Spin Hall Effect in the Presence of Spin Diffusion

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Hirsch [Phys. Rev. Lett. **83**, 1834 (1999)] recently proposed a spin Hall effect based on the anomalous scattering mechanism in the absence of spin-flip scattering. Since the anomalous scattering causes both anomalous currents and a finite spin-diffusion length, we derive the spin Hall effect in the presence of spin diffusion from a semiclassical Boltzmann equation. When the formulation is applied to certain metals and semiconductors, the magnitude of the spin Hall voltage due to the spin accumulation is found to be much larger than that of magnetic multilayers. An experiment is proposed to measure this spin Hall effect.

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Spin accumulation has been known in magnetic multilayers when a current flows in the direction perpendicular to the plane of the layers (CPP) [1–4]. One of the immediate consequences of the spin accumulation is that there is an extra resistance when the magnetization of the neighboring magnetic layers is aligned antiparallel compared to that with parallel alignment. This extra resistance can be measured by utilizing the giant magnetoresistance effect in CPP geometry [5]. For a magnetic and a nonmagnetic bilayer structure, Johnson and Silsbee [2] demonstrated that such spin accumulations can extend to long length scales of the order of a few micrometers. Recently, Hirsch [6] proposed another spin accumulation effect due to spin-orbit coupling or anomalous scattering mechanisms. When a spin-unpolarized current flows in a metal, the spin-orbit interaction produces asymmetric scattering of the conduction electrons so that electrons with one particular spin direction, e.g., spin-up electrons, have a larger probability to be scattered to the right compared to spin-down electrons. Similarly, spin-down electrons would tend to scatter to the left more than spin-up electrons. Thus a spin current will be generated in a direction transverse to the direction of the current flow. Furthermore, if a spin-polarized current is present in a semiconductor, Bulgakov *et al.* [7] have shown that the Hall-like effect can be induced by the spin-orbit coupling without an external magnetic field. This left-right asymmetric scattering is known as skew scattering and has been studied for decades [8]. Much of the experimental and theoretical work has been focused on determining the magnitude of the anomalous transverse electric field in various *ferromagnetic* metals and semiconductors. The presence of the transverse *spin current* for a *paramagnetic* metal mentioned above was first discussed by Hirsch [6]. In this paper, we derive this "spin Hall effect" from a semiclassical Boltzmann equation and extend it to the case where the spin-diffusion length is finite.

We emphasize that the finite spin-diffusion length is necessary in studying the spin Hall effect. To see this let us consider a current flow in the x direction. The spin-orbit coupling will generate a *spin-dependent* transverse current as discussed above. However, the boundary condition in the transverse-open-circuit condition requires that the current for each spin channel vanishes at the boundaries. Therefore, it is necessary for the spin current to decay when the transverse spin current approaches boundaries. The decaying of the spin current must be associated with spin relaxation, i.e., a finite spin-diffusion length is required. The second justification for us to consider the spin relaxation is that the anomalous Hall effect requires a large spin-orbit coupling, but this same spin-orbit coupling also governs the magnitude of the spin relaxation (or spin-flip scattering). Therefore, generation of the anomalous Hall effect and spin-relaxation processes must be studied on an equal footing.

We begin with a semiclassical Boltzmann equation for each spin direction in a magnetic metal

$$\mathbf{v}^{\sigma}\cdot\nabla_{\mathbf{r}}f^{\sigma}(\mathbf{v},\mathbf{r})+e\mathbf{E}_{\mathrm{ext}}\cdot\mathbf{v}\left(-\frac{\partial f^{0}}{\partial\epsilon_{F}}\right)=\int W^{\sigma\sigma}_{\mathbf{v}\mathbf{v}'}[f^{\sigma}(\mathbf{v}',\mathbf{r})-f^{\sigma}(\mathbf{v},\mathbf{r})]\frac{d^{3}\mathbf{v}'}{(2\pi)^{3}}+\int W^{-\sigma\sigma}_{\mathbf{v}\mathbf{v}'}[f^{-\sigma}(\mathbf{v}',\mathbf{r})-f^{\sigma}(\mathbf{v},\mathbf{r})]\frac{d^{3}\mathbf{v}'}{(2\pi)^{3}},\tag{1}$$

where $f^{\sigma}(\mathbf{v},\mathbf{r})$ and f^{0} are the total and the equilibrium distribution functions, respectively, $\mathbf{E}_{\mathrm{ext}}$ is the external electric field, ϵ_F is the Fermi energy, $W^{\sigma\sigma}_{\mathbf{v}\mathbf{v}'}$ and $W^{-\sigma\sigma}_{\mathbf{v}\mathbf{v}'}$ are non-spin-flip and spin-flip scattering rates. We emphasize that the spatial dependence of the distribution function is needed even if the scattering rates are spatially independent within the film, because the distribution function varies as one approaches the boundaries.

