Spin Transport in Interacting Quantum Wires and Carbon Nanotubes

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We present a general formulation of spin-dependent transport through a clean one-dimensional interacting quantum wire or carbon nanotube, connected to noncollinear ferromagnets via tunnel junctions. The low energy description of each junction is given by a conformally invariant boundary condition representing *exchange coupling*, in addition to a pair of electron tunneling operators. The effects of the exchange coupling are strongly enhanced by interactions, leading to a dramatic suppression of spin accumulation: a direct signature of spin-charge separation. Finally, backscattering induces nonequilibrium *oscillations* in the current-voltage relation.

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Recent studies on metal-ferromagnet hybrid systems have revealed new and interesting physics due to the interplay between the electronic charge and spin [1], e.g., the giant magnetoresistance effect [2]. Following the initial spin-injection proposal [3], the work of Johnson and Silsbee [4] and subsequent advances have opened the way for the field of spintronics, where the electron spin is the central element for information storage and transport [5]. Spin-dependent transport plays an important role in quantum computation proposals, and has already led to new technological applications. In this context, a detailed understanding of transport through ferromagnetic-normal-ferromagnetic devices is both of fundamental and technological interest. In such structures, the current-voltage relation is predicted to sensitively depend on the relative angle θ between the magnetization directions of the ferromagnets (FMs) [2,6,7].

Current theoretical models [6,7] are based on Fermi liquid theory, thereby ignoring the effect of interactions in the metal. As the inevitable miniaturization of spin-dependent devices proceeds, however, at least the interconnects must ultimately reach the one-dimensional (1D) quantum limit, in which Fermi liquid theory breaks down [8]. This theoretically expected change from Fermi liquid to *Luttinger liquid* (LL) behavior drastically alters transport phenomena, as has recently been verified in experiments on charge conduction in carbon nanotubes [9], which are nearly ideal 1D quantum wires (QWs) [10]. Despite these developments, spin injection into a LL has received surprisingly little attention [11]. In this paper, we present a general low-energy theory for spin transport in a LL, which directly applies to nanotubes and semiconductor QWs [12]. We assume, as expected theoretically [13] and recently observed experimentally [14] for carbon nanotubes, that spin-orbit coupling in the LL is negligible. Its inclusion is, however, straightforward.

Our analysis shows that spin transport in LLs is *qualitatively* different both from charge transport in LLs and from Fermi liquid spin transport. We focus for concrete results on the case of an end-contacted quantum wire, and assume that the distance L between contacts is sufficiently long, $\max(V,T) \gg v/L$, to ensure an incoherent stepwise transport mechanism through the tunnel barriers between each FM and the QW. (Here v is the Fermi velocity, and we put $e = k_B = \hbar = 1$.) A complete analytic solution of this problem is contained in Eqs. (2) and (12)–(14). In contrast to charge transport, we find that spin conduction occurs not only through electron transfer but also exchange. This exchange effectively gives rise to a modification of the boundary conditions at the end of the LL, e.g., for the left contact,

$$\vec{J}_R = \mathcal{R}(\Theta)\vec{J}_L + \vec{J}_{\rm tunnel}\,, \tag{1}$$

where $\vec{J}_{L/R}$ is the left/right moving spin current into/out of the contact, and $\vec{J}_{\rm tunnel}$ represents the effect of electron transfer [see Eq. (12)]. The effect of exchange coupling is given by the one-parameter $SO(3)$ matrix $\mathcal{R}(\Theta) = \exp(\Theta\Gamma)$, where $\Gamma_{\mu\nu} = \sum_\lambda \hat{m}_\lambda \epsilon_{\lambda\mu\nu}$, and $\hat{m}$ is a unit vector in the direction of magnetization of the FM. Physically, Θ represents the angle an incident spin in the LL precesses due to exchange interaction with the FM. Because of spin-charge separation in the LL, the exchange contribution is *not* suppressed by the orthogonality catastrophe affecting the tunneling current, and therefore dominates the physics in many situations. This enhancement of the exchange current does not occur in a Fermi liquid, and its observation would provide a direct experimental signature of electron fractionalization. In addition to the novel physics arising at the contact, we find that a long ballistic QW exhibits a *bulk precession* of the magnetization due to backscattering [of strength b, see Eq. (16)],

$$v\partial_x\vec{M} + \partial_t\vec{J} = b\vec{M}\times\vec{J}\,, \tag{2}$$

where $\vec{M} = \vec{J}_R + \vec{J}_L$ and $\vec{J} = \vec{J}_R - \vec{J}_L$ are the local

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magnetization and current in the QW, respectively. Equation (2) leads to *oscillations* in the nonlinear current-voltage relation. Remarkably, the latter is a purely nonequilibrium effect that arises from a marginally irrelevant backscattering interaction in the LL. The detailed character of these oscillations is also influenced by interactions.

