Confinement Scaling Laws for the Conventional Reversed-Field Pinch

J. Scheffel
Fusion Plasma Physics, Alfvén Laboratory (Association EURATOM-NFR), Royal Institute of Technology, SE-100 44 Stockholm, Sweden

D. D. Schnack
Science Applications International Corporation, 10260 Campus Point Drive, San Diego, California 92121
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A series of high resolution, 3D, resistive MHD numerical simulations of the reversed-field pinch are performed to obtain scaling laws for poloidal beta and energy confinement at Lundquist numbers approaching 10^6. Optimum plasma conditions are attained by taking the transport coefficients to be classical, and ignoring radiation losses and resistive wall effects. We find that poloidal beta scales as $\beta_\theta \propto I^{-0.40}$ and that the energy confinement time scales as $\tau_E \propto I^{0.34}$ for fixed I/N, with aspect ratio $R/a = 1.25$.

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The reversed-field pinch (RFP) is regarded as one of the leading fusion confinement schemes alternative to the tokamak. Its potential stems from the possibility of high ratio β_θ of plasma to magnetic pressure, i.e., efficient use of the magnetic field, and compact size. An interesting and important feature of the RFP plasma is a nonlinear dynamo mechanism that continuously converts poloidal flux supplied from the external circuit into toroidal flux in the plasma core in such a way as to sustain the mean magnetic field configurations against resistive diffusion [1].

In present RFP experiments the dynamo fluctuations are thought to be responsible for anomalously large radial particle and energy transport, so that the scaling laws associated with the fluctuations are of importance to an assessment of the RFP as a fusion reactor. Early experimental results at low values of the Lundquist number $S = \tau_R/\tau_A$ (where τ_R is the resistive diffusion time and τ_A is the Alfvén transit time) indicated that these fluctuation levels may decrease with increasing Lundquist number [2], implying improved confinement properties in high temperature reactor relevant regimes. Later experiments with S approaching 10^6 indicated a weaker scaling [3].

In this work we investigate the confinement properties of RFP plasmas by carrying out a series of high resolution, three-dimensional, nonlinear, resistive MHD numerical simulations whose aim is to determine the dependence of the poloidal beta β_θ and the energy confinement time τ_E on experimental parameters, such as the plasma current. These calculations include the effects of finite pressure, Ohmic heating, and convection and anisotropic heat conduction. By using plasma parameters relevant for present day RFP experiments, scaling laws that are in substantial agreement with the international RFP database, and that may be useful for predicting performance at higher temperatures and plasma currents, are obtained. Throughout we have attempted to obtain configurations with optimal confinement properties, so that the results of this study should be considered to be optimistic.

The simulations are run for low aspect ratio (major radius to plasma minor radius ratio $R/a = 1.25$) to save computing time. This particular choice is not crucial, since RFP confinement properties are predominantly governed by the stochasticity of the magnetic field in the plasma core, which in turn is a function of the level of radial magnetic field fluctuations. This level is independent of aspect ratio; larger aspect ratio results in more unstable modes at lower amplitudes which add up to a comparable fluctuation level [4]. We have verified this by performing validation runs at $R/a = 4$.

The scaling results so obtained indicate that the "conventional" RFP (i.e., with no flow or current profile control) may not extrapolate well to thermonuclear conditions, primarily because of persistent magnetic fluctuations that are large enough to make the core magnetic field stochastic. This serves to emphasize the importance of experimental techniques for enhancing confinement, such as current profile and sheared flow control [5–7], that may thus be required to establish the reactor potential of the RFP.

Fluctuation scaling is a long-standing and debated issue for the RFP. Because of the large number of resonant surfaces, the resistive instabilities that are the fundamental dynamo mechanism [1] locally destroy any nested flux surfaces: the saturated magnetic islands are large enough to overlap, and the magnetic field becomes stochastic. A magnetic field line may then wander from the core to the edge of the plasma, instead of being confined to a flux surface. This establishes channels for particles and energy to escape in the radial direction *along* the magnetic field lines. It is thus desired that the amplitudes of the characteristic magnetic fluctuations decrease sufficiently rapidly with increasing Lundquist number (increasing temperature), so that these anomalous losses may diminish to acceptable levels for reactor plasmas.