There are two ways to include the spin-orbit scattering in deriving anomalous Hall effects and the spin Hall effect. The first is to calculate the scattering rates $W^{\sigma\sigma}$ and $W^{-\sigma\sigma}$ by taking the spin-orbit scattering into account.

This leads to skew scattering terms, i.e., W's containing asymmetric terms with respect to the incoming and outgoing velocities. The distribution function is found by solving the Boltzmann equation, Eq. (1), up to second order of the spin-orbit parameter. After obtaining the distribution function, the current is calculated by evaluating the integral $\mathbf{j}^\sigma(\mathbf{r}) = \int \mathbf{v} f^\sigma(\mathbf{v},\mathbf{r})\, d^3\mathbf{v}/(2\pi)^3$. The difference between the currents derived from $f^\sigma(\mathbf{v},\mathbf{r})$ with and without spin-orbit coupling is the anomalous current. The second method is to find the distribution function by using the scattering matrices in the *absence* of the spin-orbit coupling in Eq. (1), and include the spin-orbit scattering by an anomalous term in the current, i.e.,

$$\mathbf{j}^\sigma(\mathbf{r}) = \int [\mathbf{v} + \boldsymbol{\omega}^\sigma(\mathbf{v})] f^\sigma(\mathbf{v},\mathbf{r}) \frac{d^3\mathbf{v}}{(2\pi)^3}, \qquad (2)$$

where $\boldsymbol{\omega}^\sigma(\mathbf{v})$ is the anomalous velocity which can be written in terms of the spin-orbit coupling constant α [9],

$$\boldsymbol{\omega}^\sigma(\mathbf{v}) = \frac{m\alpha}{\tau^\sigma}\, \mathbf{v} \times \boldsymbol{\sigma}, \qquad (3)$$

where τ^σ is the electron relaxation time (defined below). Both approaches have been used to discuss the extraordinary Hall effect and they were shown to be completely equivalent up to second order in the spin-orbital coupling constant [10]. Here, we adopt the second approach; i.e., we first find the distribution function without spin orbital coupling and then obtain the current and conductivity from Eq. (2).

A number of approximations have to be made in order to solve the Boltzmann equation. First we use the momentum-independent relaxation time approximation, i.e., W's in Eq. (1) are constants. Then the scattering terms are

$$\int \frac{d\mathbf{v}'}{(2\pi)^3} W^{\sigma\sigma}_{\mathbf{v}\mathbf{v}'} [f^\sigma(\mathbf{v}',\mathbf{r}) - f^\sigma(\mathbf{v},\mathbf{r})] + \int \frac{d\mathbf{v}'}{(2\pi)^3} W^{-\sigma\sigma}_{\mathbf{v}\mathbf{v}'} [f^{-\sigma}(\mathbf{v}',\mathbf{r}) - f^\sigma(\mathbf{v},\mathbf{r})] = -\frac{f^\sigma(\mathbf{v},\mathbf{r}) - \bar{f}^\sigma(\mathbf{r})}{\tau^\sigma} - \frac{\bar{f}^\sigma(\mathbf{r}) - \bar{f}^{-\sigma}(\mathbf{r})}{\tau^{\uparrow\downarrow}}, \qquad (4)$$

where $\bar{f}^\sigma(\mathbf{r}) = \int d\Omega_\mathbf{v} f^\sigma(\mathbf{v},\mathbf{r}) / \int d\Omega_\mathbf{v}$, and the non-spin-flip and spin-flip relaxation times are $1/\tau^\sigma (1/\tau^{\uparrow\downarrow}) = \int d^3\mathbf{v}'/(2\pi)^3 W^{\sigma\sigma}(W^{\uparrow\downarrow})$. Next we write the distribution function as the sum of the equilibrium and nonequilibrium ones,

$$f^\sigma(\mathbf{v},\mathbf{r}) = f^0(\mathbf{v}) + \left(-\frac{\partial f^0}{\partial \epsilon_F}\right)[g^\sigma(\mathbf{v},\mathbf{r}) + e\mu^\sigma(\mathbf{r})], \qquad (5)$$

where we have introduced two nonequilibrium terms $e\mu^\sigma(\mathbf{r})$ and $g^\sigma(\mathbf{v},\mathbf{r})$; the former is related to the angular velocity averaged distribution function, i.e., $\bar{f}^\sigma(\mathbf{r}) = (-\partial f^0/\partial \epsilon_F) e\mu^\sigma(\mathbf{r})$ so that $\int g^\sigma(\mathbf{v},\mathbf{r})\, d\mathbf{v} = 0$. By substituting Eqs. (4) and (5) into Eq. (1), one obtains

