Momentum Transport from Nonlinear Mode Coupling of Magnetic Fluctuations

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A cause of observed anomalous plasma momentum transport in a reversed-field pinch is determined experimentally. Magnetohydrodynamic theory predicts that nonlinear interactions involving triplets of tearing modes produce internal torques that redistribute momentum. Evidence for the nonlinear torque is acquired by detecting the correlation of momentum redistribution with the mode triplets, with the elimination of one of the modes in the triplet, and with the external driving of one of the modes.

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Plasmas often exhibit momentum transport and flow phenomena that are not explained by classical collisional processes. In this Letter, we present experimental evidence for a cause of rapid momentum transport in the reversed field pinch (RFP) toroidal configuration. It has been observed in RFP plasmas that the radial profile of the toroidal plasma rotation can change rapidly and spontaneously, representing a rapid radial transport of toroidal momentum [1,2]. The effect cannot be explained by a classical process since it is very fast (around 100 μs), about 2 orders of magnitude more rapid than what would be expected from classical viscosity. Prior measurements made in the Madison Symmetric Torus (MST) [3] RFP found global momentum transport to be anomalous [4]. Other measurements in MST indicate that the rotation of a mode and the flow of the plasma at the same radius track each other closely in time [5]. We will therefore treat the plasma in a single-fluid MHD model, so that any force produces changes in the momentum of the single fluid. In the RFP, magnetic reconnection (tearing instability) occurs at multiple radial locations. We find strong evidence that the momentum transport is a nonlinear effect that arises from the three-wave coupling between tearing modes.

Since this three-wave coupling results in an electromagnetic force, we begin with the Lorentz force density at a given location $\mathbf{x}$ in the plasma, which is $\mathbf{F}(\mathbf{x}) = \mathbf{J}(\mathbf{x}) \times \mathbf{B}(\mathbf{x})$, where $\mathbf{F}$ is the force density, $\mathbf{J}$ is the current density, and $\mathbf{B}$ is the magnetic field. The contribution from fluctuations to the mean force density is given by

$$\langle \mathbf{J} \times \mathbf{B} \rangle_{\text{fluct}} = \sum_{\mathbf{k}} \langle \mathbf{j}_{\mathbf{k}} \times \mathbf{b}_{\mathbf{k}} \rangle,$$

where $\mathbf{j}$ is the current density fluctuation, $\mathbf{b}$ is the magnetic field fluctuation, $\mathbf{k}$ are wave vectors of the fluctuations, and $\langle\rangle$ denotes an ensemble average approximating an average over a magnetic surface. Resistive magnetohydrodynamic (MHD) theory, in the limit of large electrical conductivity, predicts that this force vanishes for tearing modes except at the resonant surface where $\mathbf{k} \cdot \langle \mathbf{B} \rangle = 0$ [6]. Therefore, for a mode with given k, the fluctuation-driven mean Lorentz force density at its resonant surface is $\mathbf{F}_{\mathbf{k}} = \langle \mathbf{j}_{\mathbf{k}} \times \mathbf{b}_{\mathbf{k}} \rangle$. This force is responsible for the well-studied phenomenon of mode locking in which a tearing mode becomes locked to an external magnetic field structure [7–13]. If the magnetic structure is a static field error, then the mode becomes stationary in the lab frame. In this case $j_{\mathbf{k}}$ is the current density induced at the resonant surface by the field error ($j_{\mathbf{k}} \propto b_{fe,\mathbf{k}}$, the resonant error field), and the force density is proportional to the product of the magnetic fields from the field error and the tearing mode: $F_{\mathbf{k}} \propto b_{fe,\mathbf{k}} b_{\mathbf{k}}$ where $b_{\mathbf{k}}$ is the amplitude of the preexisting tearing mode.