Numerical studies of the fluctuation levels of nonlinearly evolving RFP plasmas have shown that the saturated fluctuation amplitudes decrease with increasing S.

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In one study [8] the scaling law $\delta B \propto S^{-0.22}$ was obtained for Lundquist numbers in the range $3 \times 10^3 \leq S \leq 10^5$. Here δB is the rms value of the variation of the total volume averaged magnetic field in time, including all poloidal and toroidal modes. Another study [7] obtained a fluctuation scaling of $S^{-0.18}$ in the range $2.5 \times 10^3 \leq S \leq 4 \times 10^4$. These studies were carried out for zero pressure plasmas without thermal transport. Our results with finite pressure effects and thermal transport indicate that the magnetic fluctuation level remains high for values of S approaching 10^6.

The computation of parallel transport in the RFP is difficult because of the fast time scales involved, and has received particular attention. In this work we use the algorithm developed by Sovinec [7]. It combines the semi-implicit method with subcycling to achieve accuracy and numerical stability for large time steps.

In order to address the fusion potential of the RFP, we have striven to attain upper limits of confinement states; no impurity radiation is assumed, and we use a stabilizing conducting wall at the plasma boundary and classical values for transport coefficients. These simulations were carried out for plasma currents in the range 18–252 kA and densities in the range $(0.5\text{–}7.0) \times 10^{19}$ m^{-3}. This resulted in on-axis temperatures in the range 26–105 eV and on-axis Lundquist numbers in the range $2 \times 10^4 \leq S(0) \leq 7 \times 10^5$.

The resistive MHD equations, when normalized to the Alfvén time and pinch radius, are

$$\nabla \times \mathbf{A} = \mathbf{B}, \qquad \frac{\partial \mathbf{A}}{\partial t} = \mathbf{v} \times \mathbf{B} - \frac{\eta}{S_0} \nabla \times \mathbf{B}, \quad (1)$$

$$\frac{\partial \mathbf{v}}{\partial t} = -(\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{1}{\rho}(\nabla \times \mathbf{B}) \times \mathbf{B} - \frac{\beta_0}{2\rho}\nabla p + \frac{\nu}{\rho}\nabla^2 \mathbf{v}, \quad (2)$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot (p\mathbf{v}) - (\gamma - 1)p\nabla \cdot \mathbf{v} + \frac{2(\gamma - 1)\eta}{\beta_0 S_0}(\nabla \times \mathbf{B})^2 + \frac{1}{S_0}\nabla \cdot (\kappa_\perp^i \nabla_\perp T + \kappa_\parallel^e \nabla_\parallel T). \quad (3)$$

We have defined $\beta_0 \equiv 4\mu_0 n_0 T_0 / B_0^2$ and $S_0 \equiv \tau_R/\tau_{A0} = 5.09 \times 10^{14} a B_0 T_0^{1.5}/(\mu^{0.5} n_0^{0.5} Z_{\text{eff}} \ln\Lambda)$. The dependent variables have their usual meanings. Classical, radially dependent transport parameters [9] are used; $\eta = T^{-1.5}$ (Spitzer resistivity), $\kappa_\perp^i = \kappa_{\perp 0} n^2/(B^2 T^{0.5})$, and $\kappa_\parallel^e = \kappa_{\parallel 0} T^{2.5}$. Classical ion viscosity is given by $\nu = \nu_0 T^{2.5}$, with $\nu_0 = 8.5 \times 10^4 S_0 T_0^{2.5}/(a B_0 n_0^{0.5} \ln\Lambda)$. The local Lundquist number and poloidal beta are given by $S = S_0 B T^{1.5}/n^{0.5}$ and $\beta_\theta = \beta_0 \langle p \rangle / B_\theta^2(1)$, respectively. Normalized pressure $p = nT$ obtains through using $p_0 = 2n_0 T_0 = B_0^2 \beta_0/(2\mu_0)$. The mass density is assumed to be spatially uniform and constant in time.

