Reversed Current Structure in a Z-Pinch Plasma

Ki-Tae Lee and Dong-Eon Kim
Department of Physics, Pohang University of Science and Technology, San 31 Hyoja-Dong, Pohang, Kyungbuk 790-784, Korea

Seong-Ho Kim
Department of Molecular Science and Technology, Ajou University, San 5 Wonchon-Dong, Paldal, Suwon 442-749, Korea
(Received 19 April 2000)

The current profile of a Z-pinch plasma is investigated using a one-dimensional magnetohydrodynamic code. Simulation results reveal the formation of a reversed current profile, its propagation, and an ejection of plasma at boundary region, which have been observed in previous experiments. A new physical mechanism is proposed to account for such phenomena. The physical mechanism involves the propagation of a shock wave. It is found that a reversed current profile appears when a shock wave reflected at axis expands in a compressing plasma column.

PACS numbers: 52.30.–q, 52.55.Ez, 52.65.–y

Z-pinch plasmas, produced by the magnetic compression of cylindrical plasma columns, have been widely used to produce hot and dense plasmas for the study of their basic characteristics and various applications as bright light sources as well as in nuclear fusion. Recently their applications have been extended to focusing energetic charged particles [1], guiding intense optical laser pulses [2], amplifying ultrashort wavelength radiations [3], and producing ultrahigh magnetic fields [4]. Since most of the applications rely on the temporal dynamics and spatial profiles of Z-pinch plasmas, the understanding of their detailed dynamics is very important for control and utilization.

The macroscopic dynamics of a Z-pinch plasma such as pinch time is easily described by the snowplow model [5], in which only the Lorentz force due to an axial current and its induced magnetic field is assumed to compress a cylindrical plasma column. The dynamics of a Z-pinch plasma is, however, more complicated. Especially, the current structure in a Z-pinch plasma has been discussed for a long time. Haines [6] proposed an inverse skin effect to explain experimentally observed reversed current profiles near edges of plasma columns. In his paper, he derived the formulation for a reversed current profile in a rigid conductor and argued that the same structure is possible even in a moving conductor such as a plasma, although he was not able to calculate the current profile due to the complexity of a plasma. In his theory, the effect of a shock wave was not taken into account. Experiments on the reversed current profile in Z-pinch plasmas have continued [7–10]. Jones *et al.* [10] used a holographic interferometry to follow the development of a reversed current layer and to pinpoint its location. The observed phenomena have been attributed to the inverse skin effect. Kumpf *et al.* [11] have recently employed Haines' theory in their one-dimensional magnetohydrodynamic (MHD) code to simulate a Z-pinch plasma lens for Antiproton collector (ACOL) at CERN [9]. Even though there has been a good agreement between the numerical simulation and experiment in terms of overall electron density and temperature, no reversed current profile was produced in the simulation. The physical origin of the reversed current profile is not clearly understood.

In this Letter, we present the numerical evidence of the reversed current profile in a moving conductor. To our knowledge, we identify, for the first time, the shock discontinuity and the direction of its propagation as the cause of the reversed current profile. One dimensional MHD simulations show that a reversed current layer appears at the position of a shock when the shock propagates outwards in a compressing plasma column after being reflected from axis near pinch time (a time of maximum compression). The reversed current layer moves with the shock. These numerical simulations propose a new physical mechanism for the reversed current profile, in which the propagation of a shock wave plays an important role.

For the numerical simulation, single-fluid, two-temperature MHD equations are evaluated for a hydrogen plasma. The simulation code, developed for the study of the dynamics of a Z-pinch plasma and its application to the x-ray laser [12,13], solves fluid equations together with a magnetic field equation derived from Maxwell's equations. Our code is different from Kumpf's [11] in that the current profile is calculated from the magnetic field equation instead of Haines' theory. A more detailed description on the code was presented in Ref. [12]. The experimental conditions of ACOL at CERN [9] were adopted for simulation conditions. Namely, hydrogen ions (proton) and electrons are assumed to initially have uniform distributions with equal densities of $N_o = 2 \times 10^{17}$ ions/cm^3 in a cylinder of radius of $R = 10$ cm without current density. The driving current is treated as an external source, $I(t) = I_o \sin(2\pi t/T)$ with $I_o = 400$ kA and $T/4 = 3.5$ μs.

