Evidence for a Possible E(5) Symmetry in ^{134}Ba

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Evidence for the first empirical example of a transitional dynamical symmetry at a critical point is discussed in the spectrum of ^{134}Ba. The role of such classes of symmetries in nuclear structural evolution is also discussed.

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It is the purpose of the present Letter to provide the first empirical evidence for the class of transitional symmetry at the critical point exemplified in the previous Letter [1]. This evidence stems from data on atomic nuclei but, since the symmetry involved is itself of rather general interest [1], its empirical confirmation also has wider implications.

Nuclear physics has benefited for over half a century from the existence of a set of structural paradigms that act as benchmarks describing idealized limits of structure. Examples are magic nuclei [2], harmonic vibrators [3], deformed symmetric rotors [4], and γ-unstable nuclei [5]. For collective nuclei, the last three symmetries have been codified under the umbrella of the algebraic structure U(6) in the framework of the interacting boson approximation (IBA) model [6] in terms of the U(5), SU(3), and O(6) limits.

The usefulness of these symmetries extends far beyond the few nuclei that actually realize them since they provide simple bases in terms of which nuclei close in structure can be described, and they serve as convenient termini [7] for regions of structural transition between them.

Nevertheless, these symmetries represent *stable* limits of structure, whereas most nuclei are in regions of structural change. There are numerical methods for calculating such regions, and Pan and Draayer [8] gave an elegant solution for the span of structures from O(6) to U(5) based on symplectic groups. Until now, though, we have not had analytic descriptions of structure at the phase transition point itself, where structure changes most rapidly: In particular, no description exists to date that assigns quantum numbers to specific levels or families of levels, or selection rules relating to them, for critical-point nuclei. Yet such a description could advance nuclear structure enormously by providing a tractable and essentially parameter-free description of the most difficult to treat of all nuclear systems.

The preceding Letter describes the first such symmetry worked out, namely the E(5) symmetry. It is an example of a class of symmetry applicable to a variety of systems (e.g., nuclear, molecular) undergoing second order phase transitions. In nuclei, it is applicable at the critical point of a transition from a vibrator to a γ soft nucleus. Another class of symmetry, currently being worked out [9], is applicable to a first order phase transition in two variables. In the nuclear case, this would describe a vibrator to rotor transition region in β and γ. These symmetries are based not on group theoretical descriptions that are solved algebraically but on particular potentials amenable to analytic descriptions in terms of the zeros of special functions. The present symmetry, E(5), corresponds to a flat-bottomed potential, such as can arise in a transition region where spherical and deformed minima cross (see Fig. 2 of Ref. [1]).

In the present Letter we discuss the empirical situation with regard to the E(5) symmetry, citing, in particular, the evidence from ^{134}Ba. This evidence, showing that ^{134}Ba closely follows the predictions of the E(5) symmetry, provides more than just another (in this case, analytic) fit: it demonstrates a new category of structure and establishes the existence of a new paradigm that can now be sought, tested, and exploited elsewhere. A second purpose is to demonstrate that the predictions of this symmetry can be conveniently replicated with the IBA model, to use this model to obtain E(5) predictions for finite numbers of valence nucleons (Ref. [1] gives explicit results only for the infinite-N limit), and to show the effects of perturbations to the exact predictions at the phase transitional point. This will allow us to more accurately position ^{134}Ba structurally and to develop simple signatures of passage through this kind of phase transition region.

Figure 1 of Ref. [1] shows a number of detailed predictions of the E(5) symmetry, including energies, E2 branching ratios, and absolute transition rates. Except for an overall energy scale factor and an effective charge for the E2 rates, these predictions are parameter-free. They should describe nuclei at the phase transition point of a U(5)-O(6) transition region [1]. Of course, given that nuclear properties change discretely with nucleon number, there is a very small chance that any given nucleus will land precisely at the critical point just like it is unlikely that any given nucleus will precisely satisfy any of the familiar dynamical symmetries of the IBA. Nevertheless, we will show that ^{134}Ba is close to this point and that it exhibits a number of features that emerge automatically from the E(5) symmetry.

