Azaria *et al.* Reply: We [1] have studied a special frustrated three-leg spin ladder (a 3-spin wide strip of the Kagomé lattice) which consists of two $S = 1/2$ Heisenberg chains with exchange constant $J_{\parallel}$ coupled in a zigzag way with couplings $J_{\perp}$ to an array of $S = 1/2$ localized spins. In the weak coupling limit when $\lambda = J_{\perp}/J_{\parallel} \ll 1$, it has been predicted within the bosonization approach that the system has soft singlet modes described by two critical Ising models with different velocities and a very small spectral gap for the magnetic excitations. The fate of this unusual spin liquid phase when λ increases, in particular for intermediate coupling ($\lambda = 1$), is far beyond the scope of the field theory analysis of Ref. [1].

In a recent paper, Pati and Singh [2] investigated the phase diagram of the model for intermediate couplings by the density matrix renormalization group (DMRG) approach. Using DMRG calculations for various system sizes with periodic boundary conditions, they reported a phase transition at $\lambda_c \simeq 1.2$ between a standard critical phase belonging to the universality class of the $S = 1/2$ Heisenberg chain and a spin gap phase for $J_{\perp}/J_{\parallel} < \lambda_c$. Moreover, the authors of Ref. [2] concluded that this spin gap phase cannot be a standard dimerized phase as in the Majumdar-Ghosh (MG) model [3] since a number of singlets are decreasing in energy as the system size is increased. In particular, it is very clear from Fig. 4 of Ref. [2] that there is at least three singlet states before the first triplet excitations which cannot be explained within a MG scenario. This fact leads Pati and Singh to conclude (see, in particular, the conclusion of Ref. [2]) that the nature of the spin gap phase is consistent with the findings of Ref. [1] obtained in the weak coupling limit.

In this Comment, White and Singh [4] studied the same model at the special point $J_{\parallel} = J_{\perp} = J$ by DMRG calculations using both open and periodic boundary conditions and concluded that the system is analogous to the MG model and that the field theory approach of Ref. [1] has to be reexamined.

In open systems, they terminated the cluster using a direct exchange interaction of magnitude J between the two surface chains that push singlet states above the first triplet state to be able to resolve an extremely small triplet gap $\Delta/J = 0.0104(5)$. One should notice that the introduction of this modified open end will introduce an effect of order $1/L$ (L being the linear size of the sample) which is of the same order of the estimated bulk gap. Moreover, it is easy to see within the field theory approach of Ref. [1] that a direct exchange between the surface chains will kill the criticality of the singlet modes and the low lying excitations are triplet excitations with a small gap. For periodic boundary conditions, they found without showing any numerical results that there are only two singlet states before the first triplet excitation which is in contradiction with the numerical findings of Ref. [2] discussed above.

The numerical findings of White and Singh [4] at the special point $J_{\parallel} = J_{\perp}$ do not imply that the field theory analysis of Ref. [1] expected to be valid when $J_{\parallel} \gg J_{\perp}$ has to be reexamined since the two analyses belong to different regimes of parameters of the model. To show that the field theory approach of Ref. [1] has to be revisited, the authors should perform DMRG calculations in the regime $J_{\parallel} \gg J_{\perp}$ with periodic conditions and use a wave-vector-resolved calculation to extract the number of singlet modes at low energy.

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