Figure 1(a) shows radial current density profiles at different times near pinch time ($\sim$3.47 μs). It can clearly be seen that the dip of the reversed current profile moves

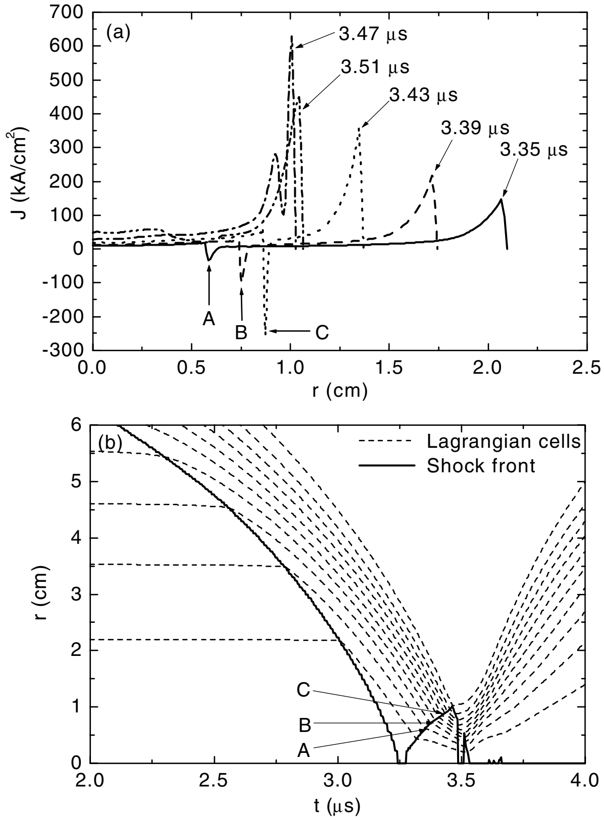

FIG. 1. (a) Radial distributions of the current density are plotted for different times near pinch time. The reversed current dips (*A*, *B*, and *C*) are seen. (b) The temporal variations of the shock position and several Lagrangian cells are plotted. The positions of the dips (A, B, and C) coincide with the positions of the shock.

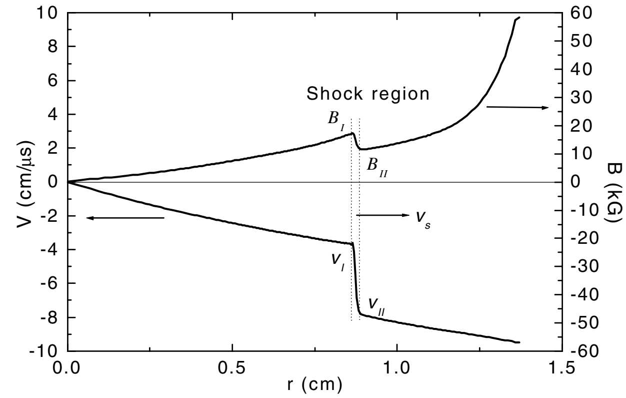

FIG. 2. Radial profiles of a plasma velocity and a magnetic field at a time of 3.43 μs are plotted. The sharp discontinuity at the shock region is marked by dotted lines.

outward in a compressing plasma column, as was observed experimentally in CERN [9]. The temporal variation of the position of a shock is plotted in Fig. 1(b) with those of several Lagrangian cells. The position of the shock wave is taken here to be the one where the shock heating term in the ion temperature equation has a maximum [12]. The shock reaches the axis at a time of 3.25 μs, is reflected, and moves outward in a compressing plasma column during the time of 3.3 to 3.5 μs. One can see that each Lagrangian cell begins to move inward when the compressing shock passes by, and slows down later when it meets the expanding shock. Examining Figs. 1(a) and 1(b), one can notice that the positions of the dip of the reversed current profile exactly coincide with those of the shock. This clearly demonstrates that the formation of a reversed current profile is related to a shock.

To understand the physical role of a shock in the formation of a reversed current profile, we examine the radial distributions of a plasma velocity and a magnetic field as plotted in Fig. 2. The sharp discontinuity can be seen both in the velocity and magnetic field profile with a finite width. The decrease in the magnetic field, which is another indication of the reversed current density, can be clearly noticed at the shock position. The magnetic field is evaluated in the MHD code by the following equation:

$$\frac{\partial B}{\partial t} + \frac{\partial}{\partial r}(vB) - \frac{\partial}{\partial r}\left[\frac{c^2}{4\pi}\frac{\eta}{r}\frac{\partial}{\partial r}(rB)\right] = 0\,, \qquad (1)$$