$$\mathbf{v} \cdot \nabla g^\sigma(\mathbf{v},\mathbf{r}) + e\mathbf{E}^\sigma(\mathbf{r}) \cdot \mathbf{v} = -\frac{g^\sigma(\mathbf{v},\mathbf{r})}{\tau^\sigma} + \frac{e\mu^\sigma(\mathbf{r}) - e\mu^{-\sigma}(\mathbf{r})}{\tau^{\uparrow\downarrow}}, \qquad (6)$$

where $\mathbf{E}^\sigma(\mathbf{r}) = \mathbf{E}_{\text{ext}} - \nabla\mu^\sigma(\mathbf{r})$. As seen from the above equation, the separation of the nonequilibrium distribution function into $e\mu^\sigma(\mathbf{r})$ and $g^\sigma(\mathbf{v},\mathbf{r})$ results in a redefinition of a spin-dependent electric field, such that one can interpret $\mu^\sigma(\mathbf{r})$ as a spin-dependent chemical potential. To explicitly derive both $g^\sigma(\mathbf{v},\mathbf{r})$ and $\mu^\sigma(\mathbf{r})$ from Eq. (6), we postulate that the spin-flip scattering rate is much smaller than non-spin-flip rates, i.e., $W^{\sigma\sigma} \gg W^{-\sigma\sigma}$, or $\tau^{\uparrow\downarrow} \gg \tau^\sigma$. This approximation is generally valid for metallic films, as shown in Refs. [1,5]. Then one can expand Eq. (6) in terms of the orders of $\tau^\sigma/\tau^{\uparrow\downarrow}$. By using $\int g^\sigma(\mathbf{v},\mathbf{r})\, d\mathbf{v} = 0$ and assuming a spherical (isotropic) Fermi surface, we find that up to $O(\tau^\sigma/\tau^{\uparrow\downarrow})$

$$g^\sigma(\mathbf{v},\mathbf{r}) = -e\tau^\sigma \mathbf{E}(\mathbf{r}) \cdot \mathbf{v} \qquad (7)$$

and

$$\nabla^2\mu^\sigma(\mathbf{r}) = \frac{\mu^\sigma(\mathbf{r}) - \mu^{-\sigma}(\mathbf{r})}{(D^\sigma)^2}, \qquad (8)$$

where $D^\sigma = v_F^\sigma \sqrt{\tau^\sigma \tau^{\uparrow\downarrow}/3}$, and v_F^σ is the Fermi velocity. If one replaces σ by $-\sigma$ in Eq. (8) and subtracts the resultant equation from Eq. (8), we can write the spin-diffusion equation in a usual form,

$$\nabla^2[\mu^\sigma(\mathbf{r}) - \mu^{-\sigma}(\mathbf{r})] = \frac{\mu^\sigma(\mathbf{r}) - \mu^{-\sigma}(\mathbf{r})}{D^2}, \qquad (9)$$

where $D^{-2} = D^{\uparrow -2} + D^{\downarrow -2}$. This spin-diffusion equation has first been used by Son *et al.* [11] and derived by Valet and Fert [1] in magnetic multilayers. Here we illustrate that this equation remains valid in the presence of the transverse electric fields and currents. By inserting Eqs. (3) and (7) into Eq. (2), we find, after some tedious simplifications,

$$\mathbf{j}^\sigma(\mathbf{r}) = C^\sigma \mathbf{E}^\sigma(\mathbf{r}) + C_h^\sigma \mathbf{E}^\sigma(\mathbf{r}) \times \boldsymbol{\sigma}, \qquad (10)$$

where $C^\sigma = e^2\tau^\sigma (k_F^\sigma)^3/6\pi^2 m$ is the Drude conductivity for spin σ and $C_h^\sigma = e^2\alpha(k_F^\sigma)^3/6\pi^2$ is the anomalous Hall conductivity.

