Stabilization of Neoclassical Tearing Modes by an Externally Applied Static Helical Field

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The effect of a static helical magnetic field on the nonlinear growth of the neoclassical tearing mode (NTM) is investigated. The NTM is found to be stabilized by an externally applied helical field of a different helicity if the field magnitude is sufficiently large, suggesting a very simple method for stabilizing the NTM. The mechanism responsible for this stabilization is the decreased fundamental harmonic pressure perturbation of the NTM in the presence of the helical field.

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1. Introduction.—For a high β tokamak plasma the perturbed bootstrap current can drive the magnetic island to grow even if the tearing mode instability factor Δ' is negative, leading to the neoclassical tearing mode (NTM) [1,2]. NTMs have been found experimentally to limit the maximal achievable pressure well below the predictions of ideal magnetohydrodynamics (MHD) calculations and to lead to the most severe limitation in the present day tokamaks [3–7]. These modes are considered to be even more dangerous for a tokamak reactor as the β value for the mode onset becomes smaller with normalized ion gyro radius ($\beta_N \sim \rho_i^*$) [7]. In recent years extensive efforts have been devoted to understand the threshold for the onset of the NTMs [7–9] and the nonlinear evolution of the single and the double NTMs [10,11].

The stabilization of the NTM is therefore of great concern. It had been shown in the recent experiments that the NTM can be stabilized by localized electron cyclotron current drive (ECCD) [7,12]. The corresponding theoretical studies have shown approximately the same results ([13] and references therein). It is found that 2% of the plasma current by ECCD is required for stabilizing the $m/n = 3/2$ mode with a 14% equilibrium bootstrap current density fraction at the $q = 3/2$ surface, where q is the safety factor. When scaled to a fusion reactor, 30 MW radio frequency (rf) wave power will be required for the stabilization of the same β value plasma [13]. If NTMs with low m numbers ($m = 2$, 3, and 4) are expected to be unstable for a higher β reactor plasma, then rf current drive at several rational surfaces is needed and the total required rf power will be quite significant. Therefore, to explore other possible methods for the stabilization is important.

In the present paper the effect of a static helical magnetic field on the nonlinear evolution of the NTMs is investigated. It is found that the NTM can be stabilized by a helical field of a different helicity if the magnitude of the helical field is sufficiently large. This finding suggests a very simple and cheap method for the stabilization. Both analysis and numerical modeling have been carried out as shown in the following.

2. Model and analytical theory.—The basic equations describing the NTM are Ohm's law, the equation of motion, and the pressure evolution equation,

$$\frac{\partial\psi}{\partial t} + \mathbf{B}\cdot\nabla\phi = E - \eta(j - j_b), \tag{1}$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\nabla^2\phi = \mathbf{e}_t\cdot(\mathbf{B}\cdot\nabla j) + \rho\mu\nabla^4\phi, \tag{2}$$

$$\frac{3}{2}\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)p = \nabla\cdot(\chi_b\nabla_b p) + \nabla\cdot(\chi_\perp\nabla_\perp p) + Q, \tag{3}$$

where $\mathbf{B} = B_{0t} + \nabla\psi\times\mathbf{e}_t$ and $\mathbf{v} = -\nabla\phi\times\mathbf{e}_t$ are the magnetic field and plasma velocity, respectively, ψ and ϕ are the magnetic flux function and the stream function, respectively, and the subscript 0 denotes an equilibrium quantity. $j = -\nabla^2\psi$ and $j_b = -g(\sqrt{\varepsilon}/B_p)dp/dr$ are the plasma and the bootstrap current density, respectively, along the $\mathbf{e}_t$ (toroidal) direction, g is a function of the minor radius r, which becomes zero as r approaches the edge assuming a collisional edge, $\varepsilon = r/R$ is the inverse aspect ratio, and B_p is the poloidal magnetic field. ρ is the plasma mass density, μ is the plasma viscosity, p is the plasma pressure, χ_b and $\chi_\perp$ are the parallel and perpendicular transport coefficients, respectively, ∇_b and $\nabla_\perp$ are the parallel and the perpendicular gradient, respectively, and Q and E are the heating power and the equilibrium electric field, respectively.