In the RFP, the current density $j_{\mathbf{k}}$ can also be generated by a nonlinear interaction involving two other modes which satisfy the sum rule $\mathbf{k}' + \mathbf{k}'' = \mathbf{k}$ [14,15]. In this case, an eddy current is nonlinearly driven at the rational surface by two tearing modes, i.e., $j_{\mathbf{k}} \propto b_{\mathbf{k}'} b_{\mathbf{k}''}$, where $b_{\mathbf{k}'}$ and $b_{\mathbf{k}''}$ are tearing modes. The force density on the surface resonant with $\mathbf{k}$ is then $F_{\mathbf{k}} \propto b_{\mathbf{k}'} b_{\mathbf{k}''} b_{\mathbf{k}}$. The physics of the mode acceleration is similar to mode locking to a field error, except that the resonant current is driven nonlinearly, rather than by a field error. However, whereas a field error can impart net momentum to the plasma, the nonlinear forces are mutual interactions between resonant surfaces. The multiple torques at different resonant surfaces sum to zero, imparting no net momentum to the plasma. Rather, they redistribute the momentum and change the flow profile. Hence, they yield an anomalous viscosity arising from within MHD theory. We observe the nonlinear torques first by measurement of the appropriate fluctuation triplet and second by a series of experiments that selectively alter the amplitude of individual modes in the triplet to test the response of the plasma rotation.

The experiments were performed in the MST RFP. In MST, as in other axisymmetric toroidal magnetic confinement devices, fluctuation activity is typically described in terms of Fourier decomposition in the angle variables, i.e., $\xi \propto e^{i(m\theta - n\zeta)}$, where ξ is a generic fluctuation, m is the poloidal mode number, θ is the poloidal angle, n is the toroidal mode number, and ζ is the toroidal angle. A set of 32 equally spaced coil forms with poloidal and toroidal coils is used to measure the tearing mode fluctuation amplitudes in MST for all three experiments to be discussed. These coils reside on the inside surface of the vacuum vessel, and have a frequency response of about 100 kHz. The signals are analog integrated and the toroidal mode spectra are extracted from the integrated data through a

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discrete Fourier transform. The $(m = 1, n = 6)$ mode is the largest of the core-resonant modes $(m = 1, n = 5\text{–}9)$, and we will use its phase velocity to represent the core rotation. To quantify the edge kinematics, we will use the velocity of the (0,1) mode which is the largest of the edge-resonant $m = 0$ modes.

In MST, under normal experimental conditions, most of the global plasma parameters exhibit large changes during sawtooth oscillations [cf. Figs. 1(a) and 1(b)]. The rotation of the core plasma and the core-resonant tearing modes periodically exhibits rapid deceleration during the crash phase of sawtooth oscillations [Fig. 1(c)]. The core rotation decelerates during a sawtooth crash in about 100 μsec. The sudden change in rotation is not due to a magnetic field error. This is demonstrated by an experiment in which a radial electric field is applied (by a biased electrode inserted into the edge plasma) to spin the edge plasma in the opposite direction to the normal (unbiased) direction of the core flow (Fig. 2) [16]. Under these conditions the core plasma rotates in the same direction as the edge, although more slowly. The core rotation speeds up during the sawtooth crash, while the edge rotation slows [Fig. 3(b)]. This effect cannot be explained by a stationary field error; in both the biased and unbiased plasma, the rotation profile is flattened at the crash (Fig. 3). Note from Fig. 3(a) that the (0,1) mode's speed in unbiased events is small. This is presumably due to strong coupling to the wall and allows for a net change in the plasma momentum.

For our experiments concerning the nonlinear torque we will focus on the rotation of the (1,6) mode. The nonlinear torque on this mode is due to three-wave interactions with the (1,5) mode or the (1,7) mode and the (0,1) mode, and also with the (1,8) and (0,2) modes, etc. We perform three experiments to test the thesis that the nonlinear torque is the cause of the sudden changes in rotation during sawtooth crashes. First, we measure correlated triple products of three modes, and observe that they change rapidly, simultaneously with the sudden change in rotation. Second, we eliminate the three-wave interaction by suppressing the (0,1) mode [by adjusting the equilibrium to eliminate the (0,1) resonant surface], and find that during the sawtooth crash the rapid change in rotation disappears. Third, we apply a static external (1,6) perturbation, and find that the other modes $(n = 5, 7, 8)$ decelerate, in addition to the directly affected $n = 6$ mode.