We now apply Eqs. (9) and (10) to a thin film geometry. Let us consider a film with its length L, width w, and thickness d as shown in Fig. 1. We assume L is sufficiently long so that the current in the x direction is uniform and that the thickness d is much smaller than the spin-diffusion

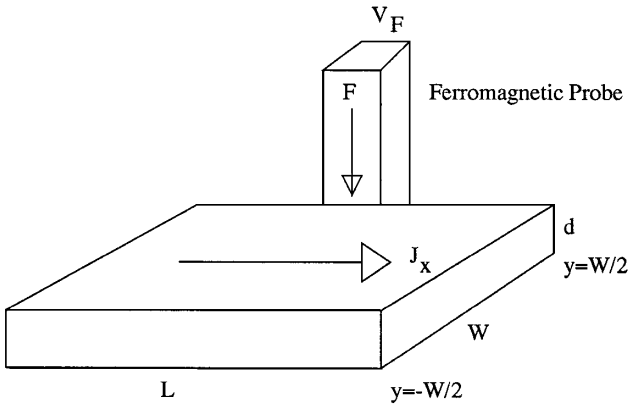

FIG. 1. The conductor of length L, width w, and thickness d carries a steady current along the x direction. Spin accumulation is detected via a ferromagnetic conductor (probe) attached in the side of the conductor. The magnetization of the ferromagnet is pointing to the direction perpendicular to the plane of the film (up or down).

length D in Eq. (9), so that one can neglect the spin current perpendicular to the film. Thus we restrict the anomalous current to the transverse direction in the film plane, and both currents and the chemical potentials depend only on y, the coordinate in the film width direction. Furthermore, we assume that the anomalous Hall conductivity is much smaller than the Drude conductivity, i.e., $C_h^\sigma \ll C^\sigma$, so that the current density in the x direction $j_x^\sigma = C^\sigma E_x$ (note that $E_x \equiv E_x^\uparrow = E_x^\downarrow$ in this case) is position independent. From Eq. (10), we find that the current density in the y direction is

$$j_y^\uparrow(y) = C^\uparrow E_y^\uparrow(y) - C_h^\uparrow E_x \tag{11}$$

and

$$j_y^\downarrow(y) = C^\downarrow E_y^\downarrow(y) + C_h^\downarrow E_x \tag{12}$$

and the spin-dependent chemical potentials are, from Eq. (9),

$$\mu^\uparrow(y) - \mu^\downarrow(y) = A\exp(y/D) + B\exp(-y/D)\,, \tag{13}$$

where A and B are integral constants to be determined by the boundary condition. In the open circuit condition, $j_y^\uparrow(y) + j_y^\downarrow(y) = 0$ for all y and $j_y^\sigma(\pm w/2) = 0$. By placing these relations into Eqs. (11)–(13) and utilizing the definition of the effective electric fields after Eq. (6), we find the spin accumulation in the transverse direction is

$$\mu^\uparrow(y) - \mu^\downarrow(y) = \frac{(C^\uparrow C_h^\downarrow + C^\downarrow C_h^\uparrow)Dj_x}{C^\uparrow C^\downarrow(C^\uparrow + C^\downarrow)} \frac{\sinh(y/D)}{\cosh(w/2D)} \tag{14}$$

and the anomalous Hall field is

$$E_y \equiv \frac{E_y^\uparrow + E_y^\downarrow}{2} = \frac{j_x}{(C^\uparrow + C^\downarrow)^2}\left[(C_h^\uparrow - C_h^\downarrow) + (C^\uparrow - C^\downarrow) \times \left(\frac{C_h^\uparrow}{C^\uparrow} + \frac{C_h^\downarrow}{C^\downarrow}\right)\frac{\cosh(y/D)}{\cosh(w/2D)}\right]. \tag{15}$$

The above two equations, (14) and (15), describe the spin and charge buildups in our thin film geometry.

We now discuss the measurement of these charge and spin accumulations. The charge accumulation, Eq. (15), has two contributions. The first term is independent of position; this is the conventional anomalous Hall effect. The second term comes from the correction due to finite spin-diffusion length. If the Hall voltage is measured near the edge of a sample, an extra Hall voltage is present. The magnitude of the additional Hall voltage is comparable to the first term (the conventional one) as long as the spin-up and spin-down conductivities are substantially different. For the transition metals, Fe, Co, Ni, and their alloys, $(C^\uparrow - C^\downarrow)/(C^\uparrow + C^\downarrow)$ ranges from 0.2 to 0.5 [12]. It would be interesting to test experimentally how the Hall field, Eq. (15), varies with the position of the Hall contact. Such experiments should yield information on the spin-diffusion length in ferromagnetic metals and on the spin polarization in these materials.