Equations (1)–(3) are the coupled equations for studying the nonlinear evolution of NTMs which result from the expansion of MHD equations and keep the terms to the order $\varepsilon^{1/2}$ [1,2]. The toroidal coupling is neglected. Equations (1)–(3) have been used to investigate the nonlinear evolution of the single and double NTMs [10,11]. We have solved Eqs. (1)–(3) simultaneously in order to study the effect of an externally applied helical field. The results of these numerical calculations are presented in the next section. In the following we give a simple analytical approach which allows an easier understanding of the effects shown by our simulations.

Since usually the transport time scale is much smaller than that of the NTM growth, Eq. (3) can be simplified as [9]

$$\chi_b\nabla\cdot(\nabla_b p) + \chi_\perp\nabla\cdot(\nabla_\perp p) = 0, \tag{4}$$

assuming there is no significant source inside the island. The convective transport term is neglected in Eq. (4) which

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is valid for the NTM in a high temperature plasma. For the case with only one Fourier component helical magnetic perturbation, Eq. (4) has been solved in detail [9], and the NTM is found to be more stable for a larger $\chi_\perp/\chi_b$.

When there are two Fourier components of helical magnetic perturbations, $\mathbf{b}_1 = \nabla\psi_1 \times \mathbf{e}_t/|\mathbf{B}_0|$ and $\mathbf{b}_2 = \nabla\psi_2 \times \mathbf{e}_t/|\mathbf{B}_0|$ of different helicities, respectively, they will lead to corresponding pressure perturbations p_1 and p_2. ψ and p are expressed in terms of Fourier series,

$$\psi = \psi_{0/0}(r,t) + \sum \psi_j(r,t)\exp[i(m_j\theta + n_j\xi)], \quad (5)$$

$$p = p_{0/0}(r,t) + \sum p_j(r,t)\exp[i(m_j\theta + n_j\xi)], \quad (6)$$

where θ and ξ are the poloidal and the toroidal angle, respectively, the summation is over j with $j = 1$ and 2, and m_j and n_j are, respectively, the poloidal and the toroidal mode number of the perturbations $\mathbf{b}_j$.

It is found from Eqs. (4)–(6) that p_1 is described by

$$\chi_b[-F_1^2 p_1 + iF_1 p_{0/0}' b_{1r} + \langle(\mathbf{b}_1\cdot\nabla)(\mathbf{b}_1\cdot\nabla p_1) + (\mathbf{b}_2\cdot\nabla)(\mathbf{b}_2\cdot\nabla p_1) + (\mathbf{b}_2\cdot\nabla)(\mathbf{b}_1\cdot\nabla p_2) + (\mathbf{b}_1\cdot\nabla)(\mathbf{b}_2\cdot\nabla p_2)\rangle_1] + \chi_\perp\nabla\cdot(\nabla_\perp p_1) = 0, \quad (7)$$

where the prime denotes $\partial/\partial r$, $F_1 = \mathbf{k}_1\cdot\mathbf{B}_0/|\mathbf{B}_0|$, $\mathbf{k}_1$ is the wave vectors of $\mathbf{b}_1$, $\langle y\rangle_1 = \int y\cos(\zeta)\,d\zeta/\pi$ with the integration from $\zeta = 0$ to 2π, and $\zeta = m_1\theta + n_1\xi$. In Eq. (7) higher harmonic perturbations are neglected which is valid if the magnitude of the perturbations is not too large [9]. p_2 is described by an equation similar to Eq. (7).

When $\psi_2 = 0$ and $p_2 = 0$, corresponding to the case with only one Fourier component, it is found from Eq. (7) that $p_1 = 0.3(w_1/w_c)^2(\partial p_{0/0}/\partial r)(r - r_{s1})$ for $|r - r_{s1}| \ll r_{s1}$ and $w_1 \ll w_c$ [9], where w_1 is the magnetic island width induced by $\mathbf{b}_1$ at the rational surface r_{s1} defined by $q(r_{s1}) = m_1/n_1$, $w_c = r_{s1}(\chi_\perp/\chi_b)^{1/4} \times (\varepsilon_{s1}s_{s1}n_1/8)^{-1/2}$, $\varepsilon_{s1} = r_{s1}/R$, $s_{s1} = rq'/q$, and the subscript $s1$ denotes taking values at r_{s1}. The parameter w_c is the so called critical island width [9]. When $w_1 \le w_c$, it can be found from Eq. (7) that the magnitude of the perpendicular transport term is of the same order as that of the parallel one near the rational surface, and the pressure is not flattened inside the island.