Our first experiment was to measure the correlated triple products of mode amplitudes that are characteristic of the nonlinear torque (cf. [14]):

$$T^{\mathrm{NL}}_{(1,6)} = \sum_{(m,n)} C^{(1,6)}_{(m,n),(m-1,n-6)} b_{(1,6)} b_{(m,n)} b_{(m-1,n-6)} \times \sin(\delta_{(m,n)} - \delta_{(1,6)} - \delta_{(m-1,n-6)}), \qquad (1)$$

where $C^{(1,6)}_{(m,n),(m-1,n-6)}$ are the nonlinear coupling coefficients, $b_{(m,n)}$ is the amplitude of the (m,n) mode, and $\delta_{(m,n)}$ is the phase of the (m,n) mode. It was not feasible to measure the coupling coefficients directly, so we present instead the three largest of the triple products in Eq. (1), without the coefficients. We have averaged the measured mode amplitudes and the sines of the phase differences over a large ensemble of about 700 similar sawtooth events in order to approximate a surface average.

The results are summarized in Fig. 4. In Fig. 4(a) is plotted the average $(m = 1, n = 6)$ mode phase velocity for the ensemble. Zero time corresponds to the time of maximum toroidal flux generation. Of particular interest is the sharp deceleration prior to the sawtooth event and slow reacceleration following it. In Fig. 4(b), the measured nonlinear mode triple products:

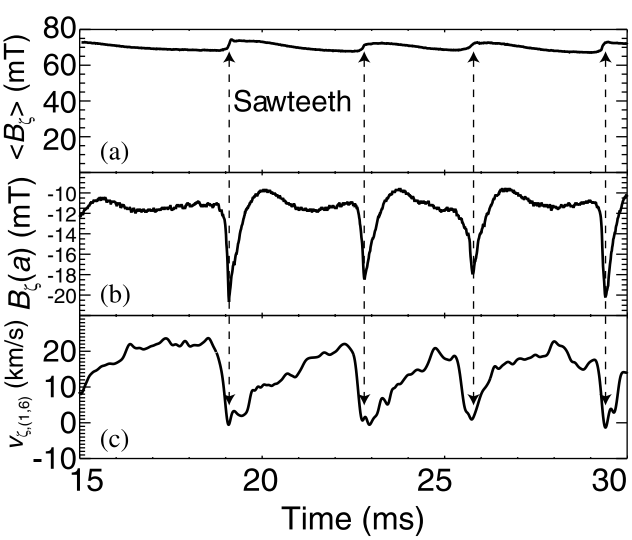

FIG. 1. (a) Volume averaged toroidal field. (b) Average toroidal field at plasma edge $(r = a)$. (c) Toroidal phase velocity of (1,6) mode, in a typical MST discharge.

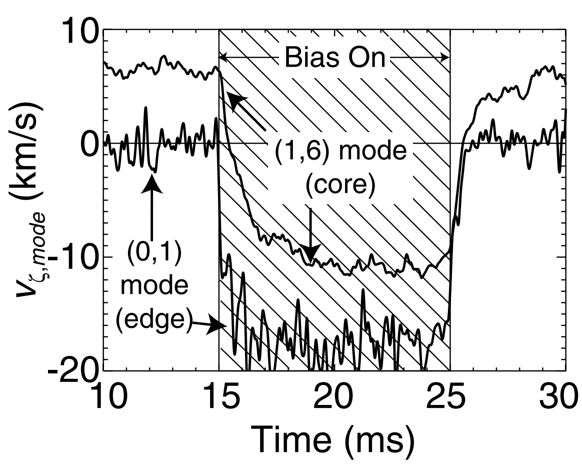

FIG. 2. Evolution of (1,6) and (0,1) mode toroidal phase velocities with edge biasing. The velocities are averaged over approximately 40 discharges.

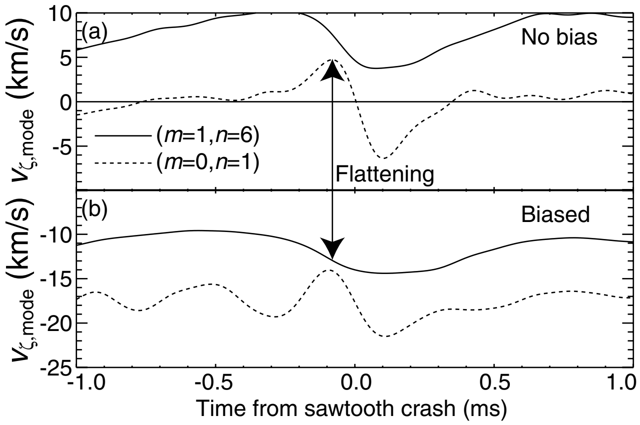

FIG. 3. Ensemble averaged toroidal phase velocities for the (1,6) and (0,1) modes in (a) normal and (b) biased cases. Note the difference in the vertical scales.