where B is the azimuthal magnetic field induced by an axial current, v the plasma velocity, c the speed of light, and η the plasma resistivity [14]. Since the inclusion of a finite plasma conductivity results in the diffusion or smoothing of the magnetic field inside plasma, the third term in Eq. (1) cannot contribute to the reversed current profile. For this reason, the plasma resistivity is neglected for an analytical analysis. The total time derivative which follows a shock with its velocity, v_s, is given by $(\frac{dB}{dt})_s = \frac{\partial B}{\partial t} + v_s \frac{\partial B}{\partial r}$. Then Eq. (1) is rewritten as

$$\left(\frac{dB}{dt}\right)_s - v_s \frac{\partial B}{\partial r} + \frac{\partial}{\partial r}(vB) = 0\,. \qquad (2)$$

Assuming that the shock discontinuity is abrupt in the radial distribution of the magnetic field and the plasma velocity, the integration of Eq. (2) over the shock region in the limit of $\Delta r \rightarrow 0$ yields

$$\frac{B_{\rm II}}{B_{\rm I}} - 1 = \frac{1-\beta}{\alpha - 1}\,, \qquad (3)$$

where $\alpha = v_s/v_{\rm II}$ and $\beta = v_{\rm I}/v_{\rm II}$. It has been known that there exists a magnetization and drift current in the shock region if there is a difference in the magnetic field across the shock [15]. The current which arises from the magnetization and drift due to the discontinuity of the magnetic field is expressed as

$$\Delta I = \lim_{\Delta r \rightarrow 0} 2\pi \int jr\,dr = \frac{c}{2}\, r_s B_{\rm I}\, \frac{1-\beta}{\alpha - 1}\,, \qquad (4)$$

where r_s is the position of the shock. As seen in Fig. 2, when a shock expands in a compressing plasma, $\alpha < 0$ and $\beta < 1$. Hence ΔI becomes negative implying that there can exist a reversed current sheet. In addition to this, it also indicates that when a shock propagates along a plasma ($\alpha > 1$), the positive accumulation of current can take place.

The positive accumulation of current can be seen in Fig. 3 which displays the current profiles when the shock propagates toward axis in the compressing phase. In this case, however, due to the significant diffusion effect of the magnetic field, the resulting peak current density is much smaller than expected from Eq. (4). For the case of the reversed current density, the amounts of the change in current across the shock at different times are compared between the analytical analysis (shock model) [Eq. (4)] and the simulation in Fig. 4. The simulation results are obtained by integrating the dipped portion of the current density over the shock region with 3 numerical grid points. The theoretical results are evaluated with plasma quantities at the end points of integration in Eq. (4). The difference between the simulation and the theory may be attributed to the neglect of the diffusion effect by a finite plasma conductivity in the theoretical model [Eq. (4)] which smooths the plasma current density and reduces the reversed current. Thus the simulation results have lower negative values than the theoretical shock model which only takes into account the magnetization and drift current.

These results show that the experimental observation of the reversed current can be explained by the propagation of a shock. Kumpf *et al.* [11] performed numerical simulations for similar experimental conditions using the expression for current density by Haines in his inverse skin effect. Even though there appeared a shocklike motion of fluid, no reversed current profile was observed in the simulation.

The ejection of plasma at a boundary region and the pinch enhancement have been also observed with the reversed current profile in several experiments [7,9,10]. These phenomena have been attributed to the outward-exerting electromagnetic force due to the reversed current profile. To examine this, the radial distributions of electron density and temperature at different times near pinch are plotted in Figs. 5(a) and 5(b), respectively. The density profile shows that the position of peak electron density moves outward just before pinch as the reversed current dip does [Fig. 1(a)], while the overall density of the inner region keeps increasing. In the temperature, it is also seen that the temperature at axis increases higher after pinch (the time of maximum compression). Figure 5(c) is the radial distribution of the velocity at a time of 3.51 μs (right after pinch), clearly showing that the plasma is divided into two regions, an expanding outer region and a compressing inner region. This results in the pinch enhancement that the density and the temperature increase further at axis, even though the plasma column begins to expand in the outer region. To figure out the driving force that causes this phenomenon, we examine the relevant forces: thermal pressure force, Lorentz force, and shock viscosity force. Figure 5(d) shows these forces at a time of 3.43 μs. It can be noticed that it is the outward thermal pressure force produced by a sharp density gradient that reduces the velocity of compression and eventually reverses the direction of plasma motion in the outer region [Fig. 5(c)].

