Natural Chaotic Inflation in Supergravity

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We propose a chaotic inflation model in supergravity. In the model the Kähler potential has a Nambu-Goldstone–type shift symmetry of the inflaton chiral multiplet which ensures the flatness of the inflaton potential beyond the Planck scale. We show that chaotic inflation naturally takes place by introducing a small breaking term of the shift symmetry in the superpotential. This may open a new branch of model building for inflationary cosmology in the framework of supergravity.

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The inflationary expansion of the early universe [1] is the most attractive ingredient in modern cosmology. This is not only because it naturally solves the long-standing problems in cosmology, that is the horizon and flatness problems, but also because it accounts for the origin of density fluctuations [2] as observed by the Cosmic Background Explorer (COBE) satellite [3]. Among various types of inflation models proposed so far, the chaotic inflation model [4] is the most attractive since it can realize an inflationary expansion even in the presence of large quantum fluctuations at the Planck time. In fact, many authors have used the chaotic inflation model to discuss a number of interesting phenomena such as preheating [5], superheavy particle production [6], and primordial gravitational waves [7] in the inflationary cosmology [1].

On the other hand, supersymmetry [8] is widely discussed as the most interesting candidate for the physics beyond the standard model since it ensures the stability of the large hierarchy between the electroweak and the Planck scales against radiative corrections. This kind of stability is also very important to keep the flatness of inflaton potential at the quantum level. Therefore, it is quite natural to consider the inflation model in the framework of supergravity.

However, the above two ideas, i.e., chaotic inflation and supergravity, have not been naturally realized simultaneously. The main reason is that the minimal supergravity potential has an exponential factor, $\exp(\frac{\varphi_i^* \varphi_i}{M_G^2})$, which prevents any scalar fields φ_i from having values larger than the gravitational scale $M_G \simeq 2.4 \times 10^{18}$ GeV. However, the inflaton φ is supposed to have a value much larger than M_G at the Planck time to cause the chaotic inflation. Thus, the above effect makes it very difficult to incorporate the chaotic inflation in the framework of supergravity. In fact, all of the existing models [9,10] for chaotic inflation use rather specific Kähler potential, and one needs a fine-tuning in the Kähler potential since there is no symmetry reason for having such specific forms of these potentials. Thus, it is very important to find a natural chaotic inflation model without any fine-tuning.

In this Letter, we propose a natural chaotic inflation model where the form of Kähler potential is determined by a symmetry. With this Kähler potential the inflaton φ may have a large value $\varphi \gg M_G$ to begin the chaotic inflation. Our models, in fact, need two small parameters for successful inflation. However, we emphasize that the smallness of these parameters is justified by symmetries and hence the model is natural in 't Hooft's sense [11].

The existence of a natural chaotic inflation model may open a new branch of inflation-model building in supergravity, since most of the model building in supergravity has been concentrated on other types of inflation models (e.g., hybrid inflation model, etc. [12]). Furthermore, we consider that future astrophysical observations [13,14] will be able to select types of inflation models.

Our model is based on the Nambu-Goldstone–type shift symmetry of the inflaton chiral multiplet $\Phi(x, \theta)$. Namely, we assume that the Kähler potential $K(\Phi, \Phi^*)$ is invariant under the shift of Φ,

$$\Phi \rightarrow \Phi + iCM_G \,, \tag{1}$$

where C is a dimensionless real parameter. Thus, the Kähler potential is a function of $\Phi + \Phi^*$, $K(\Phi, \Phi^*) = K(\Phi + \Phi^*)$. It is now clear that the supergravity effect $e^{K(\Phi+\Phi^*)}$ discussed above does not prevent the imaginary part of the scalar components of Φ from having a larger value than M_G. We identify it with the inflaton field φ. We also stress that the present model overcomes the so-called η problem [12] and it is an alternative to other inflation

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models such as D-term inflation models [15] and running inflaton mass models [16]. However, as long as the shift symmetry is exact, the inflaton φ never has a potential and hence it never causes the inflation. Therefore, we have to introduce a small breaking term of the shift symmetry in the theory. The simplest choice is to introduce a small mass term for Φ in the superpotential,

$$W = m\Phi^2. \tag{2}$$

Then, we have the potential,

$$V = e^K\left\{\left(\frac{\partial^2 K}{\partial\Phi\partial\Phi^*}\right)^{-1} D_\Phi W D_{\Phi^*} W^* - 3|W|^2\right\}, \tag{3}$$

with

$$D_\Phi W = \frac{\partial W}{\partial\Phi} + \frac{\partial K(\Phi + \Phi^*)}{\partial\Phi} W. \tag{4}$$

Here, Φ denotes the scalar component of the superfield Φ and we have set M_G to be unity. We easily see that $V \to -\infty$ as $|\varphi| \to \infty$ with $\Phi + \Phi^* = 0$ and the chaotic inflation does not take place, where $\varphi = -i(\Phi - \Phi^*)/\sqrt{2}$.