$$P_{167} \equiv \langle b_{(1,6)} b_{(1,7)} b_{(0,1)} \sin[\delta_{(1,7)} - \delta_{(1,6)} - \delta_{(0,1)}] \rangle,$$

$$P_{165} \equiv \langle b_{(1,6)} b_{(1,5)} b_{(0,1)} \sin[\delta_{(1,6)} - \delta_{(1,5)} - \delta_{(0,1)}] \rangle,$$

and

$$P_{268} \equiv \langle b_{(1,6)} b_{(1,8)} b_{(0,2)} \sin[\delta_{(1,8)} - \delta_{(1,6)} - \delta_{(0,2)}] \rangle$$

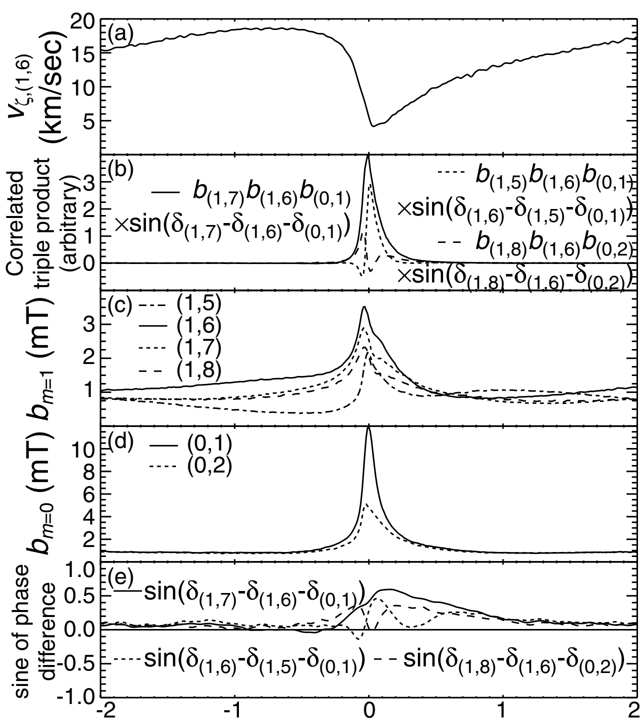

FIG. 4. Ensemble-averaged quantities for tripleproduct measurement. (a) (1,6) mode toroidal phase velocity. (b) Nonlinear triple products. (c) ($m = 1$) mode amplitudes. (d) ($m = 0$) mode amplitudes. (e) Phase factors.

are plotted. The amplitudes of P_{167} and P_{165} are about 3 times larger than P_{268} and peak during the sawtooth crash concurrent with the deceleration of the $(1,6)$ mode, whereas P_{268} changes sign at the crash. All three terms vanish during the slow reacceleration phase between crashes. The time variation of the triple products arises from changes in both the amplitudes and relative phases of the modes.

The amplitudes of the individual modes are plotted in Figs. 4(c) and 4(d). All show bursts of increased amplitude near the sawtooth event. The ensemble averages of the sines of the mode phase differences are plotted in Fig. 4(e). These quantities are nearly zero away from the sawtooth event because the modes are uncorrelated, and they become about $1/2$ during the event, as the modes become phase locked to each other.

Having found evidence for the role of the nonlinear torque in producing anomalous momentum transport during sawtooth events, we next set out to modify it. The $m = 0$ resonance was removed by changing the equilibrium magnetic field configuration such that the edge toroidal field was not reversed, implying that there is no location in the plasma at which the safety factor, q, is zero. Hence internally resonant $m = 0$ modes are absent.

The large deceleration in the $m = 1$ modes (cf. Fig. 1) is not seen, as is apparent in Fig. 5(a). The $m = 1$ modes still exhibit burst behavior, as shown in Fig. 5(b). However, the $(0,1)$ amplitude is near the noise limit for its detection, and no bursts are observed [Fig. 5(c)]. By removing the $m = 0$ channel for nonlinear coupling, we have eliminated most of the deceleration of the $m = 1$ modes. The small residual changes in the mode velocity coincident with the $m = 1$ bursts can be accounted for by other effects such as torques due to field errors.