We now turn to the derivation of these results. In the incoherent limit, we may consider each contact separately, as an initial system composed of two decoupled pieces, $H_0 = H_{\rm FM} + H_{\rm QW}$. The FM ($x < 0$), described by $H_{\rm FM}$, is polarized along direction $\hat{m}$, while the $SU(2)$ invariant QW ($x > 0$) is described by $H_{\rm QW}$. The $SU(2)$ invariance guarantees the existence of a continuity equation for spin density and current. At time $t \to -\infty$, each half is assumed at equilibrium at its own chemical potential, $\mu_{\rm FM}$ and $\mu_{\rm QW}$, and with a spin chemical potential $\vec{h}$ (see below) in the QW. We are interested in the steady state achieved at $t = 0$, long after the tunneling perturbation has been adiabatically turned on, $H(t) = H_0 + e^{\delta t}H'$ ($\delta \to 0^+$). The calculation is nontrivial primarily due to its nonequilibrium nature: the system evolves according to $H(t)$ while the initial states are distributed according to $\exp(-\beta H_0)$. Consider

$$H' = F^\dagger W \Psi + \Psi^\dagger W^\dagger F, \tag{3}$$

where F and Ψ are spin-1/2 fermion annihilation operators at $x = 0$ for the FM and the QW, respectively. Employing the projection operators $\hat{u}_s = (1 \pm \hat{m} \cdot \vec{\sigma})/2$, the 2×2 tunneling matrix W takes the form $W = \sum_s t_s \hat{u}_s$, with spin-dependent transmission coefficients t_s for the spin quantization axis parallel to $\hat{m}$. The junction is then characterized by the conductance $G = G_\uparrow + G_\downarrow$ and the polarization $P = (G_\uparrow - G_\downarrow)/G$, where $G_{\uparrow,\downarrow} = (e^2/h)|t_{\uparrow,\downarrow}|^2$ are the spin conductances [7].

From Eq. (3) and the spin continuity equation, the tunneling spin current across the junction is $\vec{J} = -i(F^\dagger W \vec{\sigma} \Psi - \Psi^\dagger \vec{\sigma} W^\dagger F)/2$. By defining $\tilde{H}_0 = H_0 + \mu_{\rm FM} N_{\rm FM} + \mu_{\rm QW} N_{\rm QW} + \vec{h} \cdot \vec{S}_{\rm QW}$, the standard perturbative result can be rewritten as

$$\begin{aligned}\langle \vec{J} \rangle = \mathrm{Re} \sum_{\alpha\beta\gamma\lambda} \int_{-\infty}^{0} dt\, e^{\delta t} (W\vec{\sigma})_{\alpha\beta} [U^\dagger(t) W^\dagger]_{\gamma\lambda} \\ \times \langle [F_\alpha^\dagger(0)\Psi_\beta(0), \Psi_\gamma^\dagger(t) F_\lambda(t)] \rangle_{\tilde{H}_0}.\end{aligned} \tag{4}$$

Thereby an intrinsically nonequilibrium expectation value is expressed in terms of an equilibrium average using the shifted Hamiltonian $\tilde{H}_0$, where the nonequilibrium nature of the problem is fully encoded in the time-dependent unitary matrix $U(t) = \exp[i(V + \vec{h} \cdot \vec{\sigma}/2)t]$, with $V = \mu_{\rm QW} - \mu_{\rm FM}$. A formula similar to Eq. (4) can easily be written down for the charge current, $I = i(F^\dagger W \Psi - \Psi^\dagger W^\dagger F)$. Thus both charge and spin current can be calculated using equilibrium correlation functions.