Our studies have shown that confinement properties are largely independent of the exact choice of viscosity. Thus, for reasons of numerical stability, we choose local viscosity to be sufficiently large to produce an effective Reynolds' number that is small enough to avoid the generation of subgrid scale turbulence and the resulting nonlinear numerical instabilities. This is required in any nonlinear fluid simulation. Since the dynamo in the RFP arises from the nonlinear interaction of several long wavelength modes that are well resolved on the grid [1], the essential physics of plasma relaxation and profile sustainment are not affected.

In all cases, we have chosen the plasma current to line density ratio to be $I/N = 2.8 \times 10^{-14}$ A m. This is close to the experimental lower limit, similar to the Greenwald limit for tokamaks. It may also be shown that $\kappa_{\parallel 0}$ cannot be chosen smaller than the value given below without further decreasing I/N. Thus we may use the classical value for $\kappa_{\parallel 0}$ to obtain the best reasonable confinement, i.e., small values of parallel heat conduction.

Numerical studies of transport scaling have not been previously performed. This may be because of the apparent difficulties arising from the number of independent parameters appearing in the problem. In contrast, in this study we have found that only two dimensionless parameters are needed to determine optimized confinement. This is because the five parameters β_0, S_0, $\kappa_{\perp 0}$, $\kappa_{\parallel 0}$, and ν_0 are not independent, since $\kappa_{\perp 0} = 10.0\beta_0\mu^{0.5}$, and $\kappa_{\parallel 0} = 32.5\nu_0\mu^{0.5}$ (expressed in SI units with μ being the ion to proton mass ratio). Thus only the two parameters β_0 and S_0 are required for computing optimal confinement within the resistive MHD model. We have assumed there exists a perfectly conducting wall at the plasma boundary, that $\Theta \equiv B_\theta(a)/\langle B_z \rangle = 1.8$ and $p(a)/p(0) = 0.1$ (in the initial state). Further, edge resistivity is modeled by superimposing a resistivity profile that is unity in the core and increases very close to the wall. A sufficient grid resolution is found to be 300 radial mesh points, 42 axial modes, and 5 poloidal modes. All calculations are run until a quasi-steady state with a self-sustaining dynamo is reached, and time-averaged values of poloidal beta and energy confinement time are determined.

Values of poloidal beta for various values of β_0 and S_0 are displayed in Fig. 1. These data are obtained by averaging over a sawtooth period, which usually lasts a few percent of the resistive time. Using linear regression analysis, the data points in Fig. 1 can be well represented as a power law (indicated as solid curves)

$$\beta_\theta = 3.65\beta_0^{0.407\pm0.011} S_0^{-0.165\pm0.007}. \quad (4)$$

In a similar manner, the on-axis temperature $T(0)$ can also be found as a function of β_0 and S_0 from the time-averaged $T(0)/T_0$ data. Using the definition of S_0 there results the

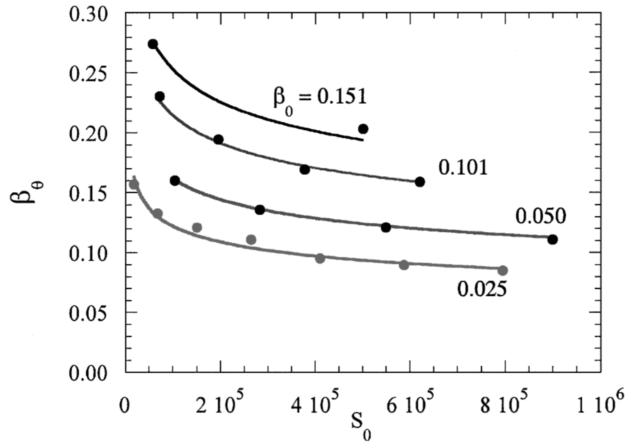

FIG. 1. Numerical results for poloidal beta (β_θ) as a function of the basic parameters β_0 and S_0. Solid lines represent the power law fit, given by Eq. (4), to data points.