In the final experiment we apply a static $n = 6$, broadband m magnetic perturbation. MST has a thick (5 cm) aluminum shell/vacuum vessel with an electromagnetic

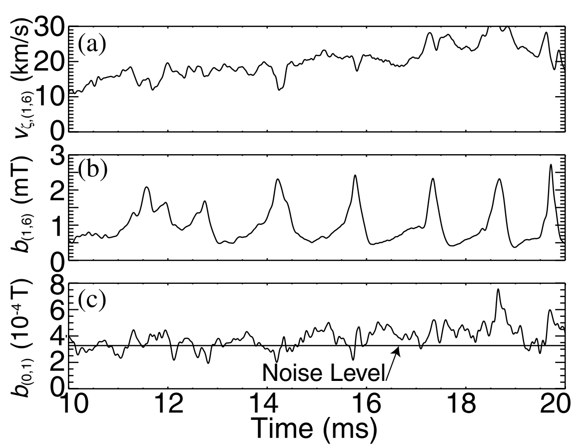

FIG. 5. Discharge with no ($m = 0$) resonance. (a) (1,6) mode toroidal phase velocity. (b) (1,6) mode amplitude. (c) (0,1) mode amplitude.

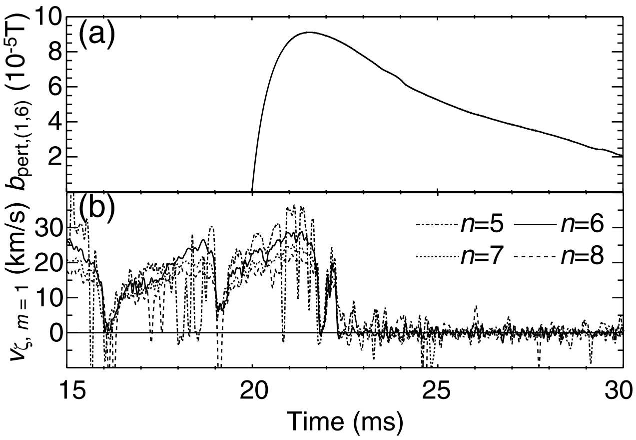

FIG. 6. Discharge with applied $n = 6$ perturbation. (a) ($m = 1$) component of ($n = 6$) perturbation. (b) ($m = 1, n = 5-8$) mode toroidal phase velocities

skin time long compared to the length of a discharge (about 50 ms). We have therefore utilized the 1 cm horizontal cut in the shell to apply an $n = 6$ perturbation. We position wires outside the vacuum vessel, such that they pass through the cut in an $n = 6$ pattern. The $n = 6$ perturbation thus produced exceeds the $n = 5, 7, 8$ perturbations (generated by imperfections in the wire placement) by about an order of magnitude. Since the width of the cut is much smaller than the plasma minor circumference the m spectrum of the perturbation is quite broad. However, for $n = 6$, only the $m = 0$ and $m = 1$ harmonics are resonant. Hence, the system is effectively selective in poloidal mode number, despite the limited poloidal access.

The stationary $n = 6$ ($m = 1$) perturbation is applied at 20 ms [Fig. 6(a)]. In response to the perturbation, all of the dominant core-resonant modes ($m = 1, n = 5, 6, 7, 8$) become locked (achieve zero phase velocity in the lab frame), as seen in Fig. 6(b). The rapid effect of the $n = 6$ perturbation on other modes is consistent with nonlinear torques acting on the modes, although other sources of anomalous viscosity may also be present to couple mode rotation. Experiments have been performed on the RFX RFP in which signatures of a nonlinear interaction are seen when a rotating ($m = 0, n = 1$) perturbation is applied [17].

In summary, we have determined experimentally that magnetic fluctuations can drive anomalous transport of momentum via internal electromagnetic forces. In the RFP, the effect arises from forces generated by three-wave nonlinear interactions predominantly involving two $m = 1$ modes and an $m = 0$ mode. We observe that the correlated triple wave products vary in time and peak during the period of strongest core mode (and plasma) deceleration. By selectively removing the (0,1) mode the sudden change deceleration of the core modes at sawteeth can be essentially eliminated. Finally, by selectively driving the (1,6) mode the rotation of the other $m = 1$ modes is affected. These active experiments further support the presence of nonlinear interactions. Other mechanisms, such as parallel particle diffusion along stochastic magnetic field lines, may produce anomalous momentum transport as well.

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