When there are two Fourier component perturbations, the pressure perturbations can be found in a similar way. Away from the rational surfaces, Eq. (7) is dominated by the first two linear terms and is reduced to

$$-F_1 p_1 + ip_{0/0}' b_{1r} \approx 0, \quad (8)$$

which lead to the outer region solution of p_1. The form of p_2 is similar to Eq. (8).

In the inner region near the rational surface r_{s1} with $|r - r_{s1}| \ll r_{s1}$, p_1 has a large radial derivative since $p_1 \sim 1/(r - r_{s1})$ as r approaches r_{s1}, so that $\partial p_1/\partial r \gg \partial p_1/r\partial\theta$. The radial slope of p_2 is much smaller than that of p_1 near r_{s1} under the assumption that r_{s2} is not close to r_{s1}. Usually the radial slopes of $\mathbf{b}_1$ and $\mathbf{b}_2$ of NTMs are much smaller than that of p_1. Thus, Eq. (7) is simplified as

$$-F_1^2 p_1 + iF_1 p_0' b_{1r} + [0.25(b_{1r}^2 + b_{2r}^2) + \chi_\perp/\chi_b]p_1'' = 0. \quad (9)$$

It is seen that the effect of $\mathbf{b}_2$ on the radial profile of p_1 is similar to that of the perpendicular transport. When $|\mathbf{b}_2| \ll |\mathbf{b}_1|$, $w_1 \ll w_c$, and $|r - r_{s1}| \ll r_{s1}$, Eqs. (8) and (9) lead to the result of the one mode case [9]. However, when $|\mathbf{b}_1| \ll |\mathbf{b}_2|$, the balancing between the third term and the first two terms in Eq. (9) leads to a new critical width given by

$$w_{ch} = r_{s1}\left(\frac{8}{\varepsilon_{s1}s_{s1}n_1}\right)^{1/2}\left(0.25b_{2r}^2 + \frac{\chi_\perp}{\chi_b}\right)^{1/4}. \quad (10)$$

For $|b_{2r}|^2 \ll 4(\chi_\perp/\chi_b)$, $w_{ch} = w_c$. While for $|b_{2r}|^2 \gg 4(\chi_\perp/\chi_b)$, $w_{ch} = 2r_{s1}(b_{2r}/\varepsilon_{s1}s_{s1}n_1)^{1/2}$ which is increased by a factor of $(0.5|b_{2r}|)^{1/2}/(\chi_\perp/\chi_b)^{1/4}$ from w_c.

For $w_1 < w_{ch}$, $|r - r_{s1}| \ll w_{ch}$ and $|\mathbf{b}_1| \ll |\mathbf{b}_2|$, it is found from Eqs. (8) and (9) that

$$p_1 = 0.3(w_1/w_{ch})^2 p_{0/0}'(r - r_s). \quad (11)$$

Utilizing Eqs. (1), (2), and (11) and following the same procedure of Ref. [9], the island growth equation is found to be

$$I_1\frac{d}{dt}(w_1) = \eta\left(\Delta' + 1.97f\frac{r_{s1}j_{0s1}w_1}{B_{ps1}s_{s1}w_{ch}^2}\right), \quad (12)$$

where f is the equilibrium bootstrap current density fraction at $r = r_{s1}$ and $I_1 = 0.82$. Similar to the perpendicular transport, $\mathbf{b}_2$ leads to a larger threshold for the onset of the NTM. When $|b_{2r}|^2 \ll 4(\chi_\perp/\chi_b)$, Eq. (12) becomes the same as Eq. (118) of Ref. [9] for the one mode case.