scaling $T_0 \propto a^{-0.5}\mu^{0.25}Z_{\rm eff}^{0.5}\beta_0^{0.25}S_0^{0.5}$, and we find

$$T(0) = 0.221a^{-0.5}\mu^{0.25}Z_{\rm eff}^{0.5}\beta_0^{-0.320\pm0.018}S_0^{0.311\pm0.011}. \tag{5}$$

The numerical parameters β_0 and S_0 can now be related to physical quantities. Using Eq. (5) and the definitions given following Eqs. (1)–(3), along with the relation for the total current $I = 2\pi a B_0\langle B_z\rangle\Theta/\mu_0$, we can eliminate β_0 and S_0 from Eq. (4) in favor of $T(0)$ and I. The result is

$$\beta_\theta = 156a^{0.04}\mu^{-0.02}Z_{\rm eff}^{-0.04}T(0)^{1.09}I^{-1.01}. \tag{6}$$

It should be noted that, in the general case, β_θ is a function of *both* I *and* I/N [we show below how $T(0)$ can be obtained as a function of I for fixed I/N]. Any attempt to obtain experimental scalings of β_θ with I or I/N *only* is thus misleading unless the other independent variable is kept constant.

Equation (4), which applies to a time-averaged steady state, suggests that β_θ is a function of the initial conditions as represented by β_0 and S_0. However, β_0 and S_0 are just functions of the normalization parameters (a, T_0, n_0, and B_0) and in steady state β_θ must not retain a memory of the initial state, since the RFP is a strongly driven, dissipative nonlinear system; it will depend only on geometry and boundary conditions. Thus, for an achieved steady state, we are free to replace T_0 with $T(0)$. The definitions of β_0 and S_0 then allow us to use Eq. (5) to relate the on-axis temperature $T(0)$ (in eV) to the total current I. The result is

$$T(0) = 0.071a^{-0.22}\mu^{0.11}Z_{\rm eff}^{0.22}I^{0.56}. \tag{7}$$

Substituting Eq. (7) into Eq. (6) yields

$$\beta_\theta = 8.8a^{-0.20}\mu^{0.10}Z_{\rm eff}^{0.20}I^{-0.40}. \tag{8}$$

Equations (7) and (8) are scaling laws for steady state on-axis temperature $T(0)$ and poloidal beta β_θ at a constant value of I/N ($= 2.8 \times 10^{-14}$ A m). For comparison, the experimental scalings $T \propto I/n^{0.5}$ (or $\propto I^{0.5}$ for constant I/N) and $\beta_\theta \propto I^{-0.93}n^{0.46} \propto I^{-0.47}$ were recently found on the Extrap T2 RFP [10]. Earlier confinement experiments on the smaller Extrap T1 RFP gave the dependence $\beta_\theta \propto I^{-0.67}n^{0.16} \propto I^{-0.51}$ [11].

We now consider the scaling of τ_E, the energy confinement time. Following [12] we define (with $V_p = 2\pi^2Ra^2$, the plasma volume)

$$\tau_E = \tfrac{3}{2}\langle p\rangle V_p/(U_{\rm loop}I). \tag{9}$$

Assuming that the input power is balanced by Ohmic dissipation, and using the definitions of poloidal beta and parallel Spitzer resistivity, there results

$$\tau_E = \frac{2.7 \times 10^{14}}{f_G Z_{\rm eff}\ln\Lambda}a^2\beta_\theta^{2.5}(I/N)^{1.5}I^{1.5}. \tag{10}$$

For simplicity profiles of density, temperature and resistivity are here taken to be constant in space. The factor f_G stems from the screw-shaped current path in the RFP, and can be approximated by $f_G \approx 2.2(5 + 6\Theta^2)/(10 + \Theta^2)$ [13]. It is sometimes assumed that both β_θ (as determined by force balance alone) and I/N remain constant with increasing current. This "standard" RFP scaling thus yields $\tau_E \propto I^{1.5}$ and $T \propto I$.