In conclusion, one-dimensional MHD simulations with a shock clearly show the formation of a reversed current profile, the ejection of boundary plasma, and the pinch enhancement, which have been experimentally observed. The existing theory could not have described a reversed current when incorporated in the MHD simulation [11]. We propose a new physical mechanism involving the shock propagation. Our study also shows that the main driving force for the ejection of plasma at a boundary is not the Lorentz force due to the reversed current, which has been considered as the driving force, but the thermal pressure force. Simulations are also carried out for other parameters: a smaller radius of ~2 mm, a shorter current pulse

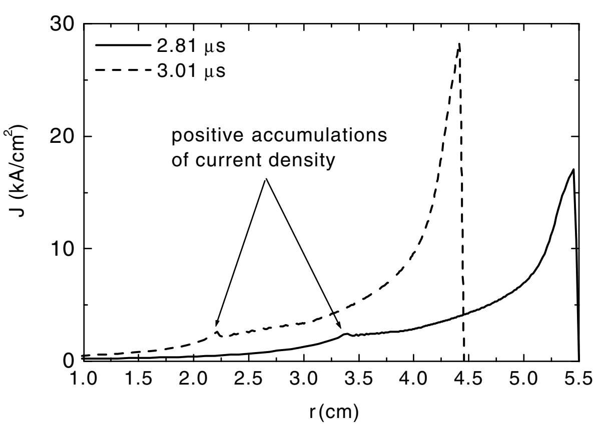

FIG. 3. Radial distributions of the current density are plotted for the times when the shock propagates inward with a plasma column. There is a positive accumulation of the current density according to Eq. (4).

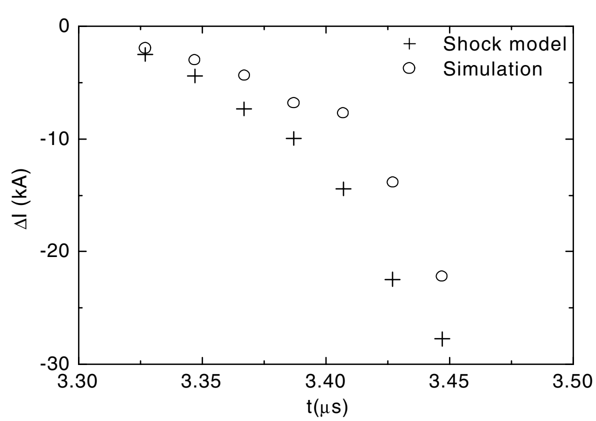

FIG. 4. Comparison of the analytical model (shock model) calculation with the simulation. Even though there is a discrepancy due to the neglect of a plasma resistivity in the analysis, the analytical model well explains the simulation result.