When $\mathbf{b}_2$ is an externally applied helical field and its magnitude is sufficiently large, according to Eq. (12) w_1 will decay due to the large critical island width w_{ch}. For $|b_{2r}|^2 \gg 4(\chi_\perp/\chi_b)$, the condition for the stabilization of w_1 by $\mathbf{b}_2$ is

$$|b_{2r}| > b_{2rc} = 0.49f\varepsilon_{s1}n_1(r_{s1}j_{0s1}/b_{ps1}) \times (w_1/r_{s1})/(-r_{s1}\Delta').$$

With $w_1/r_s = 0.04$, $r_{s1}\Delta' = -3$, $f = 0.1$, $\varepsilon_{s1} = 0.15$, $n_1 = 2$, and $r_{s1}j_{0s1}/b_{ps1} = 1$, it is found that the NTM will be stabilized if $b_{2r} = 2 \times 10^{-4}$. The required magnitude of the helical field is much smaller than that of the equilibrium poloidal field which is of the order $0.1B_{0t}$.

3. Numerical calculations.—For checking the analytical results, Eqs. (1)–(3) are solved simultaneously using an initial value code TM. This code had been used to simulate the nonlinear evolution of the single and double NTMs [10,11]. For high m NTM, the saturated island width obtained from TM agrees with the analytical result. $S = \tau_R/\tau_A = 5 \times 10^6$ and $S_\mu = \tau_R/\tau_\mu = 10$ are taken, where τ_A is the Alfvén time, $\tau_R = a^2\mu_0/\eta$ is the resistive time, $\tau_\mu = a^2/\mu$ is the viscous time, and a is the minor radius. A constant $\chi_b\tau_R/a^2 = 10^9$ is used. $\chi_\perp = 0.23a^2/\tau_R$ at the rational surface. A small $\chi_\perp$ chosen here is to minimize the stabilizing effect from finite $\chi_\perp/\chi_b$.

For the externally applied helical field not to result in a magnetic island inside the plasma and considering $\psi_{m/n} \sim r^m$ inside the plasma when neglecting plasma response, it is desirable to select the helical field mode numbers to be $m/n < 1$ with $m = 1$ or 2. Here the effect of $m/n = 2/4$ static helical field on the $m/n = 3/2$ NTM is studied. The $m/n = 2/4$ static helical field is introduced in the numerical modeling by taking the boundary condition $\psi_{2/4}(a) \neq 0$. $q = 3/2$ surface is at $r = 0.575a$. In addition to the $m/n = 2/4$ perturbation, two Fourier components ($m/n = 0/0$ and $3/2$) are included for the $m/n = 3/2$ mode, as this leads to approximately the same results as if more harmonics are included [10].

For $\psi_{2/4}(a) = 0$, the growth of the normalized $m/n = 3/2$ island width, $w \equiv w_{3/2}/a$, is shown for $f = 0.021$ (solid curve) and $f = 0.016$ (dotted curve) in Fig. 1. The $m/n = 3/2$ island is unstable for larger f and stable for smaller f, indicating that the Δ' is negative for the present q profile. When a $m/n = 2/4$ field is introduced at $t = 0$ by using the boundary condition $\Psi \equiv a^{-1}\psi_{2/4}(a)/|\mathbf{B}_0| = 10^{-4}$ and an initial profile $\psi_{2/4}(r) \sim \psi_{2/4}(a)r^2$, the time evolution of w is shown for $f = 0.048$ (dashed curve) and $f = 0.043$ (dot-dashed curve) in Fig. 1. For the same $\psi_{2/4}(a)$, $w_{3/2}$ decays for smaller f but still grows for larger f.

In Fig. 2 the time evolution of w is shown for $\Psi = 0.5 \times 10^{-3}$ (solid curve) and 10^{-3} (dotted curve) with $f = 0.21$ and $w_{3/2}(t = 0) = 0.014a$. For the same f and initial island width, a larger $\psi_{2/4}(a)$ leads to the decay of the 3/2 island. For smaller $\psi_{2/4}(a)$, w still grows but more slowly than that with $\psi_{2/4}(a) = 0$. With a larger initial island width $w_{3/2}(t = 0) = 0.031a$, w is shown by the dashed curve in Fig. 2 for $\Psi = 10^{-3}$. The dot-dashed curve is for $\Psi = 0$. It is seen that, with the same f and $\psi_{2/4}(a)$, w grows for a larger initial island width and decays for a smaller one.

Test calculations have been carried out by taking the $m/n = 2/4$ component field to be zero in calculating the 3/2 component of pressure. In this case no stabilizing effect is observed. If all the other coupling terms between $m/n = 3/2$ and 2/4 perturbations except the fourth term in Eq. (7) are set to be zero in the numerical modeling, the time evolution of $w_{3/2}$ is essentially the same as that including all the coupling terms, indicating our analysis is correct.