The energy confinement time, obtained from regression analysis of our simulation data, can be expressed in experimental variables (assuming $\ln\Lambda = 15$);

$$\tau_E = 0.019a^{2.0}\mu^{0.0}Z_{\rm eff}^{-1.0}T(0)^{2.6}I^{-1.1}. \tag{11}$$

After inserting $T(0)$ from Eq. (7), there results

$$\tau_E = 1.9 \times 10^{-5}a^{1.4}\mu^{0.29}Z_{\rm eff}^{-0.42}I^{0.34}. \tag{12}$$

The scaling of Eq. (12) with current is much weaker than that of the standard case, primarily due to the degradation of β_θ with current. It is, however, close to measured data at the Extrap T2, RFX, and MST experiments.

Equations (7), (8), and (12) are our principal results. In an extended paper, where we will also include the dependence on I/N and Θ, comparisons with individual experiments will be made. Here we briefly remark on the implications of these results for the fusion potential of the conventional RFP. Inserting parameter values relevant for a compact RFP reactor ($a = 0.6$ m, $I = 18$ MA), Eq. (12) predicts $\tau_E \approx 0.003$ s. This value for the confinement time is well below the design value of 0.2 s assumed in the TITAN reactor study [14] for the same parameters. Further, Eq. (7) implies an on-axis temperature of only about 1.1 keV as compared to the TITAN design value of 10 keV, contradicting the assumption of Ohmic heating to ignition. However, the TITAN study used $\Theta = 1.5$, as compared to our value of 1.8. This may yield somewhat better confinement due to a less stochastic plasma core. Care must also be taken when extrapolating to TITAN Lundquist numbers of 10^9, by far exceeding the range employed in our study.

For comparison with analytically and experimentally obtained scaling laws, we use Eq. (6) to eliminate $T(0)$ from

Eq. (11). We find (with $I/N = 2.8 \times 10^{-14}$ A m)

$$\tau_E = 1.0 \times 10^{-7} \mu^{0.05} Z_{\rm eff}^{-0.90} \beta_\theta^{2.4} a^{1.9} I^{1.3}. \qquad (13)$$

Connor-Taylor scaling [15], being based on constant-β_θ resistive g-mode fluctuation theory, predicts the dependence $\tau_E \propto a^2 I^{1.5} (I/N)^{1.5}$ [which is the same as the standard dependence given in Eq. (10)]. A fit to an international RFP database [16], using the form $\tau_E = c[a^2 I^{1.5} (I/N)^{1.5}]^p$, in units of s, MA, 10^{20} m^{-1}, and m, results in the parameter values $c \approx 6 \times 10^{-3}$ and $p \approx 0.87$. Adjusting the present dependence on this form, we find $c = 1.9 \beta_\theta^{2.4} a^{0.16} \mu^{0.05} Z_{\rm eff}^{-0.90}$ and $p = 0.87$. Our expression for c has a strong dependence on β_θ. Since β_θ certainly is not independent of current, the forms for τ_E suggested in [16] are therefore of questionable utility (see also [17]).

In conclusion, we have determined for the first time theoretical limits for RFP confinement behavior. Our results indicate significant degradation from the standard picture of RFP confinement, but are in good agreement with experimental results. The relatively high Lundquist numbers, comparable to those experimentally obtained, are achieved by using a fine radial grid and by careful choice of viscosity. (In an extended paper we will show that the exact choice of viscosity has little impact on the physical results such as magnetic field fluctuation levels or confinement.) The scalings are obtained by recognizing that there are no more than two governing parameters for fixed I/N. In our study an optimum conducting wall case was assumed; further $\Theta = 1.8$, $R/a = 1.25$, and density is constant. We believe that none of these restrictions have any strong influence on our conclusions.

Several effects were neglected in these simulations. A resistive wall and field errors would introduce new unstable modes, impurities would cause line radiation, fluctuations in the presence of a density gradient would cause particle transport [18,19], and hot electrons may enhance core energy losses. These would all lead to a more pessimistic picture than the one presented here.

Finally, our results emphasize the importance of experimentally demonstrating control of the RFP current profile in order to minimize the dynamo fluctuations, reduce the corresponding thermal losses, and improve energy confinement.

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