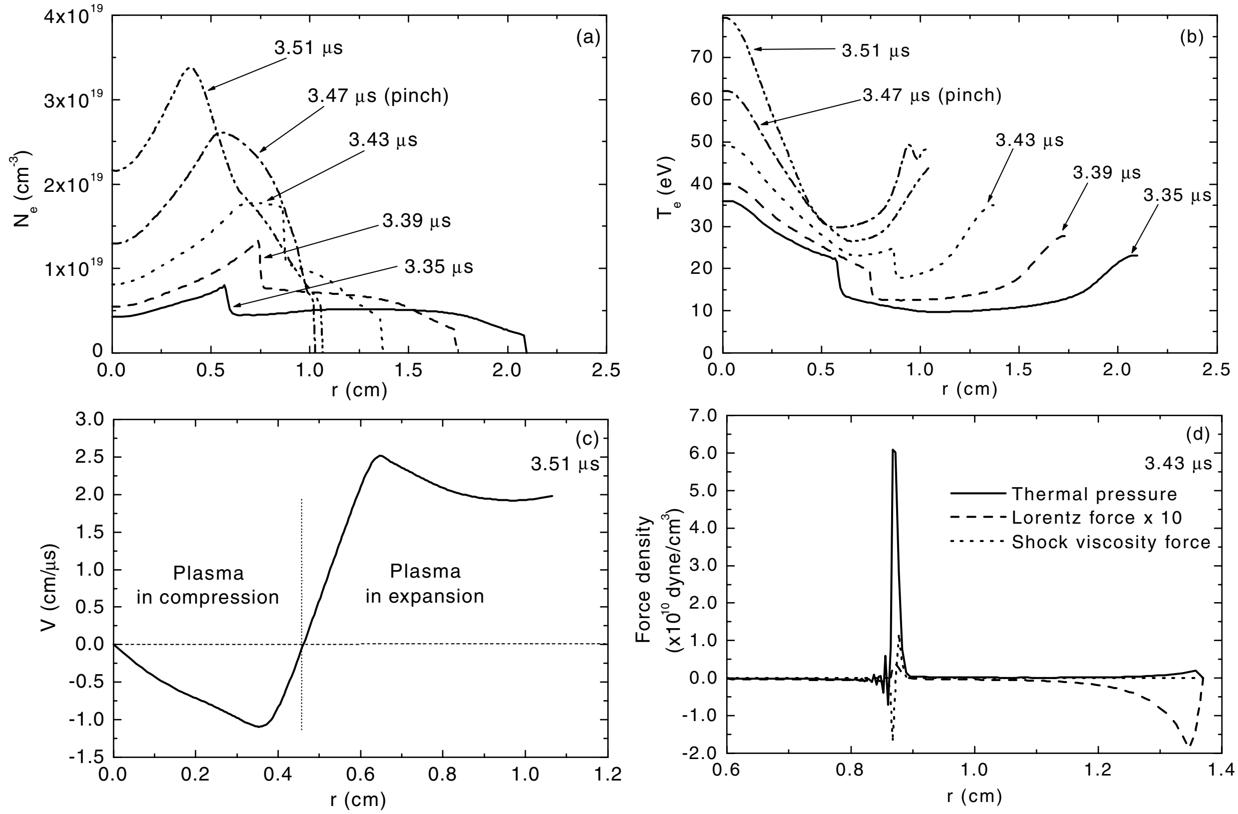

FIG. 5. Radial distributions of (a) the electron density and (b) the electron temperature at different times near pinch. The outward motion of the position of the density peak and the enhancement of the electron density and temperature can be seen just after pinch time. The radial distribution of the plasma velocity at a time of 3.51 μs is plotted in (c), which shows different directions of velocity in inner region and outer region. Various force terms are plotted at a time of 3.43 μs in (d), where the Lorentz force is multiplied by a factor of 10 for a better view.

of ~20 ns quarter period, and other species such as argon, oxygen, and carbon. In the calculation of other atomic species, an ionization balance equation with collisional-radiative rate coefficients [16] is incorporated in the MHD code. The same phenomena described in this Letter were observed for such cases as well. Thus the reversed current can be obtained in various experimental conditions only if the magnetic compression is fast enough to build a shock. In these calculations, neutral atoms are ignored by assuming initially a singly ionized plasma. The reversed current presented in this Letter appears in the phase of high temperature when no neutral atom exists. Hence the neglect of neutral atoms in the initial phase does not alter the main characteristics of the phenomena discussed in this Letter.

This work has been supported by the Brain Korea 21 project of Ministry of Education, the interdisciplinary research program (Grant No. 1999-1-111-001-5), and the Scientific Research Center project of the Korea Science and Engineering Foundation.

[1] F. Dothan *et al.,* J. Appl. Phys. **62**, 3585 (1987).
[2] Y. Ehrlich *et al.,* Phys. Rev. Lett. **77**, 4186 (1996).
[3] J. J. Rocca *et al.,* Phys. Rev. Lett. **73**, 2192 (1994).
[4] F. S. Felber *et al.,* J. Appl. Phys. **64**, 3831 (1988).
[5] N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill Book Co., New York, 1973).
[6] M. G. Haines, Proc. Phys. Soc. London **74**, 576 (1959).
[7] I. F. Kvartskhava *et al.,* Nucl. Fusion **11**, 349 (1971).
[8] I. Ya. Butov *et al.,* Sov. Phys. JETP **54**, 299 (1981).
[9] E. P. Boggasch *et al.,* IEEE Trans. Plasma Sci. **19**, 866 (1991).
[10] I. R. Jones *et al.,* Plasma Phys. **22**, 501 (1980).
[11] C. Kumpf *et al.,* Phys. Plasmas **3**, 922 (1996).
[12] K. T. Lee *et al.,* Phys. Plasmas **3**, 1340 (1996).
[13] S. H. Kim *et al.,* Phys. Plasmas **4**, 730 (1997).
[14] V. V. Vikhrev and S. I. Braginskii, *Reviews of Plasma Physics,* edited by M. A. Leontovich (Consultants Bureau, New York, 1986).
[15] G. K. Parks, *Physics of Space Plasmas* (Addison-Wesley, New York, 1992).
[16] K. T. Lee *et al.,* Phys. Rev. E **60**, 2224 (1999).