The relation between the required $m/n = 2/4$ field magnitude to stabilize the $m/n = 3/2$ mode and the bootstrap current density fraction at r_{s1} is shown in Fig. 3 for $w_{3/2}(t = 0) = 0.014a$, where the solid (empty) squares denote the unstable (stable) case. It is seen that the required $\psi_{2/4}(a)$ for the stabilization increases almost linearly as the local bootstrap current density fraction increases in agreement with Eq. (12). Taking $r_{s1}\Delta' = -m_1 = -3$ [6], the numerical results agree well with Eq. (12). With a larger initial island width, similar results to Fig. 3 are found but a larger $\psi_{2/4}(a)$ is required for the stabilization for the same f.

Reversing the phase between the 3/2 mode and the 2/4 helical field leads to the same results. Introducing a poloidal plasma rotation into the calculation and therefore a relative rotation between the island and the helical field, the result is found to be essentially not changed, in agreement with Eqs. (10) and (12). When the additional components like $m/n = 6/4$, 5/10, 8/12, 7/18 resulting from the coupling between perturbations are included in the modeling, the results are approximately the same as that with only three Fourier components, $m/n = 0/0$, 3/2,

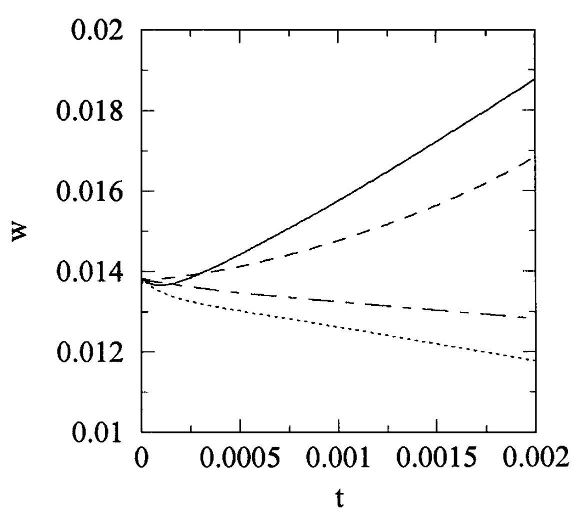

FIG. 1. w versus the normalized (to τ_R) time t for $\psi_{2/4}(a) = 0$ with $f = 0.021$ (solid curve) and $f = 0.016$ (dotted curve) and for $\Psi = 10^{-4}$ with $f = 0.048$ (dashed curve) and $f = 0.043$ (dot-dashed curve).

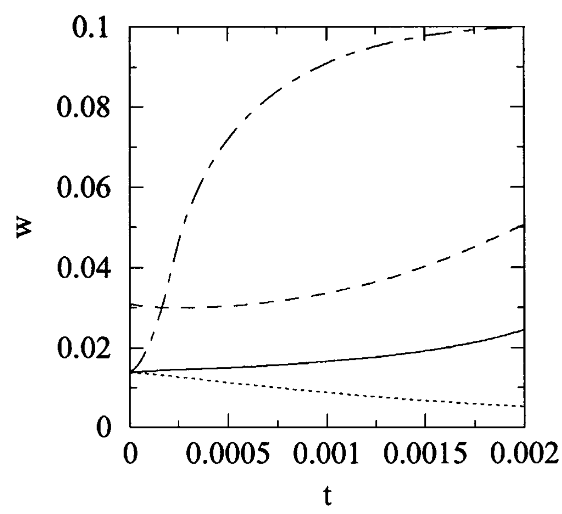

FIG. 2. w versus t for $\Psi = 0.5 \times 10^{-3}$ (solid curve) and 10^{-3} (dotted curve) with $w_{3/2}(t = 0) = 0.014a$. The dashed curve is the same as the dotted curve except $w_{3/2}(t = 0) = 0.031a$. The dot-dashed curve is for $\Psi = 0$.