To proceed, we specify the Hamiltonians $H_{\rm FM}$ and $H_{\rm QW}$. For energies well below the electronic bandwidth D, the F and Ψ *equilibrium* correlators are identical for H_0 and $\tilde{H}_0$, and, moreover, a noninteracting Fermi liquid model with constant density of states (DOS) applies to the leads. Because the lead couples to the QW only at $x = 0$, the difference in DOS for majority and minority spin carriers can be absorbed in a spatial rescaling of the Fermi fields of the FM and a suitable redefinition of the transmission coefficients (t_s) [7]. Then

$$H_{\rm FM} = -i \int_{-\infty}^{0} dx\, f^\dagger \tau^z \partial_x f, \tag{5}$$

where the spinor $f = f_{b\beta}$ is indexed by $b = (R, L)$, describing right- and left-moving modes of the FM, and by $\beta = (\uparrow, \downarrow)$ for the spin, with the boundary condition $f_R(0) = f_L(0)$. Here, the Pauli matrix τ^z acts in the R/L space. Putting $F = f(0)$, we see that F has $SU(2)$ invariant correlation functions. The low-energy description of the QW is an interacting LL model,

$$H_{\rm QW} = \int_0^\infty dx \{-i\psi^\dagger v \tau^z \partial_x \psi + u(\psi^\dagger \psi)^2\}, \tag{6}$$

where $\psi_R(0) = \psi_L(0)$ and $\Psi = \psi(0)$. Only the forward-scattering interaction u is kept in Eq. (6). Alternatively, the exponent $\alpha > 0$ for tunneling into the end of the LL ($x = 0$) serves to measure the interaction strength [8].

The $SU(2)$ invariance of Eqs. (5) and (6) implies

$$\langle [F_\alpha^\dagger(0)\Psi_\beta(0), \Psi_\gamma^\dagger(t) F_\lambda(t)] \rangle_{\tilde{H}_0} \theta(-t) = \delta_{\alpha\lambda}\delta_{\beta\gamma} i C(-t),$$

where $C(t)$ is the retarded Green's function of the operator $F_\uparrow^\dagger \Psi_\uparrow$ (the choice of spin components is arbitrary). From Eq. (4) and the corresponding expression for I, it is then straightforward to obtain

$$\begin{aligned}\langle \vec{J} \rangle = -\frac{G}{2} \sum_s [(P\hat{m} + s\hat{h}) \,\mathrm{Im}\tilde{C}(V + hs/2 + i\delta) \\ - Ps\hat{m} \times \hat{h}\, \mathrm{Re}\tilde{C}(V + hs/2 + i\delta)],\end{aligned} \tag{7}$$

$$\langle I \rangle = -G \sum_s (1 + Ps\hat{m} \cdot \hat{h}) \,\mathrm{Im}\tilde{C}(V + hs/2 + i\delta), \tag{8}$$

where $\tilde{C}(V) = \int dt\, C(t) e^{iVt}$. The terms involving [8]

$$\begin{aligned} I_\alpha(V, T) &\equiv G\, \mathrm{Im}\tilde{C}(V + i\delta) \\ &= GT(T/D)^\alpha \sinh(V/2T) \\ &\quad \times \left| \Gamma\left(1 + \frac{\alpha}{2} + i\frac{V}{2\pi T}\right) \right|^2 \end{aligned} \tag{9}$$

have a simple interpretation in terms of tunneling via Fermi's golden rule, as can be seen from the spectral representation of $\tilde{C}$. However, the appearance of $\mathrm{Re}\tilde{C}$ in Eq. (7) indicates the presence of a physical process other than tunneling. It can be shown [6,13] that it corresponds to a virtual process in which an electron near the Fermi energy in the QW hops into a state of the FM (which could be far from the Fermi energy) and hops back, thereby generating an *exchange coupling*. We thus include it from the start, $H \to H_0 + H' + H''$, with

$$H'' = -K\hat{m} \cdot \Psi^\dagger \vec{\sigma} \Psi/2 . \tag{10}$$

Since the FM possesses a nonvanishing average magnetization, the spin operator in the FM may be replaced by this average to leading approximation.

It is helpful to view both H' and H'' in a renormalization group (RG) framework, as perturbations to a decoupled fixed point described by H_0. Standard arguments give the scaling dimension of both t_s and K, $\Delta_{t_s} = 1 + \alpha/2$ and $\Delta_K = 1$. The scaling dimension Δ_K is not renormalized due to spin-charge separation in the QW. A simple calculation (for $\alpha > 0$) gives the RG scaling equations

$$\partial_\ell |t_s|^2(\ell) = -\alpha |t_s|^2, \qquad \partial_\ell K(\ell) = c(|t_\uparrow|^2 - |t_\downarrow|^2),$$

where $\ell = \ln(D/E)$, and c is a nonuniversal constant. Following the RG flow from the ultraviolet cutoff D down to energy $E \approx \max(T, V) \ll D$, we find $|t_s|^2(E) = |t_s|^2(E/D)^\alpha$, and

$$K(E) = K + \alpha^{-1} cGP[1 - (E/D)^\alpha] \gg |t_s|^2(E).$$

Therefore the tunneling spin current is much smaller than the exchange contribution. Neglecting the tunneling contribution completely, one still obtains a T-independent exchange spin current as $T \to 0$.