In this Letter, we propose instead the following small mass term in the superpotential introducing a new chiral multiplet $X(x,\theta)$:

$$W = mX\Phi. \tag{5}$$

Notice that the present model possesses $U(1)_R$ symmetry under which

$$\begin{aligned} X(\theta) &\to e^{-2i\alpha} X(\theta e^{i\alpha}), \\ \Phi(\theta) &\to \Phi(\theta e^{i\alpha}), \end{aligned} \tag{6}$$

and Z_2 symmetry under which

$$\begin{aligned} X(\theta) &\to -X(\theta e^{i\alpha}), \\ \Phi(\theta) &\to -\Phi(\theta e^{i\alpha}). \end{aligned} \tag{7}$$

The above superpotential is not invariant under the shift symmetry of Φ. However, we should stress that the present model is completely natural in 't Hooft's sense [11], since we have an enhanced symmetry (the shift symmetry) in the limit $m \to 0$. That is, we consider that the small parameter m is originated from small breaking of the shift symmetry in a more fundamental theory. We consider that as long as $m \ll \mathcal{O}(1)$, the corrections from the breaking term Eq. (5) to the Kähler potential are negligibly small. (The Kähler potential may also have the induced breaking terms such as $K \simeq |m\Phi|^2 + \ldots$. However, these breaking terms are negligible in the present analysis as long as $|\varphi| \lesssim m^{-1}$.) Then, we assume that the Kähler potential has the shift symmetry Eq. (1) and the above $U(1)_R \times Z_2$ symmetry neglecting the breaking effects,

$$K(\Phi, \Phi^*, X, X^*) = K[(\Phi + \Phi^*)^2, XX^*]. \tag{8}$$

In the following analysis we take, for simplicity:

$$K = \frac{1}{2}(\Phi + \Phi^*)^2 + XX^* + \ldots. \tag{9}$$

Dynamics of inflation.—The Lagrangian density $L(\Phi, X)$ is now given by

$$L(\Phi, X) = \partial_\mu\Phi\partial^\mu\Phi^* + \partial_\mu X\partial^\mu X^* - V(\Phi, X), \tag{10}$$

with the potential $V(\Phi, X)$ given by

$$\begin{aligned} V(\Phi, X) = m^2 e^K[&|\Phi|^2(1 + |X|^4) + |X|^2 \\ &\times \{1 - |\Phi|^2 + (\Phi + \Phi^*)^2 \\ &\times (1 + |\Phi|^2)\}], \end{aligned} \tag{11}$$

where we have neglected higher order terms in the Kähler potential Eq. (9) whose effects will be discussed later. Here, X denotes also the scalar component of the superfield X. Now, we decompose the complex scalar field Φ into two real scalar fields as

$$\Phi = \frac{1}{\sqrt{2}}(\eta + i\varphi). \tag{12}$$

Then, the Lagrangian density $L(\eta, \varphi, X)$ is given by

$$\begin{aligned} L(\eta, \varphi, X) = &\frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi \\ &+ \partial_\mu X\partial^\mu X^* - V(\eta, \varphi, X), \end{aligned} \tag{13}$$

with the potential $V(\eta, \varphi, X)$ given by

$$\begin{aligned} V(\eta, \varphi, X) = &m^2 \exp(\eta^2 + |X|^2) \\ &\times \left[\frac{1}{2}(\eta^2 + \varphi^2)(1 + |X|^4) + |X|^2\left\{1 - \frac{1}{2}(\eta^2 + \varphi^2) + 2\eta^2\left(1 + \frac{1}{2}(\eta^2 + \varphi^2)\right)\right\}\right]. \end{aligned} \tag{14}$$

Note that η and $|X|$ should be taken as $|\eta|, |X| \lesssim \mathcal{O}(1)$ because of the presence of the e^K factor. On the other hand, φ can take a value much larger than $\mathcal{O}(1)$ since e^K does not contain φ. For $\eta, |X| \ll \mathcal{O}(1)$, we can rewrite the potential as

$$V(\eta, \varphi, X) \simeq \frac{1}{2}m^2\varphi^2(1 + \eta^2) + m^2|X|^2. \tag{15}$$

At around the Planck time, we may have a region [4] where

$$\dot{\varphi}^2 \sim (\nabla\varphi)^2 \sim V(\varphi) \sim 1 \quad \text{(initial chaotic situation)}. \tag{16}$$

Here the dot represents the time derivative. In this region the classical description of the φ field dynamics is feasible because of $|\varphi| \gg \mathcal{O}(1)$ though quantum fluctuations are $\delta\varphi \simeq \mathcal{O}(1)$. Then, as the universe expands, the potential energy dominates and the universe begins inflation.