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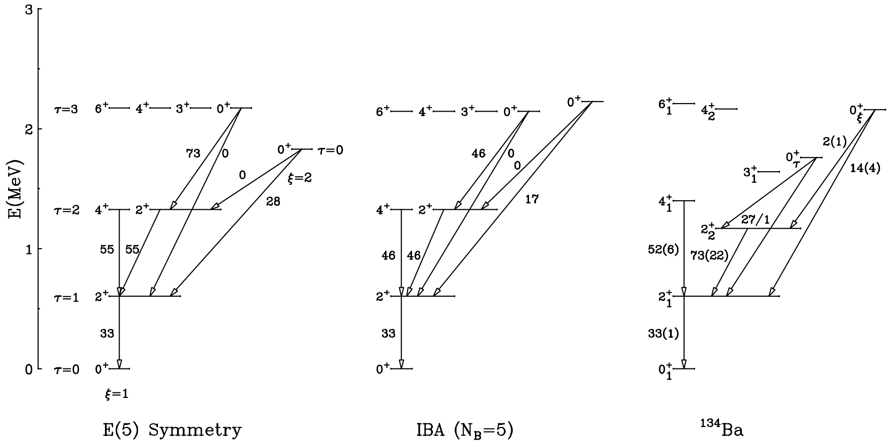

FIG. 1. Level schemes of the E(5) symmetry (left–infinite-N limit; middle–for $N = 5$) and the empirical scheme [10] for ^{134}Ba (right). Here, and in Fig. 2, we use the more descriptive shorthand labels 2_1^+ for $2_{1,1}^+$, 4_1^+ for $4_{1,2}^+$, 0_τ^+ for $0_{1,3}^+$, and 0_ξ^+ for $0_{2,0}^+$. B(E2) values in W.u. Note that only a branching ratio is known for the 0_τ^+ level.

Figure 1 (left) is an adaptation of Fig. 1 of Ref. [1] to the present case, showing the relevant subset of levels of E(5) with appropriate energy and B(E2) scales for comparison with ^{134}Ba. As discussed in Ref. [1], levels in the E(5) symmetry can be assigned explicit quantum numbers that characterize a nucleus at the critical point. ξ labels the major families of E(5) levels ($\xi = 1, 2, 3, \ldots$) and is analogous to the σ quantum number of O(6), and τ labels the phononlike levels within a ξ family ($\tau = 0, 1, 2, \ldots$), that is, the representations of the O(5) group. We label the levels by their (ξ, τ) quantum numbers. For example, the 2_1^+, 4_1^+, and 2_2^+ states are labeled $2_{1,1}^+$, $4_{1,2}^+$, and $2_{1,2}^+$, and the "bandhead" of the first excited ξ family of levels is $0_{2,0}^+$.

Figure 1 (left) shows the E(5) symmetry for the infinite-N limit. However, $^{134}_{56}$Ba$_{78}$ has 10 valence nucleons and hence its predictions will differ from the limiting case, as noted in Ref. [1]. To present the predictions for a finite nucleon number situation it is convenient to exploit the IBA model in which the E(5) symmetry corresponds to the phase transition point between U(5) and O(6) as defined by the coherent state formalism [11]. Exploitation of the IBA has the advantage that we can then simultaneously explore the complete U(5)-O(6) transition region and study perturbations to E(5). The middle panel of Fig. 1 shows the E(5) results obtained with the IBA for $N = 5$ using the IBA Hamiltonian [11] $H = \epsilon n_d + AP^\dagger P$. The critical point is defined, as given in Ref. [11], by $\epsilon/A = 2(N - 1)$, or $\epsilon/A = 8$ for ^{134}Ba. Of course, in finite nuclei, the sharpness of the phase transition will be muted.