This fact can be understood from a simple analogy to the Andreev current through a ballistic superconductor-normal-superconductor (SNS) junction [15]. Let us consider a LL connected to two insulating FMs at $x = 0$ and $x = L$, with $\hat{m} \times \hat{m}' \neq 0$. This is an equilibrium situation, which can be modeled using Eq. (6) for the LL and two copies of Eq. (10) for the contacts to the FMs. The exchange interaction operates entirely within the spin sector of the LL due to spin-charge separation. Since the charge sector is decoupled, we are free to consider it at the noninteracting point, $u = 0$. Then the resulting fictitious charge boson and the physical spin boson can be combined and refermionized into a spinful Dirac fermion η. Choosing arbitrary quantization axes $\hat{m} = \hat{x}$ and $\hat{m}' = \cos(\theta)\hat{x} + \sin(\theta)\hat{y}$, it is instructive to perform the particle-hole transformation $\eta_\downarrow \to \eta_\downarrow^\dagger$. This yields the Hamiltonian,

$$H_\eta = -iv \int_0^L dx\, \eta^\dagger \tau^z \partial_x \eta - \mathrm{Re}[K\Delta(0) + K'\Delta(L)e^{i\theta}], \tag{11}$$

where $\Delta(x) = \eta_\uparrow(x)\eta_\downarrow(x)$. Equation (11) describes a ballistic SNS junction, and, for phase twist $0 < \theta < 2\pi$ between the superconductors, supports an equilibrium current due to Andreev reflection. Since the Andreev current is $v\eta^\dagger \tau^z \eta$, the original FM-LL-FM device indeed has a nonzero spin current J_z. The analogy to a SNS junction also demonstrates that this current does not rely upon the incoherence of the two contacts.

A more general perspective on the exchange coupling can be gained by viewing the low-energy physics entirely in terms of boundary operators and boundary conditions [16]. For that purpose, we may make an arbitrary choice of short-scale physics, and let the exchange coupling act on right-movers slightly away from the junction. In this case, using the boundary condition $\psi_L(0) = \psi_R(0)$, the equations of motion for the spin currents can be integrated over the junction region to give the steady-state relation $\vec{J}_R(0^+) = \mathcal{R}(\Theta)\vec{J}_L(0^+)$ (the brackets denoting expectation values are omitted henceforth). The parameter $\Theta \approx K/v$ ultimately defines the "exchange coupling constant" of the low-energy theory. In principle, since the boundary exchange operator is exactly marginal, Θ need not be small. Then the "bulk" spin currents are $\vec{J}_L = \vec{J}_L(0^+)$ and $\vec{J}_R = \vec{J}_R(0^+) + \vec{J}_{\mathrm{tunnel}}$, and we obtain Eq. (1), with the tunneling spin current

$$\vec{J}_{\mathrm{tunnel}} = -\frac{1}{2}\sum_s (P\hat{m} + s\hat{h}) I_\alpha(V + hs/2, T). \tag{12}$$

The term proportional to $\mathrm{Re}\tilde{C}$ in Eq. (7) has been dropped, as its physical effects are included via the $SO(3)$ rotation $\mathcal{R}$. Since the magnetization far from the contact is $\vec{M} = \chi\vec{h}$ with the LL spin susceptibility χ, one can then compute the spin current $\vec{J}$ for arbitrary exchange coupling Θ. We arrive at the general result [17]

$$\vec{J} = S\chi\vec{h} + (1 - S)\vec{J}_{\mathrm{tunnel}}, \tag{13}$$

where $S = (\mathcal{R} - 1)/(\mathcal{R} + 1)$ is a real antisymmetric matrix. Similarly, the charge current is

$$I = -\sum_s (1 + sP\hat{m} \cdot \hat{h}) I_\alpha(V + hs/2, T). \tag{14}$$

From Eqs. (13) and (14), by exploiting spin and charge current conservation in order to obtain μ_{QW} and $\vec{h}$, one can then compute all transport properties in a given circuit for arbitrary parameters [13].

