_\perp) B_\perp^{\mathrm{P}}(\mathbf{y}_\perp) \\ &= -\frac{g_\perp^2}{2\pi} [\ln(r/4\lambda_\perp) + \gamma] + \mathcal{O}(r/\lambda_\perp)^2, \end{aligned} \tag{5a}$$

where $B_\perp^{\mathrm{P}} = -\Phi_0 \sum_\alpha \delta(\mathbf{x}_\perp - \mathbf{x}_\perp^\alpha)$ describes the vortices in the film with coordinates $\mathbf{x}_\perp$, $\lambda_\perp = \lambda^2/w$ is the transverse magnetic penetration depth, $g_\perp^2 = \Phi_0^2/\lambda_\perp$ is the coupling constant squared, and

$$G_\perp(\mathbf{x}_\perp) = \int dx_3\, G_\perp(\mathbf{x}_\perp, x_3) = \int \frac{d^2k_\perp}{(2\pi)^2}\, e^{-i\mathbf{k}_\perp\cdot\mathbf{x}_\perp} G_\perp(\mathbf{k}_\perp, 0), \tag{5b}$$

with $G_\perp(\mathbf{k}_\perp, 0) = 2/k_\perp(2k_\perp + \lambda_\perp^{-1})$. For small distances, the interaction is seen to be identical to that in a bulk superconductor [18], and also to that in a superfluid film. As in the bulk, the vortex coupling constant $g_\perp$ in the film is independent of the electric charge, $g_\perp^2 = \Phi_0^2/\lambda_\perp = (2\pi)^2\rho_s w/m^2$, with $\rho_s$ the bulk superconducting mass density.

The above results are valid in the mean-field region, where $\eta_\mathbf{A} = 0$. In the critical region governed by a CQCP, the value of this exponent is unity, and the correlation function becomes

$$G_\perp(\mathbf{k}_\perp, 0) = \frac{2}{k_\perp} \frac{Z_\mathbf{A}}{2k_\perp + \lambda_\perp^{-1}}, \tag{6}$$

with $Z_\mathbf{A} \sim k_\perp^{\eta_\mathbf{A}}$ the field renormalization factor. Because the magnetic vector potential and the local induction renormalize in the same way, their renormalization factor is identical. Because of this extra factor, the interaction between two vortices in the film takes the form of a $1/r$ Coulomb potential

$$V_\perp(r) = \frac{g_\perp^2}{2\pi} \frac{a}{r}, \tag{7}$$

where $a$ is some microscopic length scale which accompanies the renormalization factor $Z_\mathbf{A}$ for dimensional reasons [19].

Since the electric charge is finite at the CQCP, the penetration depth $\lambda_\perp \propto 1/\rho_s$ scales with the correlation length as $\lambda_\perp \sim \xi$. In the correlation function (6) we thus have the combination $1/(2k_\perp + \xi^{-1})$ which should be compared with $1/(\mathbf{k}^2 + \xi^{-2})$ for a bulk superconductor.

The absence of any reference to the electric charge in the renormalized and bare interaction (at least for small enough systems) implies that the same results should be derivable from our starting Hamiltonian restricted to two dimensions and with $q$ set to zero: $\mathcal{H}_\perp = (\rho_s w/2m^2)(\nabla_\perp\varphi - \boldsymbol{\varphi}_\perp^\mathrm{P})^2$. The plastic field $\boldsymbol{\varphi}_\perp^\mathrm{P}$, with $\nabla_\perp \times \boldsymbol{\varphi}_\perp^\mathrm{P} = -2\pi\sum_\alpha \delta(\mathbf{x}_\perp - \mathbf{x}_\perp^\alpha)$ describes vortices in a superfluid [15]. It is obtained from the description involving the plastic field $B_\perp^\mathrm{P}$ by a canonical transformation of the vector potential. By directly integrating out the Anderson-Bogoliubov mode, and ignoring the $k_\perp$ dependence of $\rho_s$, which is valid outside of the critical region, one easily reproduces the bare interaction potential (5a). The renormalized interaction (7) is obtained by realizing that the anomalous scaling dimension of the superconducting mass density is $d_{\rho_s} = (d + z) - 2$ [14], so that in our case $\rho_s \sim k_\perp$. In other words, the extra factor of $k_\perp$ that came in via the renormalization factor $Z_\mathbf{A}$ in our first calculation to produce the $1/r$ potential, comes in via $\rho_s$ here [20].