The experimental results for ^{134}Ba are compared on the right in Fig. 1. The overall agreement is very good. It is worth highlighting specifically a few key observables, namely the characteristic ratio $R_{4/2} \equiv \mathrm{E}(4_{1,2}^+)/\mathrm{E}(2_{1,1}^+)$, the (normalized) energy of the first excited family of intrinsic levels, $\mathrm{E}(0_{2,0}^+)/\mathrm{E}(2_{1,1}^+)$, the ratio of the energies of the first two excited 0^+ states, $\mathrm{E}(0_{2,0}^+)/\mathrm{E}(0_{1,3}^+)$, and the B(E2) ratios $\frac{B(\mathrm{E2}:4_{1,2}^+\to 2_{1,1}^+)}{B(\mathrm{E2}:2_{1,1}^+\to 0_{1,0}^+)}$, $\frac{B(\mathrm{E2}:0_{2,0}^+\to 2_{1,1}^+)}{B(\mathrm{E2}:2_{1,1}^+\to 0_{1,0}^+)}$, $\frac{B(\mathrm{E2}:0_{2,0}^+\to 2_{1,2}^+)}{B(\mathrm{E2}:0_{2,0}^+\to 2_{1,1}^+)}$, and $\frac{B(\mathrm{E2}:0_{1,3}^+\to 2_{1,1}^+)}{B(\mathrm{E2}:0_{1,3}^+\to 2_{1,2}^+)}$. These energy and transition rate predictions are fixed by the symmetry and are completely parameter-free. The comparison with experiment [10] for these quantities is summarized in Table I.

Again, the agreement is impressive. First, the ratio $R_{4/2}$ is well reproduced and clearly points to a structure intermediate between a harmonic vibrator or U(5) structure ($R_{4/2} = 2.0$) and a γ-unstable or O(6) nucleus ($R_{4/2} = 2.5$). Of particular interest as a signature of E(5) is the absolute $B(\mathrm{E2}:0_{2,0}^+ \to 2_{1,1}^+)$ value (14 W.u.) compared to the $B(\mathrm{E2}:2_{1,1}^+ \to 0_{1,0}^+)$ value of 33 W.u. In U(5) this B(E2) value would be 1.6 times the $B(\mathrm{E2}: 2_{1,1}^+ \to 0_{1,0}^+)$ value. In O(6) it would be a forbidden $\Delta\sigma = 2$ transition (see Fig. 2). The E(5) symmetry predicts a ratio of approximately 1/2, while the experimental ratio is 0.42 ± 0.12. The E2 selection rule for the decay of the $0_{1,3}^+$ level, namely an allowed transition only to the $2_{1,2}^+$ level, is also verified. However, in this case, measurement of an absolute transition rate would be a very valuable additional test. Finally, the energy of the $0_{2,0}^+$ state [relative to $\mathrm{E}(2_{1,1}^+)$] is almost exactly as given by E(5).

Several of these results could characterize a variety of anharmonic vibrator spectra and the branching ratios from the $0_{2,0}^+$ and $0_{1,3}^+$ states reflect an O(5) structure that persists

TABLE I. Comparison of key observables in ^{134}Ba with the E(5) symmetry (calculated for $N = 5$ with the IBA).

	$R_{4/2}$	$\frac{\mathrm{E}(0_{2,0}^+)}{\mathrm{E}(2_{1,1}^+)}$	$\frac{\mathrm{E}(0_{2,0}^+)}{\mathrm{E}(0_{1,3}^+)}$	$\frac{B(\mathrm{E2}:4_{1,2}^+\to 2_{1,1}^+)}{B(\mathrm{E2}:2_{1,1}^+\to 0_{1,0}^+)}$	$\frac{B(\mathrm{E2}:0_{2,0}^+\to 2_{1,1}^+)}{B(\mathrm{E2}:2_{1,1}^+\to 0_{1,0}^+)}$	$\frac{B(\mathrm{E2}:0_{2,0}^+\to 2_{1,2}^+)}{B(\mathrm{E2}:0_{2,0}^+\to 2_{1,1}^+)}$	$\frac{B(\mathrm{E2}:0_{1,3}^+\to 2_{1,1}^+)}{B(\mathrm{E2}:0_{1,3}^+\to 2_{1,2}^+)}$
E(5)	2.19	3.68	1.04	1.38	0.51	0	0
^{134}Ba	2.31	3.57	1.23	1.56(18)	0.42(12)	0.18(8)	0.037(3)