Since the initial values of the inflaton $\varphi(0)$ are determined so that $V[\varphi(0)] \sim \frac{1}{2}m^2\varphi(0)^2 \sim 1$,

$\varphi(0) \sim m^{-1} \gg 1$. [Notice that one has only to demand $\varphi(0) \gtrsim 15.0$ in order to solve the flatness and horizon problems [1].] For such large φ the effective mass of η becomes much larger than m and hence it quickly settles down to $\eta = 0$. On the other hand, the X field has a relatively light mass m and slowly rolls down toward the origin ($X = 0$). With $\eta = 0$, the potential Eq. (15) is written as

$$V(\varphi, X) \simeq \frac{1}{2} m^2 \varphi^2 + m^2 |X|^2. \tag{17}$$

Since $\varphi \gg 1$ and $|X| < 1$, the φ field dominates the potential and the chaotic inflation takes place. The Hubble parameter is given by

$$H \simeq \frac{m\varphi}{\sqrt{3}}. \tag{18}$$

During the inflation both φ and X satisfy the slow roll condition ($|\frac{V''}{V}| \ll 1, \frac{1}{2}|\frac{V'}{V}|^2 \ll 1$, where the dash represents the derivative of φ or X) and hence the time evolutions are described by

$$3H \frac{d\varphi}{dt} \simeq -m^2 \varphi, \tag{19}$$

$$3H \frac{dX}{dt} \simeq -m^2 X. \tag{20}$$

Here and hereafter, we assume that X is real and positive making use of the freedom of the phase choice. From the above equations we obtain

$$\left(\frac{X}{X(0)}\right) \simeq \left(\frac{\varphi}{\varphi(0)}\right), \tag{21}$$

where $\varphi(0)$ and $X(0)$ are the initial values of φ and X fields. Therefore, X decreases faster than φ. At the end of inflation, i.e., $\varphi \simeq 1$ ($|\frac{V''}{V}| \sim \frac{1}{2}|\frac{V'}{V}|^2 \sim 1$), X is given by

$$X \lesssim m, \tag{22}$$

where we have used $X(0) \lesssim 1$ and $\varphi(0) \sim m^{-1}$. We see that the X field becomes much smaller than 1 ($m \sim 10^{-5}$ as shown below). The density fluctuations produced by this chaotic inflation are estimated as [17]

$$\frac{\delta\rho}{\rho} \simeq \frac{1}{5\sqrt{3}\pi} \frac{m}{2\sqrt{2}} (\varphi^2 + X^2). \tag{23}$$

Since $X \ll \varphi$, the amplitude of the density fluctuations is determined only by the φ field and the normalization at the COBE scale ($\delta\rho/\rho \simeq 2 \times 10^{-5}$ for $\varphi_{\rm COBE} \simeq 14$ [3]) gives

$$m \simeq 10^{13}\ \text{GeV}. \tag{24}$$

(The spectral index $n_s \simeq 0.96$ for $\varphi_{\rm COBE} \simeq 14$.)

After inflation ends, an inflaton field φ begins to oscillate and its successive decays cause reheating of the universe. In the present model the reheating takes place efficiently if we introduce the following superpotential:

$$W = \lambda X H \bar{H}, \tag{25}$$

where H and $\bar{H}$ are a pair of Higgs doublets whose R charges are assumed to be zero and λ is a constant [18]. Then, we have the coupling of the inflaton φ to the Higgs doublets as

$$L \sim \lambda m \varphi H \bar{H}, \tag{26}$$

which gives the reheating temperature

$$T_R \sim 10^9\ \text{GeV} \left(\frac{\lambda}{10^{-5}}\right) \left(\frac{m}{10^{13}\ \text{GeV}}\right)^{1/2}. \tag{27}$$

In order to avoid the overproduction of gravitinos, the reheating temperature T_R must be lower than 10^9 GeV [19], which requires the small coupling $\lambda \lesssim 10^{-5}$. The small coupling λ is naturally understood in 't Hooft's sense [11] provided that $H\bar{H}$ is even under the Z_2 symmetry in Eq. (7).

So far we have taken the minimal Kähler potential and neglected higher order terms such as $(\Phi + \Phi^*)^4, |X|^4$, and .... Here, we make a comment on the higher terms in the Kähler potential. Since the leading quadratic terms make the expectation values of η and X fields less than 1, the inflation dynamics is almost unchanged in the presence of the higher terms. The only relevant difference comes from the $\zeta|X|^4$ (ζ: constant) term which induces the effective mass of X given by

$$m_X^2 = 2m^2 - 2\zeta m^2 \varphi^2 \simeq -2\zeta m^2 \varphi^2. \tag{28}$$

Thus, ζ should be negative to ensure the positiveness of m_X^2. If $|\zeta| \gtrsim 1$, the effective mass becomes larger than the Hubble parameter and the X quickly settles down to $X = 0$ without slow roll.

We have shown that a chaotic inflation naturally takes place if we assume that the Kähler potential has the Nambu-Goldstone–type shift symmetry of the inflaton chiral multiplet Φ and introduce a small breaking term of the shift symmetry in the superpotential Eq. (5). Unlike other inflation models the chaotic inflation model has no initial value problem and hence it is the most attractive. However, it had been difficult to construct a natural chaotic inflation model in the framework of supergravity because the supergravity potential generally becomes very steep beyond the Planck scale. Therefore, the existence of a natural chaotic inflation model may open a new branch of inflation-model building in supergravity. Furthermore, the chaotic inflation is known to produce gravitational waves (tensor metric perturbations) [7] which might be detectable in future astrophysical observations [13,14].

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