A similar change in the $r$ dependence of the interaction between two vortices upon entering a critical region has been observed numerically in the 3-dimensional Ginzburg-Landau model [21]. Near the charged fixed point of that theory, $\eta_\mathbf{a} = 1$ [22], as in our case.

This is a very pleasing coincidence as the (2 + 1)-dimensional Ginzburg-Landau model constitutes the dual formulation of the system. To appreciate the basic elements of the dual theory, note that the *dynamics* of the charged degrees of freedom is described by the effective Lagrangian

$$\mathcal{L}_{\perp,\mathrm{eff}} = \frac{\rho_s w}{2m^2}\left[\frac{1}{c^2}(\partial_t\varphi + \varphi_t^\mathrm{P})^2 - (\nabla_\perp\varphi - \boldsymbol{\varphi}_\perp^\mathrm{P})^2\right], \tag{8}$$

with $c$ the speed of sound. In accord with the above findings, we have ignored the coupling to the magnetic vector potential, so that Eq. (8) essentially describes a superfluid. Although the complete effective theory is Galilei invariant [23,24], the linearized form (8) is invariant under Lorentz transformations, with $c$ replacing the speed of light.

In the dual formulation, where the roles of charges and vortices are interchanged, the Anderson-Bogoliubov mode mediating the interaction between two vortices is represented as a photon associated with a fictitious gauge field $a_\mu$, i.e. (in relativistic notation), $\partial_\mu\varphi \sim \epsilon_{\mu\nu\lambda}\partial^\nu a^\lambda$. In 2 + 1 dimensions, a photon has only one transverse direction and thus only one degree of freedom—as has the Anderson-Bogoliubov mode. The elementary excitations of the dual theory are the vortices, described by a complex scalar field $\psi$. Specifically, the (well-known) dual theory of Eq. (8) is the Ginzburg-Landau model [5,15,16,25]

$$\mathcal{L}_\mathrm{dual} = -\tfrac{1}{4}f_{\mu\nu}^2 + |(\partial_\mu - iga_\mu)\psi|^2 - m_\psi^2|\psi|^2 - \tfrac{1}{4}u|\psi|^4, \tag{9}$$

with $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, $m_\psi$ a mass parameter, and $u$ the strength of the self-coupling. Both the gauge part as well as the matter part of the dual theory are of a relativistic form. The gauge part is because the effective theory (8) is Lorentz invariant, while the matter part is because vortices of positive and negative circulation can annihilate, and can also be created. In this sense they behave as relativistic particles. As was pointed out in Ref. [5], the speed of "light" in the gauge and matter part in general differ.

The interaction potential (5a) between two external vortices is now being interpreted as the 2-dimensional Coulomb potential between charges. The observation concerning the critical behavior of the Ginzburg-Landau model implies that the qualitative change in $V(r)$ upon entering the critical region is properly represented in the dual formulation.

Whereas in the conducting phase the charges are condensed, in the insulating phase the vortices are condensed [4]. In the dual theory, the vortex condensate is represented by a nonzero expectation value of the $\psi$ field, which in turn leads via the Anderson-Higgs mechanism to a mass term for the gauge field $a_\mu$. Because $(\epsilon_{\mu\nu\lambda}\partial^\nu a^\lambda)^2 \sim (\partial_\mu \varphi)^2$, the mass term $a_\mu^2$ with two derivatives less implies that the Anderson-Bogoliubov mode has acquired an energy gap. That is to say, the phase where the vortices are condensed is indeed an insulator. Since electric charges are seen by the dual theory as flux quanta, they are expelled from the system as long as the dual theory is in the Meissner state. Above the critical field $h = \nabla_\perp \times \mathbf{a} = h_{c_1}$ they start penetrating the system and form an Abrikosov lattice. In the original formulation, this corresponds to a Wigner crystal of the charges. Finally, when more charges are added and the dual field reaches the critical value $h_{c_2}$, the lattice melts and the charges condense in the superfluid phase described by the effective theory (8).

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