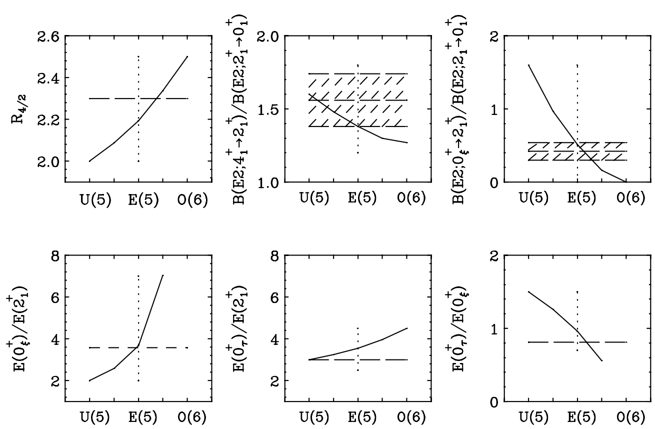

FIG. 2. Variation of key observables from U(5), through the E(5) symmetry (denoted by a vertical dotted line), to O(6) for $N = 5$. Solid curves are the predicted values. The experimental values for ^{134}Ba are indicated by horizontal dashed lines or ranges (experimental uncertainties).

throughout the transition region. Nevertheless, the confluence of all these data in one nucleus points convincingly to a near-E(5) description of ^{134}Ba.

It is interesting, though, to go a little further and see if the small discrepancies that do exist in Table I can be conveniently corrected by a small deviation from E(5). To this end, we carried out a series of calculations across the entire U(5)-O(6) transition region (for $N = 5$). The transition region is achieved by varying the ratio ϵ/A in H. The results, along with the data, are shown in Fig. 2. The data [including the ratio $B(\mathrm{E2}: 4^+_{1,2} \rightarrow 2^+_{1,1})/B(\mathrm{E2}: 2^+_{1,1} \rightarrow 0^+_{1,0})$ if the experimental uncertainties are taken into account] suggest a structure compatible with or just slightly to the right [O(6) side] of the critical point. Overall, a very consistent picture emerges providing strong support for the existence and properties of E(5).

Finally, we note that there are other candidates for an E(5) symmetry in some isotopes of Zn and Pd, but that much more data [especially B(E2) ratios and absolute transition rates] are needed before any definitive description is possible.

In summary, we have presented the first evidence for a new class of symmetry in nuclei, the E(5) symmetry discussed in the preceding Letter [1]. This type of symmetry denotes, not a stable limit of structural evolution, but the critical point of the U(5)-O(6) phase transition where structure is actually changing rapidly and where, with competing degrees of freedom, existing models are most challenged. E(5) provides a new paradigm of structure and identifies specific values for certain observables as signatures for such a structure. Moreover, the fact that the IBA has been shown to replicate E(5) expands the usefulness of this symmetry beyond the infinite-N expressions found in Ref. [1]. Also, given that few nuclei are likely to actually realize such a symmetry exactly, this use of the IBA allows one to systematically study perturbations to E(5). We have shown that ^{134}Ba is close to exemplifying E(5) symmetry and, in fact, have been able to localize ^{134}Ba at, or very slightly to the O(6) side of, the E(5) critical point. Finally, this has enabled us to suggest further experiments on ^{134}Ba to pin down the comparison with E(5).

Symmetries such as E(5) are applicable to other types of physical systems and hence the work of Ref. [1] and the current Letter may hopefully spur applications in molecular and other areas of physics as well.

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