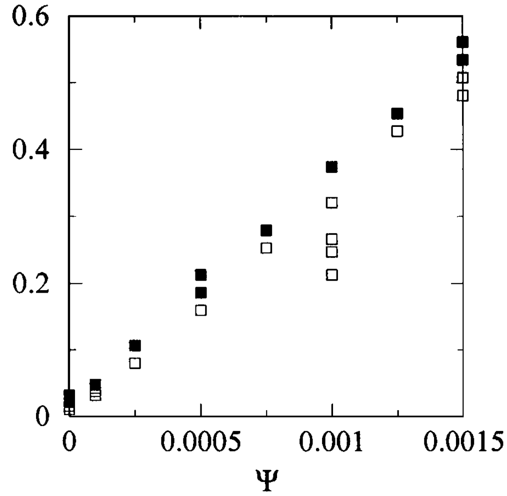

FIG. 3. Stabilization diagram in the Ψ-f plane, where the solid (empty) squares denote the unstable (stable) case with $w_{3/2}(t = 0) = 0.014a$.

and 2/4. With a larger $\chi_\perp$, the 3/2 mode is found to decay faster. The effect of the $m/n = 2/6$ or 2/8 helical field on the $m/n = 3/2$ mode is found to be the same as that of the $m/n = 2/4$ field. From the numerical modeling the $m/n = 2/1$ NTM is also found to be stabilized by a sufficiently large $m/n = 2/4$ or 2/3 helical field.

4. Discussion and summary.—The numerical and the analytical results shown above indicate that the NTM can be stabilized by an externally applied static helical magnetic field due to the larger critical island width w_{ch}. These results suggest that the NTMs are unlikely to cause an enhanced transport by creating many small islands at their corresponding rational surfaces because the less unstable mode will be stabilized by the neighboring more unstable one due to the same mechanism found here. We have indeed found from numerical simulations that a NTM is suppressed by another nearby more unstable NTM, in agreement with the experimental observations of the suppressing of one NTM by another of a different helicity [14].

Previously there had been studies on the anomalous transport caused by the ergodic magnetic field lines due to multiple helicity magnetic perturbations. In these studies the $m/n = 0/0$ pressure perturbation is of concern. In our work, however, the fundamental harmonic pressure perturbation of the NTM is important because it provides the main drive for the NTM growth through the corresponding perturbed bootstrap current. When the selected helical field has no resonant surface inside the plasma, $p_2 = ib_{2r}(\partial p_{0/0}/\partial r)/F_2$ is approximately valid everywhere inside the plasma. From the transport equation for the $m/n = 0/0$ pressure component similar to Eq. (7), it can be found that the effect of a helical field on the $m/n = 0/0$ component pressure is much smaller than that on the fundamental harmonic pressure perturbation near the rational surface of the NTM. In fact, a small nonresonant helical field can cause a significant change in p_1 profile only around the rational surface due to the large local radial derivative of p_1 there. Our numerical calculation results also indicate that there is no significant change in $p_{0/0}$ with a small magnitude of the helical field required for stabilizing the NTM.

It is seen from present results that even for a plasma with a high fraction of bootstrap current, the NTM can be stabilized by an externally applied static helical field with its magnitude much smaller than that of the equilibrium poloidal field. There are apparently great advantages in using this stabilizing method because essentially no power is required and the technology is simple. However, the helical field can stabilize only the NTMs driven by the perturbed bootstrap current (negative Δ') rather than the conventional tearing mode instability driven by an unfavorable current density gradient (positive Δ'). For fully controlling the tearing modes' instabilities and the disruptions, another method like localized ECCD is also necessary. The effects of diamagnetic drift and toroidal mode coupling are neglected in the present work. However, toroidal mode coupling is usually important between m/n and $(m - 1)/n$ or $(m + 1)/n$ modes, and one can choose the helical field mode numbers to be $m/n \ll 1$, so that $(m - 1)/n$ or $(m + 1)/n$ components are not resonant with the NTM. The inertia term $\rho(\mathbf{v} \cdot \nabla)\nabla^2\phi$ in Eq. (2) is found to be destabilizing but not sufficient to destabilize the NTM.

In summary, in the present paper the NTM is found to be stabilized by an externally applied static helical magnetic field of a different helicity if the magnitude of the helical field is sufficiently large, suggesting a very simple method for stabilizing the NTM. The stabilization is due to the increased critical island width by the helical field.

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