We now specialize to a FM-LL-FM device with identical contacts at $T = 0$ and applied voltage V within $v/L \ll V \ll D$. For algebraic simplicity, we require $P \ll 1 + \alpha$. For a tunneling contact, one expects $\Theta \ll 1$ [6], and Eq. (13) then yields

$$\vec{J} = -(\Theta\chi/2)\hat{m} \times \vec{h} + \vec{J}_{\mathrm{tunnel}}.$$

Under these conditions, it is straightforward to obtain the θ-dependent FM-LL-FM current-voltage relation,

$$I(\theta) = \frac{GV}{2}(V/D)^\alpha \left(1 - P^2 \frac{\tan^2(\theta/2)}{\tan^2(\theta/2) + Y_\alpha(V)}\right), \tag{15}$$

where $Y_\alpha(V) = 1 + (2\Theta\chi/G)^2(1 + \alpha)^{-2}(V/4\pi D)^{-2\alpha}$. For $\alpha \to 0$, this reproduces the result of Ref. [7]. Notably, unless the magnetizations of the FMs are antiparallel ($\theta = \pi$) or the exchange coupling vanishes ($\Theta = 0$), the *spin accumulation effect* [1], in which the current is reduced due to pileup of spin in the QW, is strongly *suppressed* by the voltage dependence of Y_α. Physically, this suppression is caused by the exchange coupling which is much more efficient in relaxing the injected spin polarization compared to the tunneling current.

Finally, we turn to backscattering electron-electron interactions of the form

$$H_b = -b \int_0^L dx\, \vec{J}_L \cdot \vec{J}_R \,. \quad (16)$$

For a carbon nanotube, with the lattice spacing a and the tube radius R, one may estimate $b \approx ae^2/R$ [10]. Since H_b is marginally irrelevant in a single-channel QW, it is usually neglected in the LL model (6). Nevertheless, as is shown here, *dynamical* effects caused by (16) can be important. The equations of motion away from the contacts give Eq. (2) and $v\partial_x \vec{J} + \partial_t \vec{M} = 0$. In the steady state, we have conserved spin current $\vec{J}$, and a *bulk precession equation* for $\vec{M}$. Since $\vec{M} = \chi \vec{h}$, the vector $\hat{h}$ must then precess around $\vec{J}$.

To get sizable consequences, detailed analysis [13] shows that it is essential to have small exchange couplings. For simplicity, we put $\Theta = 0$ below. For the FM-LL-FM device,

$$\hat{h}(0) \cdot \hat{h}(L) = \cos(\Delta\varphi), \qquad \Delta\varphi = bJL/v \,. \quad (17)$$

Since $\Delta\varphi \propto L$, precession will then always be significant for a sufficiently long QW. The computation of $\vec{h}(0)$ and $\vec{h}(L)$ leads to the self-consistency equation,

$$(1 - x^2)\cos^2\!\left(\frac{\pi x}{\cos(\theta/2)}\, \frac{V}{\Delta V}\, (V/D)^\alpha\right) = \sin^2(\theta/2)\,, \quad (18)$$

where $\Delta V = 8\pi v/[GPbL\cos(\theta/2)]$. Here solutions $x = x_n$ ($n \ge 0$) in the interval $0 \le x \le \cos(\theta/2)$ have to be found. The current through the device is then

$$I_n(V) = \frac{GV}{2}\,(V/D)^\alpha \{1 - P^2[1 - x_n^2(V)]\}\,. \quad (19)$$

The solution I_n corresponding to n full precession periods exists only for voltages $V > V_n \equiv D[\Delta V/D]^{1/(1+\alpha)} n^{1/(1+\alpha)}$. Using typical parameters appropriate for a 1 μm long single-wall nanotube, one finds $V_1 \approx 0.1$ to 1 V. In general, the current-voltage relation could then be multivalued, where, in the regime $V_n < V < V_{n+1}$, the solution $I_n(V)$ is expected to be realized. Hence, the current-voltage relation becomes *oscillatory*, with sawtoothlike oscillations. The observation of several periods could provide a direct and accurate measurement of α via the V_n.

In conclusion, we have presented a general formalism for spin-dependent transport through interacting 1D conductors. An experimental check of the theory should be possible by measuring the current-voltage relation for a ferromagnet-nanotube-ferromagnet device. The approach is general enough to apply to numerous other problems. Several interesting extensions currently under investigation are the description of bulk-contacted wires, inclusion of the subband degree of freedom of single-wall nanotubes, LL to LL contacts, and the ballistic-diffusive crossover potentially relevant for multiwall nanotubes.

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