In a paramagnetic metal, however, the Hall voltage vanishes since $C^\uparrow = C^\downarrow \equiv C/2$ and $C_h^\uparrow = C_h^\downarrow \equiv C_h/2$. However, the spin accumulation, Eq. (14), survives since it is directly related to the spin-diffusion length and exists even in a paramagnetic conductor:

$$\mu^\uparrow - \mu^\downarrow = Dj_x \frac{C_h}{C^2} \frac{\sinh(y/D)}{\cosh(d/2D)}\,. \tag{16}$$

Two limiting cases are highly interesting. In the limit of a small film width, i.e., $w \ll D$ as was considered by Hirsch, Eq. (16) reduces to $\mu^\uparrow - \mu^\downarrow = j_x w C_h/2C^2$ using $y = w/2$. This result is equivalent to that obtained by Hirsch [6]. When the film width is much larger than the spin-diffusion length, the spin accumulation reaches a maximum of $j_x D C_h/2C^2$ at $y = \pm w/2$, independent of the film width. This result makes the measurement of the spin accumulation easier, since one can measure it in a macroscopic sample. In Fig. 1, we show how the measurement of this spin accumulation can be made by attaching a conducting ferromagnetic probe to the sample. By putting this ferromagnetic probe on the side boundary of the sample, the average chemical potential (weighted by the spin polarization of the ferromagnet) will match the chemical potentials of the sample [13]. Thus if the magnetization of the ferromagnet is parallel to the spin-up direction of the accumulation, the chemical potential

V_F of the ferromagnet away from the interface between the ferromagnet and the sample is $V_F = \frac{(1+P)}{2}\mu^{\uparrow}(y = w/2) + \frac{(1-P)}{2}\mu^{\downarrow}(y = w/2)$, where P is the spin polarization of the ferromagnet. Likewise if one reverses the magnetization of the ferromagnetic probe, the chemical potential of the ferromagnet becomes $V_{AF} = \frac{(1-P)}{2}\mu^{\uparrow}(y = w/2) + \frac{(1+P)}{2}\mu^{\downarrow}(y = w/2)$. The voltage difference between these two measurements is

$$\delta V \equiv V_F - V_{AF} = P[\mu^{\uparrow}(y = w/2) - \mu^{\downarrow}(y = w/2)] = P j_x D C_h / C^2. \quad (17)$$

As long as the contact dimension is not larger than the scale of the spin-diffusion length, the magnetic probe will not disturb the spin accumulation calculated above. Also, we have chosen a wirelike magnetic probe so that the magnetization of the probe is either pointing up or down: these two stable magnetization directions can be easily interchanged by applying a magnetic field. The measurement of the chemical potentials should be done after the magnetic field is removed. In this way, there will be no ordinary Hall effect involved in the probe or in the sample.

Finally, let us estimate the size of the spin accumulation. For a typical transition metal, the ratio of the anomalous Hall conductivity to the conductivity is of the order of 10^{-2}. If we take the conductivity 0.1 $(\mu\Omega\ \mathrm{cm})^{-1}$, j_x 10^7 A/cm^2, spin-diffusion length 0.1 μm, and we use Fe as the ferromagnet probe which has P about 50% [12], the voltage difference between two magnetization directions would be $\delta V = 2.5\ \mu$V. This is easily measurable via conventional transport measurement methods. We notice that this voltage is much larger than that Hirsch proposed in his paper—there he proposed to use higher order effects to measure the spin accumulation.

For a doped semiconductor, the effect would be much larger compared to that of metals estimated above. There are two reasons for the increase: the much smaller conductance and somewhat longer spin-diffusion length in semiconductors [14]. To estimate the spin accumulation, let us assume that the conductivity of the semiconductor is of the order of 1 $(\Omega\ \mathrm{cm})^{-1}$ and the current density is of the order of 10^4 A/cm^2. If we continue to use $C_h/C = 0.01$ and 1 μm spin-diffusion length, we find that the voltage difference between two magnetization directions would be 2.5 mV, 3 orders of magnitude higher compared to that of metals. Such large voltages may find application in the magnetic sensor and memory industry.

If the film is magnetic, both spin and charge accumulation will occur as seen from Eqs. (14) and (15). The charge accumulation can be measured via conventional Hall measurement so that one can determine the spin-dependent Hall coefficient $C_h^{\uparrow}$ and $C_h^{\downarrow}$. The spin accumulation is again measured via a ferromagnetic probe as shown in Fig. 1. However, one needs to control independently the magnetization directions of the sample and of the probe. This might introduce some experimental difficulties in achieving parallel and antiparallel magnetization between the probe and the sample, because they are generally coupled magnetically.

In summary, the charge and spin accumulations in the transverse direction are formulated in the presence of anomalous scattering and spin relaxation. It is found that a Hall measurement can reveal spin polarization information in the sample, both magnetic and nonmagnetic. The spin accumulation can be measured via a ferromagnet probe. The measured voltage should be much larger than that of spin injection experiments [2] where a current generated in a ferromagnet is injected into a normal metal.

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